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Calabi-Yau Supermoduli Space, Field Strength Duality and Mirror Manifolds

Sergio Ferrara*
CERN, Geneva, Switzerland
and
Department of Physics
University of California, Los Angeles, CA 90024

and

Mark Bodner and Ana Cristina Cadavid†
Department of Physics
University of California, Los Angeles, CA 90024

Abstract

Calabi-Yau moduli spaces have a super extension due to their relation to type II superstring compactifications. Supermoduli can be defined as N=2 target space vector-supermultiplets containing both the moduli and their vector field bosonic superpartners. A change of basis in the cohomology vector space corresponds to (modular) duality transformations of target space vector field strengths. Target space two-forms will take values in the cohomology spaces of Calabi-Yau manifolds, and are therefore related to the Kähler class and complex structure deformations, which may be relevant for the description of mirror symmetries of (2,2) superconformal field theories.

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† ICSC - World Laboratory Scholarship.

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Recently, new insight on the moduli space of Calabi-Yau threefolds [1,2] has been gained by using both target space supersymmetry [3,4,5,6,7,8,9] and world-sheet n=2 superconformal symmetry [10,11] arguments. In particular the moduli space of the Kähler class and the complex structure deformations has been shown to be (locally) a product space [3,4,5,6],

$$M_1 \times M_2 \tag{1}$$

of two special Kähler manifolds of complex dimension $h_{(1,1)}$ and $h_{(2,1)}$ respectively. The $h_{(1,1)}$ and $h_{(2,1)}$ are the two independent Hodge numbers which specify a given Calabi-Yau threefold [1,2]. The two manifolds M_1 and M_2 are special Kähler in the sense that their Riemann tensor satisfies the following constraint [12]

$$R^{1,2}_{i\bar{l}j\bar{m}} = G^{1,2}_{i\bar{l}}G^{1,2}_{j\bar{m}} + G^{1,2}_{i\bar{m}}G^{1,2}_{j\bar{l}} - Q^{1,2}_{ijp}(G^{-1})^{p\bar{p}}\bar{Q}^{1,2}_{\bar{l}\bar{m}\bar{p}}. \tag{2}$$

$Q^{1,2}_{ijp}$ is a covariantly constant tensor with respect to a Kähler connection $K_i = \partial_i K$. This implies that the object $W^{1,2}_{ijp} = (\exp -K)Q^{1,2}_{ijp}$ is a holomorphic section of the line bundle specified by the gauge field $K_i dx^i$, and satisfies $\bar{\partial}W_{ijp} = 0$. W_{ijp} is then a holomorphic function of the moduli [1,2,4,8,13] and is related to the Yukawa couplings of the $\overline{27}$ families for the $(1,1)$ moduli, and to the Yukawa couplings of the 27 families for the $(2,1)$ moduli.

For a particular choice of coordinates on M_1, M_2, the Kähler metric which solves equation (2) is given by [14],

$$K^{1,2} = -\log Y^{1,2}$$

$$Y^{1,2} = 2(f^{1,2} + \bar{f}^{1,2}) - (M^{p(1,2)} - \bar{M}^{\bar{p}(1,2)})\left(\frac{\partial}{\partial M^{p(1,2)}}f^{1,2} - \frac{\partial}{\partial \bar{M}^{\bar{p}(1,2)}}\bar{f}^{1,2}\right) \tag{3}$$

where the $f^{1,2}$ are two holomorphic "functions" $f^1 = f^1(M^1), f^2 = f^2(M^2)$ on M_1, M_2. In the same coordinate basis the Yukawa couplings are given by

$$W^{1,2}_{ijp} = \partial_i\partial_j\partial_p f^{1,2} = f^{1,2}_{ijp} \tag{4}$$

In the Kaluza -Klein theory of $10D$ supergravity[1] compactified on a Calabi-Yau space, explicit formulas for $G^{1,2}_{i\bar{j}}$ and $W^{1,2}_{ijp}$ can be derived [4,5,6,10]. For the $(1,1)$ moduli [2,4] we have

$$G_{i\bar{j}} = -\partial_i\partial_{\bar{j}} \log V \tag{5}$$

where V is the internal space volume [2,4,6], and

$$V = \int J \wedge J \wedge J \tag{6}$$

$$G_{i\bar{j}} = -\partial_i\partial_{\bar{j}} \log(d_{lmp}\lambda^l\lambda^m\lambda^p) \ ; \ \lambda^l = Im(M^{l\,(1,1)}) \tag{7}$$

[1]This theory is regarded as the large volume limit of the effective theory of critical superstrings compactified on an "internal" $(2,2)$ superconformal field theory $[1-10]$.

where J is the Kähler class form, given in terms of a basis of $(1,1)$ moduli with coefficients λ^l, and the d_{lmp} are given by the intersection matrices of three $(1,1)$ forms.

For the $(2,1)$ moduli [5,6] the metric is given by

$$G_{IJ} = -\partial_I\partial_J \log i(\Omega,\bar{\Omega}) \tag{8}$$

$$(\Omega,\bar{\Omega}) = \int \Omega \wedge \bar{\Omega} \tag{9}$$

where $\Omega(Z)$ is a holomorphic 3-form as a function of the parameters Z which deform the complex structure.[2]

We can use a real integral cohomology basis for H^3, given by the mutually dual three-forms α_I, β^I with $I = 0, \ldots h_{(2,1)}$, to express the holomorphic 3-form.

$$\Omega(Z) = Z^I\alpha_I + iF_I(Z)\beta^I \tag{10}$$

$$\Omega(\lambda Z) = \lambda\, \Omega(Z) \ ; \ F(\lambda Z) = \lambda^2 F(Z) \tag{11}$$

where λ is a constant, $F_I = \partial_I F$ and F are related to the holomorphic function appearing in equation (3) by

$$F(Z) = (Z^0)^2 f(M^I = \frac{Z^I}{Z^0}) \tag{12}$$

The Dolbeault cohomology basis in $H^{(3,0)}$ and $H^{(2,1)}$ is given by [6,8]

$$\Omega(Z)$$

$$\Phi_I(Z) = \Omega_I - \frac{(\Omega_I,\bar{\Omega})}{(\Omega,\bar{\Omega})}\Omega \tag{13}$$

$$Z^I\Phi_I = 0$$

where

$$\Omega_I(Z) = \frac{\partial}{\partial Z^I}\Omega = \alpha_I + iF_{IJ}\beta^J$$

$$\Omega_I \in H^{(3,0)} + H^{(2,1)} \tag{14}$$

A variety of relations can be established as follows. Obviously

$$(\Phi_I,\Omega) = (\Phi_I,\bar{\Omega}) = (\Phi_I,\Phi_J) = (\Omega,\Omega) = 0 \tag{15}$$

[2]Since $K = -\log i(\Omega,\bar{\Omega})$, this means that Ω is a holomorphic section of a line bundle. In physical terms this means that Ω has a non-trivial Kähler weight i.e. it is charged with respect to the $U(1)$ gauge field whose field strength is the Kähler form of the Kähler manifold in question [8,9]. Local supersymmetry also requires that the fermions including the gravitino are fields which carry a non-trivial Kähler weight, i.e. they are defined on the same $U(1)$ bundle. This is not so in rigid supersymmetry in which case the first two terms on the right-hand side of equation (2) are absent [9].

In addition

$$(\Phi_I,\bar{\Phi}_J) = (\Omega,\bar{\Omega})\partial_I\partial_J \log i(\Omega,\bar{\Omega}) = -4i(\exp -K)\partial_I\partial_J K \quad (16)$$

$$(\Omega,\bar{\Omega}) = -i(Z^I\bar{F}_I + \bar{Z}^I F_I) = -4i(\bar{Z}NZ) \quad (17)$$

The period matrix N is given as

$$N_{IJ} = \frac{1}{4}(F_{IJ} + \bar{F}_{IJ}) \quad (18)$$

Also we have

$$(\Omega_I,\bar{\Omega}) = -4i(N\bar{Z})_I \quad (19)$$
$$(\Omega_{IJ},\bar{\Omega}) = -4iF_{IJK}\bar{Z}^K \quad (20)$$
$$(\Omega_I,\bar{\Omega}_J) = -4iN_{IJ} \quad (21)$$

In heterotic theories with $N=1$ target space supergravity the moduli M can be extended to $N=1$ chiral superfields. In type II $N=2$ local supersymmetry, the appropriate multiplet is the field strength multiplet, i.e. a chiral $N=2$ superfield[3] [15],

$$W(M,\theta) = M + \ldots \bar{\theta}^i\sigma^{\mu\nu}\theta^j\varepsilon_{ij}F^-_{\mu\nu} + \ldots \bar{\theta}\theta\bar{\theta}\theta\Box M^* \quad (22)$$

We call the W superfield a supermodulus in perfect analogy to the supermoduli of Riemann surfaces. This superfield will determine the corresponding supermoduli in the type IIB theory for $(2,1)$ forms and in the type IIA theory for $(1,1)$ forms.

In the type IIB theory, equation (22) implies that we can introduce a three-form for $N=2$ super fields

$$\Omega(W) = W^I\alpha_I + i\frac{\partial}{\partial W^I}F(W)\beta^I \quad (23)$$

such that

$$\Omega(W)_{\theta=0} = \Omega(Z) \quad (24)$$

The θ^2 component of equation (23) will include a term

$$\bar{\theta}^i\sigma^{\mu\nu}\theta^j\varepsilon_{ij}\mathcal{F}^-_{\mu\nu} \quad (25)$$

where the five-form $\mathcal{F}^-$ is given by

$$\mathcal{F}^- = F^{-I}\alpha_I + 2iG^-_I\beta^I \quad (26)$$

[3] $F^{\mp}$ denote the (anti) self-dual four-dimensional field strengths.

with

$$G^-_I = L_{IJ}F^{-I} \quad (27)$$

The matrix $L_{IJ}(Z,\bar{Z})$ can be computed by requiring that the five-form $\mathcal{F}^-$ does not depend on the choice of the α_I,β^I cohomology basis. This implies that $\mathcal{F}^-$ is invariant under symplectic transformations $Sp(2h_{(2,1)}+2,Z)$. The restriction of Sp to the symplectic modular group is required by the meaning in $H^3(Z)$ of (α,β) as an integral cohomology basis. For a non-integral cohomology basis the Sp transformation is given over the reals, as $Sp(2h_{(2,1)}+2,R)$. Given a symplectic transformation for the (α,β) bases as[4]

$$\delta\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \begin{pmatrix}-B^T & -C\\ D & B\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} \quad (28)$$

with a constant real matrix B and two constant real symmetric matrices C and D, we may compute the corresponding (infinitesimal) symplectic transformations for the coefficients that leave equations (9) and (26) invariant under a change of bases. It is found that $(-iF_I, Z^I)$ and $(-2iG^-_I, F^{-I})$ undergo the same transformations as the bases' elements. Our vector field strengths' transformations can be viewed as generalizations of the duality transformations in electromagnetism, in which the electric and magnetic fields are interchanged. These generalized duality transformations are discussed at length in ref. [16].

Similarly we can fix the (infinitesimal) symplectic transformations for L_{IJ} as

$$\delta(iL_{IJ}) = \frac{C_{IJ}}{2} - B^T_{IK}(iL)_{KJ} - (iL)_{IK}(B + D(iL))_{KJ} \quad (29)$$

We find that the last equation is satisfied for an L_{IJ} given by

$$L_{IJ} = 2\left[\frac{F_{IJ}}{4} - \frac{(N\bar{Z})_I(N\bar{Z})_J}{\bar{Z}N\bar{Z}}\right] \quad (30)$$

with N given in equation (18).

In the process of fixing the invariance of $\mathcal{F}^-$ under symplectic transformations we find a series of relations given below:

$$\delta F_{IJ} = -iC_{IJ} - B^T_{IK}F_{KJ} - F_{IK}B_{KJ} + iF_{IK}D_{KL}F_{LJ} \quad (31)$$

$$\delta(N\bar{Z})_I = -(B^T_{IJ} - iF_{IL}D_{LJ})(N\bar{Z})_J \quad (32)$$

$$\delta(\bar{Z}N\bar{Z}) = i(N\bar{Z})D(N\bar{Z}) \quad (33)$$

$$\delta(\partial_I \log(\Omega,\bar{\Omega})) = -(B^T_{IJ} - iF_{IK}D_{KJ})\partial_J \log(\Omega,\bar{\Omega}) \quad (34)$$

[4] Although in this paper we use "small" symplectic transformations, which are not appropriate for integral cohomology, the results we get for the relevant quantities are unaffected by this limitation.

We can expand $\mathcal{F}^-$ in the Dolbeault cohomology basis (which of course depends on the deformation parameters of the complex structure). Because of equations (26), (27) and (30), $\mathcal{F}^-$ has only components along the $(2,1)$ and $(0,3)$ forms:

$$\mathcal{F}^- = F^{-I}(b_I^J \Phi_J + c_I \bar{\Omega}) = \hat{F}^{-I}\Phi_I + T^- \bar{\Omega} \tag{35}$$

with

$$b_I^J = \delta_I^J - \frac{(N\bar{Z})_I \bar{Z}^J}{(\bar{Z}N\bar{Z})} \; ; \; \bar{Z}^I b_I^J = 0$$

$$c_I = \frac{(N\bar{Z})_I}{(\bar{Z}N\bar{Z})} \; ; \; \bar{Z}^I c_I = 1 \tag{36}$$

If we take the imaginary part of the five-form $\mathcal{F}^-$ we obtain a real conserved quantity that corresponds to the original self-dual 10-dimensional five-form in $(M_4 \times K_6)$, given by the field strength $(\mathcal{F} = \mathcal{F}^*)$ of type IIB supergravity [17]:

$$\mathcal{F} = i(\mathcal{F}^- - \mathcal{F}^+) = iF^{-I}(b_I^J \Phi_J + c_I \bar{\Omega}) - iF^{+I}(\bar{b}_I^J \bar{\Phi}_J + \bar{c}_I \Omega) \tag{37}$$

Furthermore,

$$\partial^\mu \mathcal{F}_{\mu\nu} = 0, \tag{38}$$

as a consequence of the Bianchi identities and the equations of motion of the vector fields[5] in four dimensions [12,14]:

$$\partial^\mu (F^{+I}_{\mu\nu} - F^{-I}_{\mu\nu}) = 0$$
$$\partial^\mu (G^+_{I\mu\nu} + G^-_{I\mu\nu}) = 0 \tag{39}$$

The coefficient $F^{-I}C_I$ for the $(0,3)$ form is precisely the graviphoton component [19] which enters into the gravitino transformation law [14],

$$\delta\psi^i_{\mu\alpha} = D_\mu \epsilon^i_\alpha + (\sigma \cdot T^-)_{\alpha\beta}(\gamma_\mu)_{\beta\dot{\alpha}} \epsilon^{ij} \epsilon_{j\dot{\alpha}} + \dots \tag{40}$$

with

$$T^- = F^{-I} \frac{(N\bar{Z})_I}{\bar{Z}N\bar{Z}} \tag{41}$$

The components along the $(2,1)$ forms $F^{-I}b_I^J$ correspond to the matter multiplet vectors,

$$\hat{F}^{-I}_{\mu\nu} = F^{-I}_{\mu\nu} - T^-_{\mu\nu}\bar{Z}^I \tag{42}$$

[5] We note that the holomorphic function $F(Z) = \frac{1}{2}F_I Z^I$ as well as the vector four-dimensional Lagrangian term, $F^+_{\mu\nu}G^{+\mu\nu} + h.c.$, are not invariant under symplectic transformations. They are given by a non-invariant product of components of three forms: $(\int \Omega \wedge \alpha_I)(\int \Omega \wedge \beta^I)$. On the other hand the objects $(\bar{Z}N\bar{Z})$, $\hat{F}^{-I}\Phi_I$, $T^-\bar{\Omega}$ are symplectic invariants. The subgroup of the symplectic transformation for which $F(Z)$ is left invariant (up to an imaginary quadratic polynomial) corresponds to (duality) isometries of the moduli space [4,7,14]. Examples of such transformations have been obtained for instance, for moduli space of Z_N orbifolds [4,18].

and appear correspondingly in the gaugino transformation law [14],

$$\delta\Lambda^I_{i\alpha} = (\sigma \cdot \hat{F}^{-I})_{\alpha\beta}\varepsilon_{ij}\epsilon^j_\beta + \dots \tag{43}$$

In this way space-time supersymmetry is seen to be deeply related to the cohomology classes of three forms of Calabi-Yau manifolds. T^- and F^{-I} are nothing but the dressed form of the graviphoton and gauge vector field strengths, due to the deformations of the complex structure.

A similar description to the one presented can be given for the vector multiplets in the type IIA theory. This allows us to justify with a geometrical argument the recent formal developments [7] which give the appropriate treatment of $(1,1)$ and $(2,1)$ forms to understand the mirror symmetry [20] of Calabi-Yau moduli space. In fact for mirror Calabi-Yau manifolds $C, \tilde{C}$ the moduli space of the complex structure deformations on C and the Kähler class deformations on $\tilde{C}$ coincide. The same is true interchanging C with $\tilde{C}$.

Let us consider the vector fields in the type IIA theory. They are given by the ten-dimensional fields $A_\mu, A_{\mu\nu\rho}$ corresponding to a zero form and to $h_{(1,1)}$ forms on the Calabi-Yau space. The field strengths for the two kinds of vector fields, together with their dual expressions are given by:

$$F_{\mu\nu}\,,\; (G^*)_{\mu_1\mu_2\mu_3\mu_4\mu_5\mu_6\mu_7\mu_8},\; G_{\mu\nu} = \frac{\partial \mathcal{L}}{\partial F_{\mu\nu}} \tag{44}$$

$$F_{\mu\nu\rho\sigma},\; (G^*)_{\mu_1\mu_2\mu_3\mu_4\mu_5\mu_6},\; G_{\mu\nu\rho\sigma} = \frac{\partial \mathcal{L}}{\partial F_{\mu\nu\rho\sigma}} \tag{45}$$

where $\mathcal{L}$ is the type IIA supergravity Lagrangian.

The four objects displayed in equations (44) and (45) can be seen on $M_4 \times K_6$ as the product of vector field strength two-forms on M_4, and of zero-, six-, two- and four-forms on the Calabi-Yau manifold. More precisely

$$F_{\mu\nu} \mapsto H^0 \;\; F_{\mu\nu\rho\sigma} \mapsto H^2 \tag{46}$$

$$(G^*)_{\mu_1\mu_2\mu_3\mu_4\mu_5\mu_6\mu_7\mu_8} \mapsto H^6 \;\; (G^*)_{\mu_1\mu_2\mu_3\mu_4\mu_5\mu_6} \mapsto H^4 \tag{47}$$

Note that because of the symmetry of the Hodge diamond on a Calabi-Yau space $h_{(0,0)} = h_{(3,3)} = 1$ and $h_{(1,1)} = h_{(2,2)}$. This is completely analogous, as observed in ref. [7], to $h_{(0,3)} = h_{(3,0)} = 1$ and $h_{(2,1)} = h_{(1,2)}$ for the three-form cohomology. This relation is obtained by interchanging the rows and columns in the Hodge diamond, or equivalently the even forms with the odd forms.

However we stress that this is not a mere analogy. Indeed the $H^{(0,0)} + H^{(3,3)}$ cohomology corresponds to the graviphoton and its dual in the type IIA theory, just as the $H^{(3,0)} + H^{(0,3)}$ cohomology is related to the graviphoton in the type IIB theory. Similarly the matter vector multiplets' field strengths and their duals correspond in the type IIB theory to the $H^{(2,1)} + H^{(1,2)}$ cohomology, and to the $H^{(1,1)} + H^{(2,2)}$ cohomology in the type IIA theory.

Let us introduce a dual cohomology basis for even forms in a Calabi-Yau space [7]. Given e_I, e^I for $I = 0 \dots h_{(1,1)}$ with

$$e_0 \in H^0, \;\; e_A \in H^2 \tag{48}$$

$$e^0 \in H^6, \;\; e^A \in H^4 \tag{49}$$

We can then use the enlarged basis to define complex vector spaces $H^0 + iH^6$ and $H^2 + iH^4$ with complex dimensions 1 and $h_{(1,1)}$ respectively. This is analogous to $H^{(3,0)}$ and $H^{(2,1)}$ for the three-form cohomology. One can make e_I, e^I anticommuting by introducing appropriate Grassmann variables, μ, ν,

$$\alpha_I = \nu e_I$$

$$\beta^I = \mu e^I$$

$$\nu\mu = -\mu\nu, \; \mu^2 = \nu^2 = 0 \tag{50}$$

Just as in the treatment of the type IIB theory, we can define a $(1,1)$ supermodulus $W(Z,\theta)$ such that for $\theta = 0$ [6]

$$\mho(W)_{\theta=0} = \mho = Z^I \alpha_I + iF_I \beta^I \tag{51}$$

As before the $\bar\theta\sigma_{\mu\nu}\theta$ component of W yields

$$\mathcal{F}^- = F^{-I}\alpha_I + 2iG_I^- \beta^I \tag{52}$$

Let us constrain equation (52) to the graviphoton alone, by setting $F(Z) = (Z^0)^2, Z^A = 0$. We obtain

$$\mathcal{F}^{-0}_{graviphoton} = F^{-0}(\alpha_0 + i\beta^0) \tag{53}$$

which shows that the self-dual graviphoton field strength is a complex combination of a zero- and a six-form as is seen from equations (44) through (47).

Exactly as before the matter self-dual gauge field strengths are elements of $H_2 + iH_4$. A basis on $H_2 + iH_4$ is given by

$$\Psi_I(Z) = \frac{\partial \mho}{\partial Z^I} - \frac{(\mho_I, \bar\mho)}{(\mho, \bar\mho)}\mho \tag{54}$$

This formula allows us to introduce for $(1,1)$ forms, arbitrary holomorphic functions $F(Z)$ and not just a cubic polynomial, that is valid only in the large radius limit. This is exactly what happens in string theory due to world-sheet instanton corrections [10,13,20,21,22]. Note that in the large radius limit (Z^A large) we have

$$F(Z) = i\frac{1}{3!}d_{ABC}Z^A Z^B Z^C \tag{55}$$

[6]Note that $F_I = \partial_I F$ because the forms $\frac{\partial \mho}{\partial Z^I} \in (H^{(0,0)} + iH^{(3,3)}) + (H^{(1,1)} + iH^{(2,2)})$, and therefore have vanishing interaction with $(H^{(0,0)} + iH^{(3,3)})$: $\int \mho \wedge \frac{\partial \mho}{\partial Z^I} = 0$.

where d_{ABC} are the intersection matrices given in terms of topological quantities on the Calabi-Yau space [2]. The cubic nature of the holomorphic function implies that the moduli space has at least $h_{(1,1)}$ Abelian isometries [4], given by $Z^A \mapsto Z^A + c^A, F^A_{\mu\nu} \mapsto F^A_{\mu\nu} + F^0_{\mu\nu}c^A$. These invariances are due [4] to the perturbative Peccei-Quinn symmetry of the antisymmetric tensor field $B_{i\bar j}$.

As stated initially we are able to give a more geometrical meaning to the symmetry between mirror Calabi-Yau manifolds, $C(h_{(1,1)}, h_{(2,1)})$ and $\tilde C(\tilde h_{(1,1)} = h_{(2,1)}, \tilde h_{(2,1)} = h_{(1,1)})$. Furthermore the mirror symmetry implies that the vector spaces and hence the cohomology classes of $H^2 + iH^4$ and $\tilde H^{(2,1)}$ are "isomorphic", just as the $H^{(2,1)}$ and $\tilde H^2 + i\tilde H^4$. In view of this (up to a global symplectic transformation) we have

$$\Omega(Z_{21}) = \tilde\mho(\tilde Z_{11}) \tag{56}$$

$$\mho(Z_{11}) = \tilde\Omega(\tilde Z_{21}) \tag{57}$$

$$\Phi_I(Z_{21}) = \tilde\Psi_I(\tilde Z_{11}) \tag{58}$$

$$\Psi_I(Z_{11}) = \tilde\Phi_I(\tilde Z_{21}). \tag{59}$$

It is important to note that the mirror manifolds have the property that one may choose the holomorphic functions for the $(1,1)$ and $(2,1)$ mutually equal so that the Yukawa coupling for the 27 family in C is equal to the Yukawa coupling for the $\bar{27}$ family in $\tilde C$, and vice versa [20].

In conclusion we have shown that the discussion of deformation theory of Calabi-Yau moduli space takes a deeper meaning by introducing the concept of the $N = 2$ supermodulus. Each modulus is accompanied by a field strength which takes values in the cohomology vector spaces of the Calabi-Yau manifolds.

The interest in considering vector field strengths starts with the fact that the presence of the graviphoton raises the number of variables to $h + 1$, where h is one of the Hodge numbers. Moreover, in this discussion it appears evident that a unified treatment of even and odd harmonic forms is naturally explained, since from the point of view of $N = 2$ space-time supergravity there is no difference between $(1,1)$ and $(2,1)$ forms. Such a difference shows up only when the σ-model perturbative analysis is made. Since a small σ-model coupling corresponds to a large volume of the internal manifold, the perturbative approach corresponds to Kaluza-Klein compactifications from ten-dimensional supergravities. The existence of quantum Calabi-Yau spaces with mirror symmetry [20] seems to imply that there is an isomorphism between the integral cohomologies of $(2,1)$ and $(1,1)$ forms, or equivalently an isomorphism between the self-dual vector spaces $H^{(2,1)} + H^{(1,2)}$ and $H^{(1,1)} + H^{(2,2)}$. This would imply that the deformation theory of the former space is identified with the deformation theory of the latter space, as presented in equations (56)-(59).

Furthermore from a σ-model point of view, the mirror symmetry relates classical to non-perturbative quantities. This relation violates the perturbative Peccei-Quinn symmetry associated to the antisymmetric tensor fields and is therefore a purely string phenomenon which may be associated to the quantum symmetries [23] of the strongly coupled string theory.

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