Muon $g - 2$ and Tests of Relativity

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After a brief introduction to the muon anomalous moment $a \equiv (g-2)/2$, the pioneering measurements at CERN are described. This includes the CERN cyclotron experiment, the first Muon Storage Ring, the invention of the "magic energy", the second Muon Storage Ring and stringent tests of special relativity.

1. Introduction

Creative imagination. That is what science is all about. Not the slow collection of data, followed by a generalisation, as the philosophers like to say. There is as much imagination in science as in art and literature. But it is grounded in reality; the well tested edifice of verified concepts, built up over centuries, brick by brick. All this is well illustrated by the muon $(g - 2)$ theory and measurements at CERN.

It also illustrates the reciprocal challenges. Theorists come up with a prediction, for example that light should be bent by gravity: how can you measure it? Eddington found a way. Conversely experiments show that the gyromagnetic ratio of the electron is not 2, but slightly larger: then the theorists are challenged to explain it, and they come up with quantum electrodynamics and a cloud of virtual photons milling around the particle. How can we check this? And so on. By reciprocal challenges the subject advances; step by step. And of course, some of the ideas turn out to be wrong; they are quietly dropped.

Over the years the muon $(g-2)$ has proved to be a marker, a lighthouse, a fixed reference that theories must accommodate; and many zany speculations have come to grief on this rock.

In this review I will not recap the detail which is given in the published papers and the many reviews.[1, 2] Instead I try to highlight the main creative steps, how they were reached, plus the many precautions needed to make the experiments work: and to give the correct answer.

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The muon $(g-2)$ at CERN has a unique record. The number published at an early stage always turned out to be correct; it was verified by the next experiment; the new number always fell inside the one sigma error bars of the previous. The final CERN measurement was confirmed by the later experiment at Brookhaven. At one stage our number disagreed with the theory, but we published anyway. The theorists then revised their calculations, and they agreed with us.

The gyromagnetic ratio g is the ratio of the magnetic moment of a system to the value obtained by multiplying its angular momentum by the Larmor ratio $(e/2mc)$. For an orbiting electron $g = 1$. When Goudschmit and Uhlenbeck[3] postulated the spinning electron with angular momentum $(h/4\pi)$ to explain the anomalous Zeeman effect, it was surprising that its magnetic moment, one Bohr magneton, was twice the expected value: the gyromagnetic ratio for the electron was apparently 2. Later, Dirac[4] found that this value came out as a natural consequence of his relativistic equation for the electron.

Another surprise was to come. Experimentally[5] the magnetic moment of the electron was in fact slightly larger, so $g = 2(1 + a)$ with a being defined as the "anomalous magnetic moment". In its turn the anomaly was understood[6] as arising from the quantum fluctuations of the electromagnetic field around the particle. The calculation of this quantity,[7] in parallel with measurements of increasing accuracy, has been the main stimulus to the development of quantum electrodynamics. For the electron, astonishingly, theory and experiment agree for this pure quantum effect to 0.02 ppm (parts per million) in a, the limit being set by our independent knowledge of the fine structure constant α.

For the muon, the $(g-2)$ value has played a central role in establishing that it behaves like a heavy electron and obeys the rules of quantum electrodynamics (QED). The experimental value of $(g-2)$ has been determined by three progressively more precise measurements at CERN and a recent experiment at Brookhaven,[8] now achieving a precision of 0.7 ppm (parts per million) in the anomaly $a \equiv (g-2)/2$. In parallel, the theoretical value for $(g-2)$ has improved steadily as higher order QED contributions have been evaluated, and as knowledge of the virtual hadronic contributions to $(g-2)$ has been refined.

At CERN, theorists and experimenters work in close proximity and interact. So CERN theorists have made important contributions to the calculation of the muon $(g-2)$, starting with Peterman[9] who corrected an error in the $(\alpha/\pi)^2$ term, and continuing with Kinoshita, Lautrup and de Rafael.[10] Kinoshita[7] in particular was alerted to the problem during a tour of the experiments arranged by John Bell in 1962 and spent the rest of his career calculating higher and higher orders for the electron and the muon.

The story starts in 1956 when the magnetic anomaly $a \equiv (g-2)/2$ of the electron was already well measured by Crane *et al.*[11] Berestetskii *et al.*[12] pointed out that the postulated Feynman cutoff in QED at 4-momentum transfer $q^2 = \Lambda^2$, would reduce the anomaly for a particle of mass m by

$$\delta a/a = (2m^2/3\Lambda^2). \tag{1}$$

Therefore, a measurement for the muon with its 206 times larger mass would be a far better test of the theory at short distances (large momentum transfers). (At present the comparison with theory for the electron is 35 times better than for the muon; but to be competitive it needs to be 40,000 times better! The muon is by far the better probe for new physics).

In 1956, parity was conserved and muons were unpolarised, so there was no possibility of doing the experiment proposed by Berestetskii. But in 1957 parity was violated in the weak interaction and it was immediately realised that muons coming from pion decay should be longitudinally polarised. Garwin, Lederman and Weinrich,[13] in a footnote to their classic first paper confirming this prediction, used the $(g-2)$ precession principle (see below) to establish that its gyromagnetic ratio g must be equal to 2.0 to an accuracy of 10%. This was the first observation of muon $(g-2)$ 57 years ago.

In 1958, the Rochester conference took place at CERN; Panofsky[14] reviewing electromagnetic effects said that three independent laboratories, two in the USA and one in Russia, were planning to measure $(g-2)$ for the muon. In the subsequent discussion, it was clear that leading theorists expected a major departure from the predicted QED value, either due to a natural cutoff (needed to avoid the well known infinities in the theory) or to a new interaction which would explain the mass of the muon. Feynman in 1959 told me that he expected QED to breakdown at about 1 GeV momentum transfer. At that time renormalisation was regarded as a quick fix to deal with infinite integrals, not a real theory.

2. Principle

The orbit frequency ω_c for a particle turning in a magnetic field B is

$$\omega_c = (e/mc)B/\gamma. \tag{2}$$

While for a particle at rest or moving slowly, the frequency at which the spin turns is

$$\omega_s = g\ (e/2mc)\ B. \tag{3}$$

At low energy ($\gamma \sim 1$) if $g = 2$ these two frequencies are equal, so polarised particles injected into a magnetic field would keep their polarisation unchanged. But if $g = 2(1+a)$, then the spin turns faster than the momentum and the angle between them increases at frequency ω_a given by

$$\omega_a = \omega_s - \omega_c = a(e/mc)B. \tag{4}$$

This is the $(g-2)$ principle discovered by Tolhoeck and DeGroot[15] and used so successfully by Crane for the electron.

Equation (4) for the $g - 2$ precession is true even at relativistic velocities.[15, 16] Significantly, there is no factor γ in this equation so at high energies the muon lifetime is dilated but the precession is not slowed down. With relativistic muons many $(g - 2)$ cycles can be recorded and the measurement becomes more accurate.

The magnetic field is measured by the proton NMR frequency ω_p and the experiment gives the ratio $R = \omega_a/\omega_p$. The ratio $\lambda = \omega_s/\omega_p$ in the same field is known from other experiments: careful studies of muon precession at rest and the hyperfine splitting in muonium.[17] Combining (3) and (4) a is calculated from

$$a = \frac{\omega_a}{\omega_s - \omega_a} = \frac{R}{\lambda - R}. \tag{5}$$

The $(g - 2)$ experiments are essentially measurements of the frequency ratio $R = \omega_a/\omega_p$. If the value of λ changes a should be recalculated.

3. 6 m Magnet with the CERN Cyclotron 1958–1962

In 1957 parity violation was discovered, muon beams were found to be highly polarised and, better still, the angular distribution of the decay electrons could indicate the muon spin direction as a function of time. The possibility of a $(g - 2)$ experiment for muons was envisaged, and groups at Berkeley, Chicago, Columbia, and Dubna started to study the problem.[14] Compared with the electron, the muon $(g - 2)$ experiment was much more difficult because of the low intensity, diffuse nature, and high momentum of available muon sources. The lower value of (e/mc) made all precession frequencies 200 times smaller, but the time available for an experiment was limited by the decay lifetime, 2.2 μs. Therefore, large volumes of high magnetic fields would be needed to give a reasonable number of precession cycles.

The main problem was how to inject muons into a magnetic field so that they made many turns. For the electron, Crane used a thermionic source already inside the solenoidal field and the spin was measured by scattering on a foil also inside the field. At CERN the muons were born inside the cyclotron (paradoxically already in a strong magnetic field); they came out and we needed to get them back in. To inject into a static field requires some kind of perturbation, usually a pulsed magnet which kicks the particle into a new direction (as used in most accelerators); otherwise the particle will exit the field after less than one turn.

Another option is a degrader in which the particle loses energy and so turns more sharply in the field. In a uniform field, the particle will then make one turn and return to the degrader. To inject successfully requires a horizontal field gradient, so that the orbit turns more sharply on one side than the other. The orbit then "walks" at right angles to the gradient and misses the degrader after one turn. We used a beryllium block about 10 cm thick to minimise multiple scattering and the edge was curved to fit the expected orbit.

In 1958 CERN acquired its first digital computer, the Ferranti Mercury with a programming language rather similar to Fortran, called Mercury Autocode. This was soon put to use[18] for tracking pions and muons coming out of the CERN cyclotron with a view to installing optimised beam pipes through the shielding. The program followed the tracks step by step in the horizontal plane and also included vertical focusing effects due to field gradients. It was put to work to follow muons turning in the horizontal plane of a long bending magnet with specially designed transverse gradients. Using a degrader, it was fairly easy to get the muons into the field. But could they be ejected? This was the key question, answered eventually by the computer.

To measure the spin angle one has to stop the muons in some block, wait for them to decay and record the distribution of the emitted electrons. But if the block is inside the magnet, the muons at rest will continue to rotate so the new spin direction will be scrambled. One must get the muons into the field, let them make many turns, and then get them out before stopping them in a field free region.

The problem is complicated by a fundamental theorem for particles turning in a magnetic field. In slowly varying fields, the flux through the orbit is an invariant of the motion. So the experts argued that once the muons were trapped in the field, it would be impossible to get them out. The experiment would fail.

At the end of the magnet the field decreases: inevitably there is a longitudinal gradient. When the particle reaches this point it feels the longitudinal gradient and moves sideways, to the side of the magnet where it either hits something or walks back along the fringing field to the beginning. It is not ejected.

What about using a very large transverse gradient? Then with a large step size, the particle will arrive suddenly at the end of the magnet and come out without moving sideways . . . as it does in a normal beam line. Ah yes, said the experts, but in a large gradient there will be strong alternating gradient focusing, the beam will blow up vertically and the particles will be lost.

The computer could address this question. It followed the muons for many turns, from injection in a medium gradient, through a transition to a very weak gradient where they made many turns, then gradually into a strong gradient with a very large step size, all the way to the end of the magnet where, it turned out, they were ejected successfully without any excessive vertical focusing!

Successful storage requires vertical focusing. Otherwise the particles will spiral up or down into the poles. A muon turning in a linear gradient is focused on one side of the orbit and defocused on the other. This gives a net focusing effect, but far too small to be useful. One needs to add a parabolic term so that the field decreases outwards on both sides of the orbit. Storage, Fig. 2, for up to 18 turns was demonstrated in a small magnet, Fig. 1, borrowed from the University of Liverpool. This result and the ejection calculation gave the lab confidence to order a special magnet 6 m long which could store the muons for many turns.

Fig. 1. First experimental magnet in which muons were stored at CERN for up to 30 turns. Left-to-right: Georges Charpak, Francis Farley, Bruno Nicolai, Hans Sens, Antonino Zichichi, Carl York, Richard Garwin.

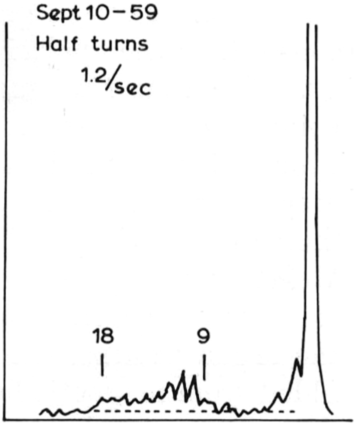

Fig. 2. First evidence of muons making several turns in the experimental magnet, shown in Fig. 1. The time of arrival of the particles at a scintillator fixed inside the magnet is plotted horizontally (time increases to the left). The first peak (right side) coincides with the moment of injection. The equally-spaced later peaks correspond to successive turns. Owing to the spread in orbit diameters and injection angles, some muons hit the counter after nine turns, while others take 18 turns to reach the same point (Charpak *et al.*, unpublished).

The 6 m magnet had removal poles 5 cm thick, which could be rolled out and shimmed. Hundreds of thin layers of iron were held in place by aluminium covers and specially shaped by trial and error to give the required field shape, a titanic task executed by Zichichi and Nicolai. At injection the step size was 1.2 cm to give reasonable clearance from the degrader. Moving along the magnet, the gradient was gradually reduced so that the muons advanced only 4 mm per turn and spent longer in the field. At the far end a very large gradient increased the step to 11 cm per turn.

The theorem mentioned above, that the flux through the orbit is an invariant of the motion, was used to good effect. If the average field varies along the magnet, the orbit will move sideways in the gradient, to keep the flux constant, so the particles can be lost. This was checked with a flux coil 40 cm diameter (the size of the orbit) which could be moved along the magnet. The coil was connected to a fluxmeter and any deviation from constancy was corrected with a special set of "longitudinal" shims. This was particularly important in the transition regions where the gradient was changing. Moving the flux coil sideways measured the lateral gradient. The theorem also implied that we could calibrate the field with NMR at the centre of the magnet; and the result would be valid everywhere.

An overall view of the final storage system[19] is shown in Fig. 3. The magnet pole was 6 m long and 52 cm wide, with a gap of 14 cm. Muons entered on the left through a magnetically shielded iron channel and hit a beryllium degrader in the injection part of the field. Here the step size s was 1.2 cm. Then there was a transition to the long storage region, where $s = 0.4$ cm. Finally, a smooth transition

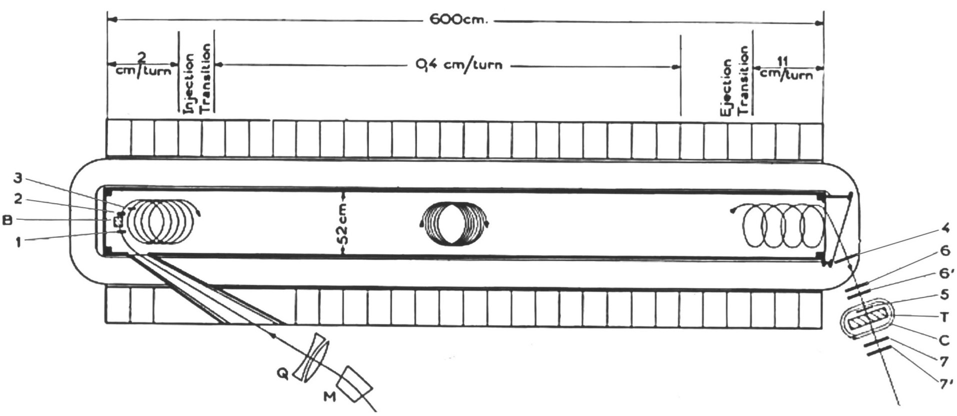

Fig. 3. The 6 m bending magnet used for storing of muons for up to 2000 turns. A transverse field gradient makes the orbit walk to the right. At the end a very large gradient is used to eject the muons which stop in the polarisation analyser. Coincidences 123 and $466'5\bar{7}$, signal an injected and ejected muon respectively. The coordinates used in the text are x (the long axis of the magnet), y (the transverse axis in the plane of the paper), and z (the axis perpendicular to the paper).

was made to the ejection gradient, with $s = 11$ cm per turn. The ejected muons fell onto the polarisation analyser Fig. 4, where they decayed to e^+.

The muons were trapped in the magnet for 2–8 μs depending on the location of the orbit centre on the varying parabolic gradient. About one muon per second was stopped finally in the polarisation analyser, and the decay electron counting rate was 0.25 per second.

To obtain the anomalous moment a from Eq. (4) one must measure the time a muon has spent in the field and the spin angle before and after storage. Time was measured with a 10 MHz clock, started when a muon came out of the magnet and stopped by a delayed signal from a muon at the entrance. An elaborate veto system rejected events with two signals close to each other at either end, so there was no chance of confusion leading to incorrect times.

The spin angle was measured by the polarisation analyser, Fig. 4. The same counters were used to signal a muon stopping in the central absorber E and to record the subsequent decay electron emitted either backwards or forwards. The ratio of backward (B) to forward (F) counts measures the asymmetry, but this is not sensitive to the transverse angle. Therefore the muon spin was flipped through

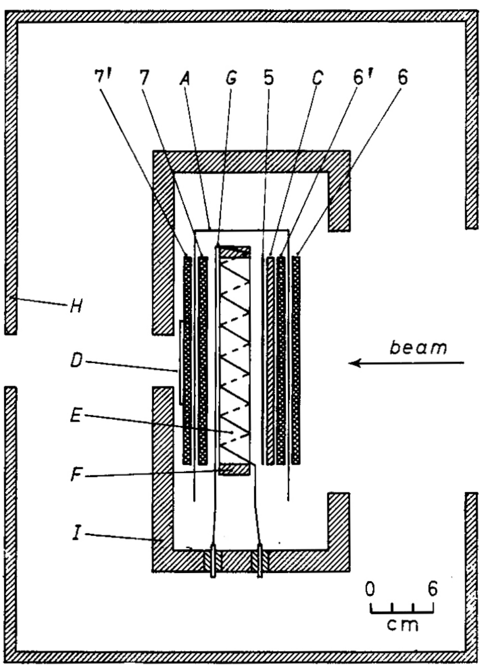

Fig. 4. Polarisation analyser. When a muon stops in the liquid methylene iodide E a pulse of current in coil G is used to flip the spin through $\pm 90°$. Backward or forward decay electrons are detected in counter telescopes 66′ and 77′. The static magnetic field is kept small by the double iron shield H, I and the mu-metal shield A. The muon must pass the thin scintillator 5, backed by plexiglass C. D is a mirror used for alignment.

±90 degrees by a short pulse of vertical field applied to the absorber every time a muon stopped. The ratio

$$A = \frac{F_+ - F_-}{F_+ + F_-} \tag{6}$$

for forward counts with +90 and −90 flipping was then a measure of the transverse spin component. Similar data was obtained from the backward telescope. The flipping angle should be consistent, but its exact value is not important.

For this to work, the absorber in which the muons stopped had to be non-conducting (no metals) and not depolarising, which ruled out most plastics. Luckily liquid methylene iodide had the right properties. A double iron shield plus an inner mumetal shield was used to reduce the magnetic field in the absorber.

The direction of the arriving muons was measured with a venetian blind made of parallel slats of scintillator used to veto the event. The only particles recorded were those that got through the spaces between the slats without touching any of them.

When the polarisation analyser was used to study the muons coming out of the cyclotron the transverse angle was found to vary rapidly with muon momentum (range). This could create an error because the band of momentum selected by the storage magnet could be very different. The effect was eliminated by passing the muon beam through a long solenoid with field parallel to the beam. This rotated all transverse spin components through 90°, horizontal into vertical and vice versa. Because of vertical symmetry inside the cyclotron the result was no spin-momentum correlation in the horizontal plane.

For muons that had been through the magnet, the analyser recorded the asymmetry A as a function of the time t the particle had spent in the field. This showed a sinusoidal variation due to the $(g - 2)$ precession in the magnet.

$$A = A_0 \sin\theta_s = A_0 \sin\{a(e/mc)Bt + \phi\} \tag{7}$$

where ϕ is an initial phase determined by measuring the initial polarisation direction and the orientation of the analyser relative to the muon beam.

The experimental data are given in Fig. 5, together with the fitted line obtained by varying A_0 and a in Eq. (7). Full discussion of the precautions needed to determine the mean field B seen by the muons, and to avoid systematic errors in the initial phase ϕ, are given in Ref. 19. The first experiment gave ±2% accuracy in a and this was later improved to ±0.4%. The figures agreed with theory within experimental errors. The corresponding 95% confidence limit for the photon propagator cut-off, Eq. (1), was $\Lambda > 1.0\,\text{GeV}$.

This was the first real evidence that the muon behaved so precisely like a heavy electron. The result was a surprise to many, because it was confidently expected that g would be perturbed by an extra interaction associated with the muon to account for its larger mass. When nothing was observed at the 0.4% level, the muon became accepted as a structureless point-like QED particle, and

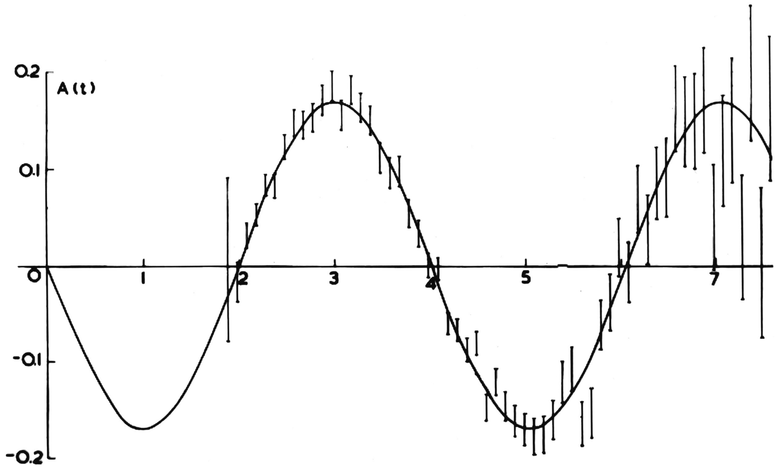

Fig. 5. Asymmetry A of observed decay electron counts as a function of the storage time t. The time t spent in the magnet depended on the transverse position of the orbit on the parabolic magnetic field. The muons that were stored for $7.5\,\mu$s made 1600 turns in the magnet and then emerged spontaneously at the far end. The sinusoidal variation results from the $(g-2)$ precession; the frequency is measured to $\pm 0.4\%$.

the possibility of finding a clue to the $\mu - e$ mass difference now appeared more remote.

In retrospect this experiment was quite remarkable. We poured muons into the magnet at one end, they were trapped inside for almost 2000 turns (2.5 km) and then came out at the other end, all of their own accord: no pulsed fields, no kickers. Nothing like this has ever been done, before nor since.

4. First Muon Storage Ring 1962–1968

4.1. *Overview*

The muon $(g-2)$ experiment was now the best test of QED at short distances. To go further and to search again for a new interaction, it was desirable to press the experiment to new levels. Relativistic particles with dilated lifetimes were available from the CERN PS and there is no factor γ in Eq. (4) so in principle high energy muons would give more precession cycles and greater accuracy. Storing muons of GeV energy in a magnetic field and measuring their polarisation required totally new techniques. Farley[20] proposed to measure the anomalous moment using a muon storage ring. Simon van der Meer designed the magnet and participated in the whole experiment.

Time dilation in a straight path was well established. But no one had proved it for a two way journey, out and return or a circular orbit. The twin paradox (clock paradox) was still a puzzle and some people did not believe it. Notably

Herbert Dingle,[21] who had written a short but excellent textbook on relativity, lost faith and carried on a campaign against it. The predictions of special relativity were clear: the twin who suffers acceleration ends up younger. But perhaps this was not the whole story, acceleration was said to be equivalent to gravity and the gravitational redshift could change time. So perhaps there was no time dilation in a circular orbit and the experiment would fail. It was a leap in the dark. Luckily there were no Dingles on the committee.

The experiment is made possible by four miracles of Nature. (First, identify your miracle, then put it to work for what you wish to do.) The first miracle is that it is easy to inject muons into a storage ring. One simply injects pions for a few turns; they decay in flight and some of the muons will fall onto permanently stored orbits. The easy way to inject pions is to put the primary target of the accelerator inside the storage magnet and hit it with high energy protons, thus producing the pions inside the ring. The second miracle is that stored muons come from forward decay, so they are strongly polarised. The third miracle is that when the muons decay the electrons have less energy; bent by the field they come out on the inside of the ring and hit the detectors. The higher energy electrons must come from forward decay: so as the spin rotates, the electron counting rate is modulated by the $(g-2)$ frequency ($\sim 270\,\mathrm{kHz}$). One simply reads it off.

An advantage of this method is that it works equally well for μ^+ and μ^-. Most muon precession experiments can only be done with μ^+, because stopped μ^- are captured by nuclei and largely depolarised. $g-2$ can be measured for μ^- as well as μ^+.

It was later realised that the injected muons would be localised in azimuth (injection time 10 ns, rotation time 52 ns), so the counting rate would also be modulated at the much faster rotation frequency ($\sim$20 MHz). This would enable the mean radius of the stored muons to be calculated, leading to a precise knowledge of the corresponding magnetic field.

With the primary target of the accelerator inside the storage ring there would be a huge background in the counters. Would this swamp the observations? A test inside the PS tunnel revealed radiation lasting for many milliseconds and decaying roughly as $1/t$. This could only come from neutrons banging around inside the building from wall to wall. A theory of neutron slowing down[22] gave a reasonable fit to the data. The typical neutron velocity after a time t is obtained by dividing the width of the room by t. This paper is widely used where short lived radioactive isotopes are studied, e.g. at ISOLDE.

We later discovered that the main background in the counters came from neutrons trapped inside the plexy light pipes, creating Cherenkov light after an (n,γ) process. Adopting air filled light pipes with white walls reduced the effect.

The first Muon Storage Ring[23] was a weak-focusing ring (Fig. 6) with $n = 0.13$, orbit diameter 5 m, and a useful aperture of $4\,\mathrm{cm} \times 8\,\mathrm{cm}$ (height $\times$ width); the

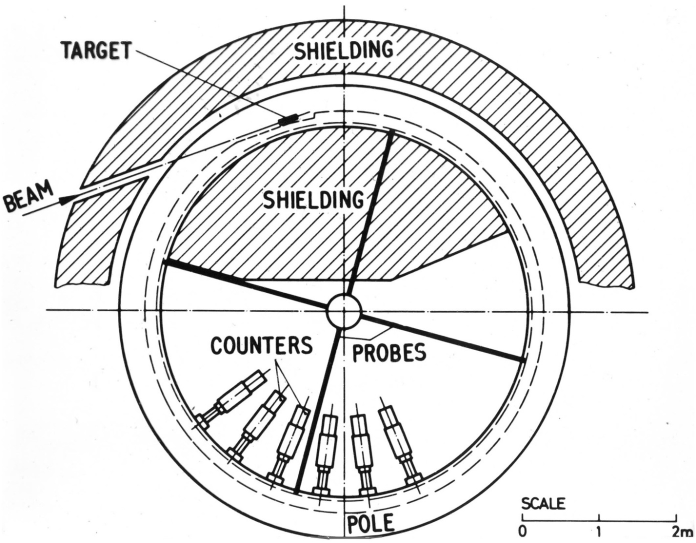

Fig. 6. First Muon Storage Ring: diameter 5 m, muon momentum 1.3 GeV/c, time dilation factor 12. The injected pulse of 10.5 GeV protons produces pions at the target, which decay in flight to give muons.

muon momentum was 1.28 GeV/c corresponding to $\gamma = 12$ and a dilated lifetime of 27 μs. The mean field at the central orbit was $\bar{B} = 1.711$ T. The injection of polarised muons was accomplished by the forward decay of pions produced when a target inside the magnet was struck by 10.5 GeV protons from the PS. The proton beam consisted of one to three radio-frequency bunches (fast ejection), each ~10 ns wide, spaced 105 ns. As the rotation time in the ring was chosen to be 52.5 ns, these bunches overlapped exactly inside the ring. Approximately 70% of the protons interacted, creating, among other things, pions of 1.3 GeV/c that started to turn around the ring. The pions made about four turns before again hitting the target, and in each turn about 20% decayed.

Typically the pions go round the magnet with momentum 1−2% above the nominal central momentum. Muons with the top energy follow the same orbit as the pions and will eventually hit something and be lost. But muons with 1−3% lower momentum fall onto permanently stored trajectories. Because they come from almost forward decay the polarisation is of order 97%.

This was the theory. But in practice the muon polarisation was found to be much lower, around 30%. A high energy pion only has a short track inside the storage region but it can decay at a large angle and inject a stored muon with small polarisation. It is a rare process, but there were very many higher energy pions and a majority of the stored muons were born in this way... low average polarisation.

4.2. *Muon decay in flight*

There was no need to get the muons out of the field to study their spin. Just observe their decay in flight. The highest energy electrons in the lab have the same momentum as the muons, and are trapped in the field. But those with lower energy are bent more and exit the ring on the inside. Here they hit one of the lead-scintillator detectors in which they produce a shower and the light output is proportional to the electron energy. By selecting pulse height in the detector, one selects a band of decay electron energies. By recording the high energy particles, one selects forward decays: as the spin rotates the number is modulated by the $(g-2)$ frequency.

When the muon decays the electron energy is boosted by the Lorentz transformation. The broad rest-frame spectrum becomes a falling triangle with a large number at low momentum dropping to zero at the end point which is equal to the stored muon momentum, Fig. 7. To have this maximum momentum in the lab, the electron must be emitted exactly forward and have the top energy in the muon frame; so the asymmetry for these particles in the lab is $A = 1$. These particles carry the maximum information about the muon spin, but there are none of them. At lower energy a mixture of rest frame electron energies and decay angles can contribute, the number rises and the asymmetry falls, Fig. 7. To have high energy in the lab, the electron must be emitted forwards in the muon frame.

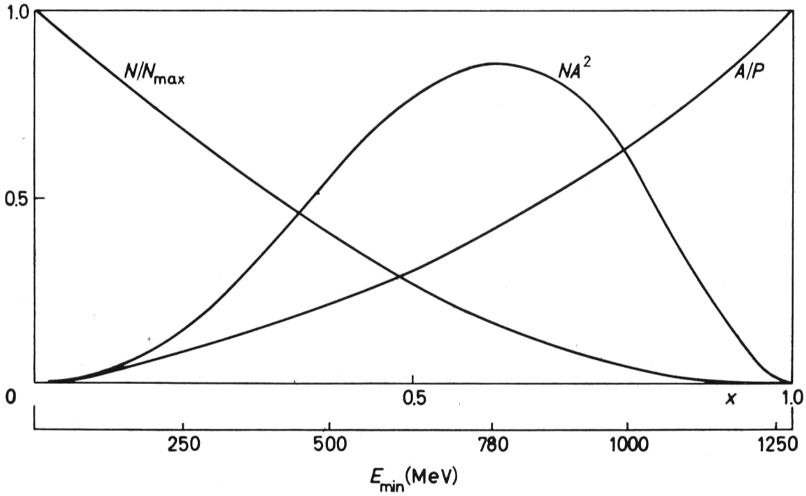

Fig. 7. Integral energy spectrum N of decay electrons hitting a detector: asymmetry coefficient A and NA^2 versus electron energy threshold $E_{\rm min}$. The maximum of NA^2 occurs when $E_{\rm min}$ is about 0.65 times the stored muon energy.

4.3. *Experimental details and results*

The dilated muon lifetime was now 27 μs so the muon precession could be followed out to storage time $t = 130\,\mu$s as shown in Fig. 8. Data for t less than 20 μs could not

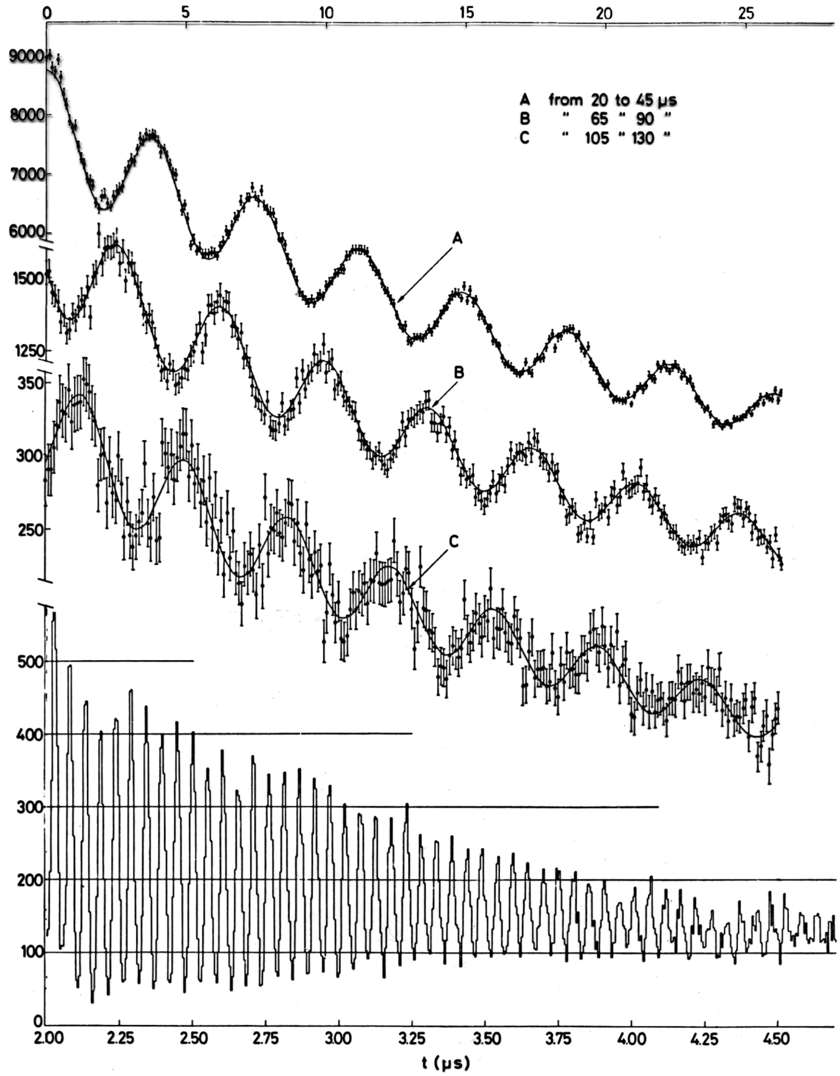

Fig. 8. First Muon Storage Ring: decay electron counts as a function of time after the injected pulse. The lower curve 1.5–4.5 μs (lower time scale) shows the 19 MHz modulation due to the rotation of the bunch of muons around the ring. As it spreads out the modulation dies away. This is used to determine the radial distribution of muon orbits. Curves A, B, and C are defined by the legend (upper time scale); they show various sections of the experimental decay (lifetime 27 μs) modulated by the $(g-2)$ precession. The frequency is determined to 215 ppm, $\bar{B}$ to 160 ppm leading to 270 ppm in a.

be used because of background due to neutrons and other effects created when the protons hit the target in the ring. The initial polarisation angle of the muons is not needed for the measurement: one just fits the oscillations that are seen. With thirty $(g-2)$ cycles to fit the accuracy in ω_a was now much better. Fitting a frequency ω to exponentially decaying oscillations the error is

$$\delta\omega/\omega = \frac{\sqrt{2}}{\omega\tau A\sqrt{N}} \tag{8}$$

where N is the total counts, τ the dilated lifetime and A the amplitude of the oscillations (asymmetry). To get good accuracy one should increase the number of cycles per lifetime by using high magnetic field and high energy, and maximise the product NA^2. The best value of NA^2 was obtained by accepting decay electrons above 780 MeV.

The magnetic field was measured between runs with the vacuum chamber removed at 288 positions in the azimuth and ten radii. During the runs it was monitored by four plunging NMR probes which could be driven into the centre of the ring. The radial magnetic gradient needed for vertical focusing implied a field variation of $\pm 0.2\%$ over the horizontal aperture of the storage ring (8 cm), so a major problem was to know the mean radius of the ensemble of muons that contributed to the data.

The muons are bunched at injection so there is a strong modulation of the counts at the rotation frequency, as seen in the lower curve of Fig. 8. Because of their various radii and rotation periods, they gradually spread around the ring, and the modulation dies away. The envelope of the modulation is the Fourier transform of the frequency spectrum, or equivalently of the radial distribution. By making the inverse transform one recovers the radial distribution of the muon equilibrium orbits. Using this and the map of the magnetic field, the mean field for the muon population is readily calculated. A conservatively assigned error of ± 3 mm in radius implied an error of 160 ppm in the field.

This method of finding the muon radius has an elegant advantage: it uses the same electron data that are used for fitting the $(g-2)$ frequency. Muons at larger radii have less chance of sending an electron into the counters than muons on the inside of the ring; so there can be a bias. Here the same detectors are used for both measurements, so there is no bias. Further details, together with checks to ensure that the measurement at early times was representative of the muon population at later times when the $(g-2)$ precession was measured, are given in Ref. 23 and the review article Ref. 2.

To calculate a from ω_a using (5) one needs the value of λ. At that time the best measurement was the measurement by Hutchinson of μ^+ precession in water. The result[23] was

$$a = (116\ 616\ \pm 31) \times\ 10^{-8}\ (270\,\text{ppm}). \tag{9}$$

Initially, this was 1.7 standard deviations higher than the theoretical value, suggesting that there was more to be discovered about the muon. In fact the discrepancy came from a defect in the theory. Theorists had originally speculated that the contribution of the six $(\alpha/\pi)^3$ diagrams involving photon–photon scattering in the QED expansion[7] for a would be small, and perhaps these terms would cancel exactly; but they had never been computed. The experimental result stimulated Aldins, Kinoshita, Brodsky and Dufner[24] to make the calculation and they obtained the surprisingly large coefficient of 18.4! The theory then agreed with the measurement, to the great satisfaction of the experimental team,

$$a_{\text{exp}} - a_{\text{th}} = 240 \ \pm 270 \text{ ppm}. \tag{10}$$

The limit for the Feynman cutoff (1) was now $\Lambda > 5\,\text{GeV}$.

Time dilation in a circular orbit was spectacularly confirmed. After this there was no serious doubt about the twin (clock) paradox: it was an uncomfortable fact. The measured lifetime was just $1.2 \pm 0.2\%$ shorter than the expected value of 26.69 μs, probably due to imperfections in the magnetic field and a slow loss of muons. A more precise verification of the Einstein time dilation was obtained with the second muon storage ring.

5. Second Muon Storage Ring 1969–1976

The success of the muon storage ring and the apparent difference from theory justified a larger ring to achieve better accuracy. This project was master-minded by Emilio Picasso, aided by John Bailey. Higher energy would increase the muon lifetime and a larger aperture would improve the statistics. But there was a fundamental limitation: the magnetic gradient needed for vertical focusing was 50 ppm per millimetre and it would be impossible to locate the muons more precisely.

5.1. *Electric focusing*

After much discussion between Bailey, Farley and Picasso[25] it was decided to use a uniform magnetic field with no gradient and focus the particles vertically with an electric quadrupole field spread all around the ring. The vertical field focuses the particles while the horizontal component defocuses, slightly offsetting the semicircular focusing effect of the magnet. Overall it has the same effect as a magnetic gradient. A voltage of 10–20 kV would be required.

The horizontal electric field would bend the orbit; but in the muon rest frame it would transform to a vertical magnetic field, which would turn the spin. How would this affect the $(g - 2)$ precession? Stray electric fields had been a major worry for the electron $(g-2)$ measurement. The change in $(g-2)$ frequency[16] for an electric field E is

$$\Delta f/f = (\beta - 1/a\beta\gamma^2)(E/B). \tag{11}$$

One observes that at a particular energy given by $\beta^2\gamma^2 = 1/a$, or equivalently when $\gamma = \sqrt{1+1/a}$, the electric field has no effect. This is the so-called "magic" energy[25] which is 3.1 GeV for muons. Here electric quadrupoles do not change the spin motion: one can use them with impunity.

The fourth miracle of Nature, mentioned above, is that the magic energy was conveniently accessible with the CERN PS and a reasonable step up from the previous storage ring. The muon lifetime was increased to 64 μs.

What about the spread in momentum? At the centre of the aperture the muons would have the magic energy exactly, but in any case the electric field there would be zero. At smaller radii the field would be inwards and the energy less than magic, the $(g-2)$ frequency would be reduced. At larger radii both effects would be opposite, so the frequency again reduced. The frequency change would be parabolic with a maximum at the centre of the aperture: the average correction was only 1.7 ppm. The pitch correction for muons oscillating vertically was re-evaluated by Farley, Field and Fiorentini[26] and extended to focusing by electric fields.

5.2. *Electric quadrupoles and scraping*

It turned out that operating the electric quadrupoles in the strong magnetic field was not easy. The configuration is similar to a Penning gauge for measuring small pressures. Electrons are trapped and oscillate up and down, gradually increasing the ionisation of the residual gas. This happened in the ring and led to sparks, flashover, electric breakdown. The effect was worse when μ^- were studied. But Frank Krienen discovered that several milliseconds were required for the ionisation to build up, and we only needed the voltage for less than a millisecond while the muons were stored. By turning off the quadrupoles between fills the problem was solved.

Muon losses during the storage time can change the mean spin angle, if those that are lost started with a different spin angle from those that remain. This was not a serious error for the $(g-2)$ measurement, but for the measurement of the lifetime it was essential to reduce the late-time muon losses to a minimum. This was done by shifting the muon orbits at early times both vertically and horizontally in order to "scrape off" the muons which passed near the edge of the aperture and were most likely to be lost.

The orbits were shifted by applying asymmetric voltages to opposite quadrupole plates at injection time, and then gradually bringing them back to normal. The result was that the aperture of the ring was reduced both vertically and horizontally during scraping, then gradually restored to normal with a time constant of about 60 turns, slow enough not to excite extra oscillations. The net result was to leave a clear space of a few millimetres around the stored muons. Any slow growth of oscillation amplitudes, would not cause muons to be lost.

A lost muon would hit something, lose energy and come out on the inside of the ring. A muon telescope sampled the lost muons. It was calibrated with no scraping when the losses were large enough to change the lifetime and then used to measure the losses when they were small.

5.3. *Ring magnet*

The major component of the new experiment was the 14 m diameter ring magnet. We needed to know the field on the muon orbit to a few ppm; but there was no way to measure it with NMR while the muons were there. One needed to stop the run, turn off the magnet, extract the vacuum chamber, then turn the magnet back on and survey the field. This process would have to be repeated many times. Guido Petrucci brilliantly designed a ring magnet that could be turned on and off and always came back to the same field.[27] This could only be achieved with some very special precautions, including:

- Temperature controlled room
- Independent temperature controlled concrete base with internal water pipes
- Coils not touching the iron, independently supported from the floor and able to deflect elastically to accommodate thermal expansion
- 40 separate iron yokes close to each other but not mechanically connected supporting quasi continuous poles
- 40 individual NMR probes with feedback loops to 40 compensating coils.

Usually the coils of a large magnet are strapped to the iron. The strong magnetic forces and thermal expansion makes the coils move, sliding and slipping whenever the magnet is turned on. Magnets always squeak and creak. The movement implies change: the field never repeats exactly. Petrucci's design avoided this. His ring made no noise. After a warmup period of two days, during which the field changed by about 5 ppm, the field averaged over the muon orbit reached a steady value, always the same to ± 1 ppm.

The 40 pole pieces were touching but because of the gaps in the yokes the field was 400 ppm less at the junctions. This did not significantly perturb the orbits nor the measurement of the average field seen by the muons. With the field stabilised at 40 points no azimuthal harmonics could develop. Overall, this magnet was mechanically far more stable than the BNL ring built later with superconducting coils.

5.4. *Pion injection*

Instead of injecting protons which gave a large background, a beam of momentum selected pions was brought into the ring just outside the muon storage region. This required a pulsed inflector to kick them onto a tangential orbit. As the inflector was a closed concentric line, the leakage of the pulsed field into the muon storage region was very small. It was measured with pick up coils to compute a small correction.

The pions had slightly higher momentum and after half a turn they passed through the centre of the aperture. $\pi-\mu$ decay in this region launched the stored muons. They came from forward decay so the polarisation was high. With the pions matched to the ring acceptance, this gave many more muons, and the background in the counters was far less. Detectors for the decay electrons could be positioned all the way around the ring.

With zero magnetic gradient, the average value of magnetic field did not depend on the assumed radial distributions of muons. Even in extreme cases the average magnetic field was the same within less than 2 ppm, compared with the 160 ppm uncertainty in the previous experiment and the new statistical accuracy of ~7 ppm. The $(g-2)$ frequency was essentially independent of the distribution of muons within the storage region. However, an accurate value for the mean radius (and momentum) was needed for checking the Einstein time dilation (see below).

5.5. *Radial distribution*

As before, the radial distribution of the muons was obtained by analysing the pattern of counts at early times when the data is modulated by the rotating bunch. Now in Fig. 9 the rotation signal and the $(g-2)$ modulation can be seen together!

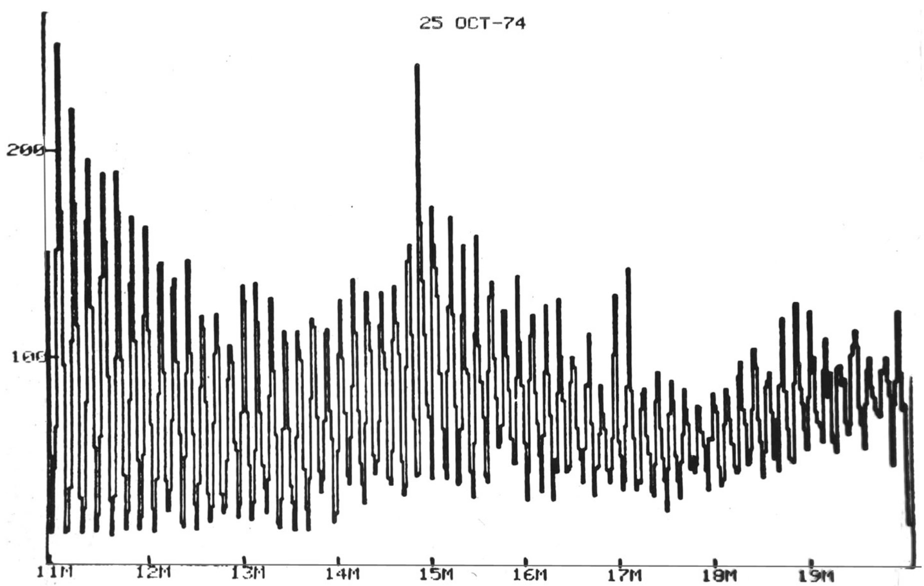

Fig. 9. Counting rate vs. time (11 to 20 μs) showing both the rotation frequency and the $(g-2)$ modulation, (online computer output for one run). The rotation signal dies away as the bunch spreads around the ring. The Fourier transform of the rotation data gives the radial distribution of the muons.

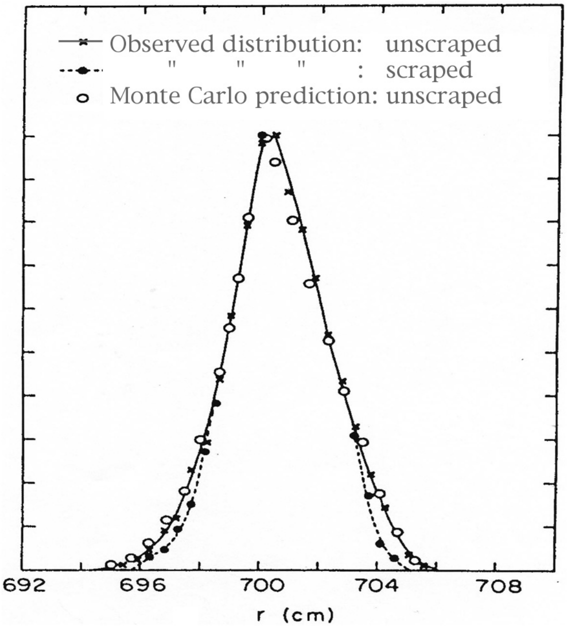

Fig. 10. Fourier transform of rotation data scraped (black dots) and unscraped (crosses), compared to prediction (open circles).

The computed radial distributions are in Fig. 10. The unscraped data agrees well with the prediction and the narrowing of the distribution by scraping is clearly seen. The mean rotation frequency ω_{rot} gives the relativistic γ factor:

$$\gamma = 2\lambda\omega_{\text{rot}}/g\omega_p \tag{12}$$

in which ω_p is the proton frequency corresponding to the magnetic field, $\lambda = \omega_s/\omega_p$ is known from mu precession at rest and muonium[17] and g is of course known from this experiment to better than 1 in 10^8. Equation (12) is used in checking the time dilation (see below).

The radial distribution is used to calculate the electric field correction (1.7 ppm) and pitch correction.[26] For $n = 0.135$, $v = 4\,\text{cm}$, $r = 700\,\text{cm}$, the pitch correction was 0.5 ppm. The statistical error in the mean radius was typically 0.1–0.2 mm.

5.6. *Results*

Figure 11 gives the combined decay electron counts versus storage time for the whole experiment, now showing the $(g-2)$ precession out to $534\,\mu\text{s}$ with a strictly exponential decay. As the muon lifetime at rest is $2.2\,\mu\text{s}$ that was quite remarkable. A maximum likelihood fit was made to the data to obtain the $(g-2)$ frequency ω_a.

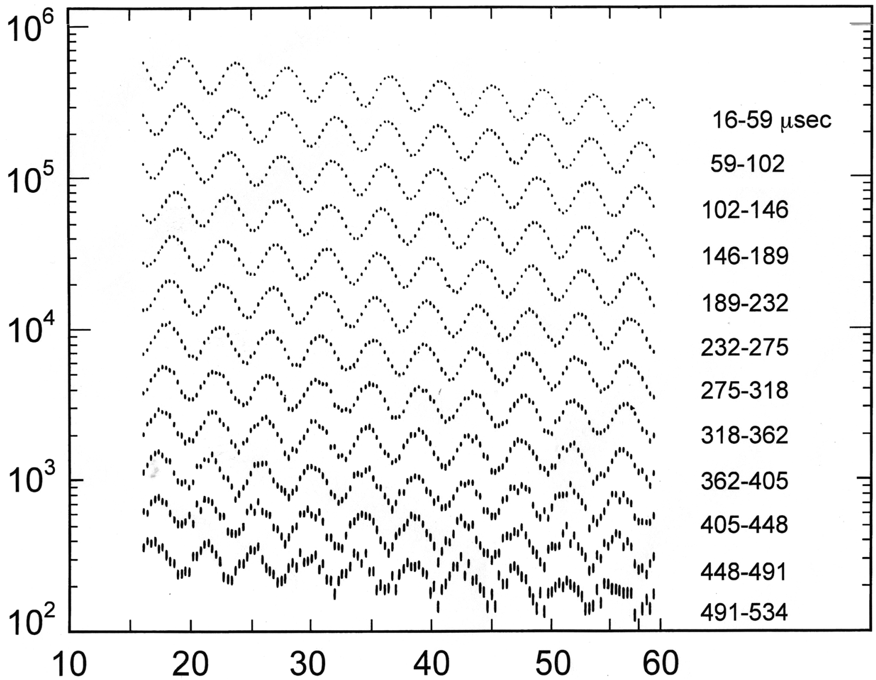

Fig. 11. Second Muon Storage Ring: decay electron counts versus time after injection. Range of time for each line is shown on the right (in microseconds).

Nine separate runs were made over a period of two years and fitted separately. As the field was determined in terms of the proton resonance frequency ω_p, the measurement of the $(g - 2)$ precession frequency ω_a is expressed as the ratio $R = \omega_a/\omega_p$. The nine R values, six for μ^+ and three for μ^- were consistent ($\chi^2 = 7.3$ for eight degrees of freedom). The overall mean value was the essential result of the experiment:

$$R = \omega_a/\omega_p = 3.707\,213\ (27) \times 10^{-3}(7\,\text{ppm}). \tag{13}$$

The error was 7.0 ppm statistical from ω_a plus 1.5 ppm from ω_p.

The corresponding value of the anomaly is given by Eq. (5) using the current result for λ.[17] The result is slightly different from that published in Ref. 28 because the value of λ has changed. Combining the data for μ^+ and μ^-,

$$a = 1\ 165\ 923\ (8.5) \times 10^{-9}\ (7\,\text{ppm}) \tag{14}$$

in agreement with the theory. The 95% confidence limit for the Feynman cutoff (1) was increased to $\Lambda = 23\,\text{GeV}$.

6. Summary

In summary, the cyclotron measurement confirmed QED and established the muon as a heavy electron. The first storage ring discovered the $(\alpha/\pi)^3$ term in the QED expansion (scattering of light by light). The second verified the hadronic loops in the cloud of virtual particles around the muon, which contribute about 50 ppm to the anomalous moment.

In $(g-2)$ two worlds collide. The theorist is surrounded by esoteric concepts, wave functions, amplitudes, complex formulae many pages long. He evaluates endless integrals and after painstaking calculation comes up with a number. The experimenter deals with nanoseconds, huge magnets, racks of electronics, mazes of cables, and flashing lights. After years of effort he comes up with a number. These two worlds have nothing in common. And yet they agree on the same answer, accurate to parts per million. How is this possible? This is the deep enduring mystery of $(g-2)$.

7. Tests of Relativity

7.1. *Einstein's second postulate*

CERN's direct test of the second postulate of special relativity,[29] that the velocity of light is independent of the motion of the source is not widely known. Gamma rays from the PS target have been shown to come from the decay of π^0 in flight. Gammas of 6 GeV were selected with a lead glass Cherenkov counter. They must come from the forward decay of π^0 with energy at least 6 GeV, so the source velocity was greater than $0.99975c$. Would the velocity of these gammas be greater than normal?

Gamma ray time of flight is normally impossible, because they only interact once. But the PS beam is bunched in time by the RF driver, so the π^0 are bunched and the gamma rays also. Bunches of gammas are sweeping across the lab: they can be timed relative to the phase of the RF. When the detector is moved, the relative phase changes. If the displacement corresponds to one RF time period, then the relative phase should again be the same. This provides a sensitive test that the gamma ray velocity is the same as the velocity of light, independent of the calibration of the timing circuits.

The data are shown in Fig. 12. Position B for the detecter is one RF wavelength further away from the accelerator than position A, and the timing curves look the same. The velocity of gammas from the moving source was found to be the same as the standard velocity of light to 1 part in 10^4. This confirms the second postulate to high accuracy at very high velocities. It is also the best measurement of the velocity of any gamma rays.

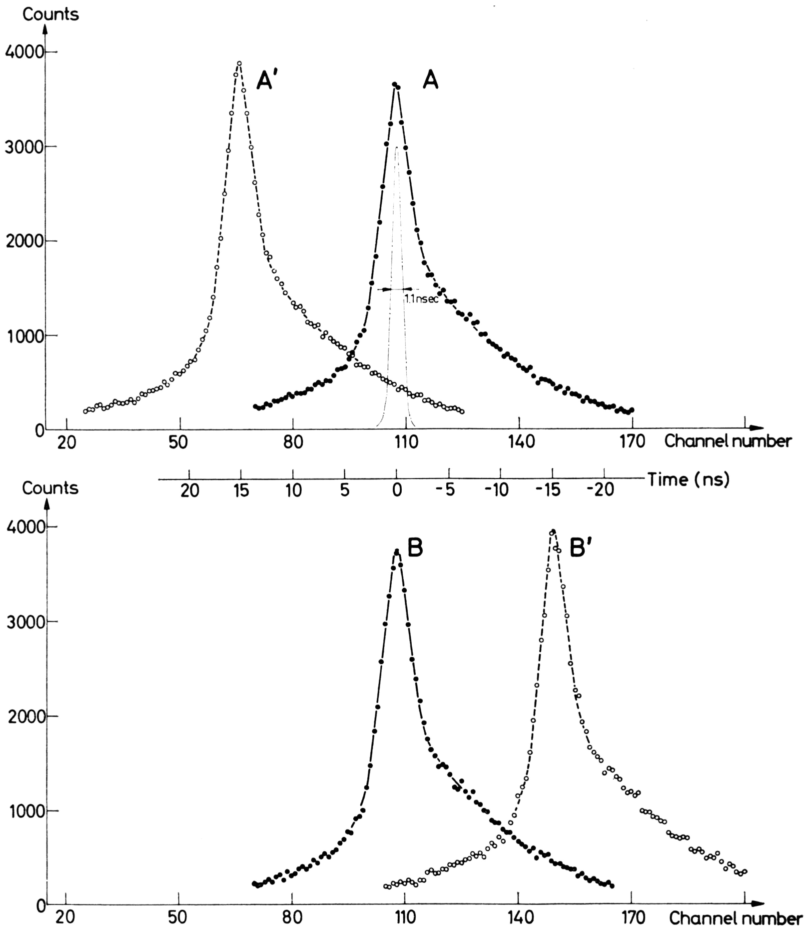

Fig. 12. Gamma arrival time vs PS radiofrequency. Positions A and B are one RF wavelength apart. A′ and B′ are offset by 4.5 m: the peaks move correspondingly 15 ns. Comparing A with B gives the velocity of gammas from the moving π^0.

7.2. *Muon lifetime in flight*

The muon lifetime in a circular orbit is a stringent test of relativity. It can also measure the life of μ^- which cannot be measured at rest and therefore tests the CPT invariance of the weak interaction.

The twin paradox was discussed in Einstein's first paper.[30] It is a paradox because, if only relative motion is important, one can ask which twin moves and which remains at rest? The difference is that to return to the same point, one twin must have suffered some acceleration which the other (older) twin did not. It seems that, according to relativity, the one with a history of acceleration finishes younger than the sessile partner; a result which is hard for the human mind to grasp, though people driving fast sports cars do seem to be younger than the average.

Time dilation was established by the first muon storage ring. With the second we measured it accurately.

The scraping system described above minimised the losses. A correction was made for the residual loss rate ($\sim$0.1% per lifetime) measured with the calibrated loss detector. The rotation frequency gave the radial distribution as shown in Fig. 10 and Eq. (12) gave the mean value of $\gamma = 29.327\,(4)$. Multiplying by the lifetime[31] at rest 2.19711 (8) μs gave a predicted lifetime of 64.435 (9) μs to be compared to the experimental value 64.378 (26) μs. So the Einstein time dilation was verified to 0.9 ± 0.4 parts per thousand. Further details are given in Bailey *et al.*[32]

This is the best reported measurement of time dilation in a circular orbit. The lifetime of negative muons was the same as μ^+.

In Memoriam

This review is dedicated in warm appreciation to the memory of my esteemed colleagues Emilio Picasso, Simon van der Meer and Frank Krienen.

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