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A Comparison of Jet Production Rates on the Z^0 Resonance to Perturbative QCD

DELPHI Collaboration

Abstract

The production rates for 2-, 3-, 4- and 5-jet hadronic final states have been measured with the DELPHI detector at the e^+e^- storage ring LEP at centre of mass energies around 91.5 GeV. Fully corrected data are compared to $O(\alpha_S^2)$ QCD matrix element calculations and the QCD scale parameter $\Lambda_{\overline{MS}}$ is determined for different parametrizations of the renormalization scale μ^2. Including all uncertainties our result is $\alpha_S(M_Z^2) = 0.114 \pm 0.003[stat.] \pm 0.004[syst.] \pm 0.012[theor.]$.

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1. Introduction

The investigation of multijet final states produced in high energy $e^+ e^-$ annihilation allows direct and detailed tests of perturbative QCD. In the past jet analyses using infrared safe jet finding algorithms were performed by the JADE [1], TASSO [2], AMY [3] and by the MARK II [4] Collaborations at centre of mass energies below M_Z and by the MARK II [5] and OPAL [6] Collaborations at the Z^0- resonance. At LEP energies one expects a decrease of hadronization corrections [5] so that the prediction for 2-, 3- and 4-parton final states in second order of the running coupling α_S [7] can be more directly compared to the experimental jet rates.
This letter presents an analysis of multijet hadronic final states using data taken with the DELPHI detector at the LEP collider at an average centre of mass energy of 91.5 GeV. The QCD scale parameter $\Lambda_{\overline{MS}}$ is fitted for various assumptions, and the scale μ^2 at which α_S should be evaluated, is determined under conditions where four jet contributions are important.

2. The Detector

Only charged particles have been used for the analysis. About 25 % of the data were taken at a reduced magnetic field of 0.7 T for which only information of the Time Projection Chamber (TPC) has been taken into account [8]. The rest of the data was taken at the nominal magnetic field of 1.23 T and included additional information from the Inner Tracking Detector (ID) and the Outer Tracking Detector (OD). A detailed description of the DELPHI detector is given in [9].

Tracks in the TPC have been reconstructed from up to 16 space points at radii between 36.5 and 106.2 cm with a space point precision for the high field data of about 300 μm in $r\phi$ and 900 μm in z. The ID, a cylindrical drift chamber, covers polar angles of approximately 20° to 160° at radii between 12 and 28 cm. It comprises a jet chamber section providing 24 $r\phi$ coordinates surrounded by 5 trigger layers measuring both $r\phi$ and z coordinates. In the OD, which covers polar angles of $40° - 140°$ at radii between 198 and 207 cm, 5 layers of drift tubes provide precise $r\phi$ coordinates and 3 of them also provide crude but fast z information to be used in the trigger system.

The barrel trigger for hadronic events was based on two independent sets of scintillation counters and on a 'track trigger' made by coincidences of the ID and OD chambers. The overall trigger efficiency including the scintillator and track trigger was found to be greater than 99.9 % [10].

3. Event selection and jet analysis

In order to prepare a clean sample of multi-jet events well contained inside the active region of the detector we performed the selection of charged particle tracks and of hadronic events according to the criteria described in [8]. Accordingly hadronic events were selected by requiring that (a) there were at least 5 charged particles with momenta above 0.2 GeV/c, (b) the total charged energy detected exceeded 15 GeV, (c) each of the two hemispheres $\cos\theta < 0$ and $\cos\theta > 0$ contained a total charged energy $E_{ch} = \sum E_i$ larger than 3 GeV, where E_i are the particle energies (assuming the π mass) and (d) the polar angle θ of the sphericity axis was in the range $40^\circ \leq \theta \leq 140^\circ$. In addition we selected "momentum balanced" events by requesting the absolute value of the sum of the three-momenta of the charged particles $|\sum \vec{p}_i^{ch}|$ to be less than 30 GeV/c. This cut removes about 8 % of the events, but does not change the results for multi-jet rates by more than the quoted errors.

After applying these selection criteria 1727 events measured at a solenoid field of 0.7 T and 4990 events measured at 1.23 T, both at a mean centre of mass energy of 91.5 GeV remained. The background due to beam-gas scattering and $\gamma\gamma$-interactions is less than 0.1 % of the selected samples; the background due to $\tau^+\tau^-$ events is estimated to be about 0.2 % [8].

4. Evaluation of experimental jet rates

Jets are reconstructed in hadronic events by using the y-cluster jet finding algorithm originally introduced by the JADE Collaboration [1]. For each event the squares of the scaled invariant masses for each pair of charged particles i and j

$$y_{ij} = \frac{2E_iE_j(1-\cos\theta_{ij})}{E_{vis}^2} \tag{4.1}$$

are evaluated. Here E_i, E_j are the energies and θ_{ij} the angle between the momentum vectors of the two particles. E_{vis} is the total charged energy of the event (pion mass assumed). The particle pair with the lowest value y_{ij} is selected and replaced by a pseudo-particle with four momentum $(p_i + p_j)$, hereby reducing the multiplicity by one. In successive steps the procedure is repeated until the scaled invariant masses of all pairs of pseudo-particles or particles are larger than a given cut resolution y_c. The remaining pseudo-particles or particles are called jets.
The relative production rates $R_{i-jet} = \frac{\sigma_{i-jet}}{\sigma_{tot}}$ for 2-, 3-, 4- and 5-jets as a function of y_c, where σ_{tot} is the total hadronic cross-section, are our main experimental results. Monte Carlo simulations including acceptance and kinematical cuts, the detector

resolution, interactions of particles with the material of the detector, other detector imperfections and also QED corrections have been performed in order to correct the measured jet rates. We used the JETSET parton-shower model [11] which is known to reproduce well the measured hadronic distributions [8] to obtain a correction factor for all values of y_c studied

$$C_i(y_c) = \frac{R^{gen}_{i-jet}(y_c)}{R^{acc}_{i-jet}(y_c)} \tag{4.2}$$

After multiplication with $C_i(y_c)$ the jet rates are normalized to one. Here the 'generated' jet rates (R^{gen}) were reconstructed from final state charged particles in events generated without photon radiation and without tracking through the detector and the 'accepted' rates (R^{acc}) were reconstructed from charged particles after tracking through the fully simulated DELPHI detector. In the latter case effects of QED corrections were included.

Since the 0.7 T data and the 1.23 T data were measured with different detector components, the correction factors had to be evaluated separately for each data set. Fig. 1 shows the correction for the 1.23 T data. After correction the two resulting data sets are compared in Fig. 2. Since the two analyses are independent, the good agreement is indicative of small systematic errors.

The correction factors C_i are also computed by using the HERWIG parton shower generator [12] and the JETSET matrix element version [11]. The variance of the C_i values was taken as one contribution to the systematic error. Further contributions arise due to imperfect detector calibration and alignment. The influence of the selection criteria was also investigated. In total, due to the systematic error the 3-jet rate is uncertain within a shift of 4% either up or down. After correction the data sets were merged according to their statistical weights. The results are presented in Table 1.

The statistical errors $\Delta R_{i-jet}(y_c)$ quoted in Table 1 are highly correlated. For the fitting of QCD parameters to the experimental jet rates it would thus be necessary to construct the full error matrix for the evaluation of the quantity χ^2. To avoid this difficulty we also evaluate the differential distributions D_{2-jet} and D_{3-jet} defined as [13]:

$$\begin{aligned} D_{2-jet}(y_c) &= \frac{R_{2-jet}(y_c+\Delta y_c)-R_{2-jet}(y_c)}{\Delta y_c} \\ D_{3-jet}(y_c) &= \frac{R_{3-jet}(y_c+\Delta y_c)-R_{3-jet}(y_c)}{\Delta y_c} + D_{2-jet}(y_c) \end{aligned} \tag{4.3}$$

In these distributions the information of the transition from i+1 jet to i jet enters only once. The quantities $D_{i-jet}(y_c)$ are proportional to the number of events in the interval y_c to $y_c + \Delta y_c$ which change between the jet-multiplicities i+1 and i, D_{3-jet} is therefore mainly given by the differential 4-jet rate. For $y_c \geq 0.10$ the distribution D_{3-jet} is zero within our present statistics. The D_{2-jet} and D_{3-jet} data are presented in Table 2.

5. Jets in second order QCD

The comparison of the experimental 2-, 3- and 4-jet rates with QCD is based on the $O(\alpha_S^2)$ expansion of perturbation theory with and without an optimized scale [14]. At a renormalization scale μ^2 the strong coupling strength α_S is related to the QCD scale parameter $\Lambda_{\overline{MS}}$ by [15]

$$\alpha_S(\mu^2) = \frac{12\pi}{b_0 \ln\left(\frac{\mu^2}{\Lambda^2_{\overline{MS}}}\right)} \left[1 - \frac{b_1 \ln\left(\ln\left(\frac{\mu^2}{\Lambda^2_{\overline{MS}}}\right)\right)}{b_0^2 \ln\left(\frac{\mu^2}{\Lambda^2_{\overline{MS}}}\right)}\right] \tag{5.1}$$

with $b_0 = 33 - 2N_f$, $b_1 = 918 - 114N_f$ and $N_f = 5$ being the number of quark flavours produced. The experimental evaluation of $\Lambda_{\overline{MS}}$ is the main result of the comparison of the data with QCD predictions. Perturbation theory in second order does not specifiy the right scale μ^2. In agreement with earlier work [16], we set $\mu^2 = f\,Q^2$, with $f \leq 1$ and Q^2 being the square of the center of mass energy, and consider the scale factor f as a parameter to be determined by experiment.

The parton rates $R_{i-parton}(y) = \frac{\sigma_{i-parton}(y)}{\sigma_{tot}}$ with $i = 2, 3, 4$, describing the fraction of final states with i partons at a given resolution cut y as defined in [14], can be written as:

$$\begin{aligned} R_{2-parton}(y) &= 1 + C_1^{(2)}(y)\alpha_S(\mu^2) + C_2^{(2)}(y,f)\alpha_S^2(\mu^2) \\ R_{3-parton}(y) &= C_1^{(3)}(y)\alpha_S(\mu^2) + C_2^{(3)}(y,f)\alpha_S^2(\mu^2) \\ R_{4-parton}(y) &= C_2^{(4)}(y)\ \alpha_S^2(\mu^2) \end{aligned} \tag{5.2}$$

with

$$\begin{aligned} C_1^{(2)}(y) &= K_1^{(2)}(y) - r_1 \\ C_2^{(2)}(y,f) &= K_2^{(2)}(y) - r_1(K_1^{(2)}(y) - r_1) - r_2 + (K_1^{(2)}(y) - r_1)(b_0/12\pi)\ln f \\ C_1^{(3)}(y) &= K_1^{(3)}(y) \\ C_2^{(3)}(y,f) &= K_2^{(3)}(y) - r_1 K_1^{(3)}(y) + K_1^{(3)}(y)(b_0/12\pi)\ln f \\ C_2^{(4)}(y) &= K_2^{(4)}(y) \end{aligned} \tag{5.3}$$

The quantities $r_1 = 1/\pi$ and $r_2 = 1.4089/\pi^2$ are the first and second order α_S expansion coefficients of the total cross-section. The coefficients $K_2^{(2)}$ and $K_2^{(3)}$ have been evaluated by Kramer and Lampe in the KL' parton recombination scheme [17] and $K_2^{(4)}$ by A. Ali et al. [18,19]. Numerical values of $K_2^{(i)}$ are now available for values of the resolution cut y ranging from 0.01 to 0.25 [20]. The coefficients $C_2^{(2)}$ and $C_2^{(3)}$ explicitly depend on the scale factor f and render the jet rates R_2 and R_3 renormalization scale invariant.

The $O(\alpha_S^2)$ expansion of the parton rates equation (5.2) can not be directly compared to the measured jet rates since fragmentation effects are not considered in equation (5.2). At present it is not possible to evaluate hadronization in a model independent way. However, studies involving several options of the JETSET event generator show that hadronisation effects for jet rates measured with the y-cluster algorithm are small at LEP energies and can be safely taken into account according to

$$R_{i-jet}(y_c, f, \Lambda_{\overline{MS}}) = \sum_j M_{ij}(y = y_c) R_{j-parton}(y, f, \Lambda_{\overline{MS}}) \tag{5.4}$$

with i = 2, 3, 4, 5 and j = 2, 3, 4. The transition coefficients M_{ij} containing all hadronization effects are computed by generating 100 000 events with an option of the JETSET Monte Carlo containing the second order QCD matrix elements of Kramer and Lampe (introduced by N.Magnussen into JETSET [11,20]) and using string fragmentation. Thus the KL' parton recombination scheme (applied for the calculation of the coefficients of equation (5.3), which will be used for the fits to the data) is used for evaluating of the transition coefficients M_{ij}.

We checked that even when only charged hadrons are used in the JADE cluster algorithm the close agreement between jet- and parton-rates evaluated at the same values of y_c (the experimental resolution cut) and y (the resolution cut for resolved partons) is still maintained.

A first series of fits is performed to the measured differential distribution D_{2-jet} for $0.05 \leq y_c < 0.25$ and to $R_{2-jet}(y_c = 0.05)$. Thus the maximum information contained in the data is used without introducing additional correlations. In this y_c-region the contribution of the 4-jet rate is negligible and hadronization corrections are very small. Setting the renormalization scale $\mu^2 = Q^2$ yields $\Lambda_{\overline{MS}} = 180^{+33}_{-30}$ MeV corresponding to $\alpha_S = 0.114 \pm 0.003$ with $\chi^2/NDF = 10.9/10$. Figure 3a shows the differential distribution D_{2-jet} for the merged data set and the results of this fit. In Figure 3b the corresponding jet rates are compared to the data. Table 3 summarizes the results for $\Lambda_{\overline{MS}}$ from fits to the same data assuming several fixed values of the scale factor $f = \mu^2/Q^2$. For f-values ranging from 1 to 0.001 no significant difference in the quality of the fits (χ^2/NDF) is observed. The corresponding values $\Lambda_{\overline{MS}}$ vary by about a factor of two. A fit assuming $\mu^2 = yQ^2$ [21] yields approximately the same value of χ^2/NDF (Table 3).

The influence of the systematic uncertainty of the data has been evaluated by changing the 3-jet rate $R_{3-jet}(y_c)$ by $\pm 4\%$. This changes $\Lambda_{\overline{MS}}$ by $+43 - 38$ MeV for fits with $\mu^2 = Q^2$ corresponding to a change in α_S of ± 0.004. Ignoring hadronization effects by setting $M_{ii} = 1$ and $M_{ij} = 0$ for $i \neq j$ results in a change of $\Lambda_{\overline{MS}}$ of less than 10 MeV.

In a second series of fits the low y_c region $0.02 \leq y_c \leq 0.05$ was included. At $y_c = 0.02$ the 4-jet rate contributes about 10 %. With decreasing y_c hadronization corrections become larger and model dependent [5]. From fits to the differential distribution D_{2-jet} for $0.02 \leq y_c < 0.25$, D_{3-jet} for $0.02 \leq y_c < 0.08$, $R_{2-jet}(y_c = 0.02)$ and $R_{3-jet}(y_c = 0.25)$ one observes that scales like $\mu^2 = Q^2$ or $\mu^2 = y\,Q^2$ are unable to describe the data. Therefore $\Lambda_{\overline{MS}}$ and $f = \mu^2/Q^2$ are simultaneously determined. One obtains $\Lambda_{\overline{MS}} = 92^{+10}_{-9}$ Mev, $f = 0.0010^{+0.0005}_{-0.0003}$ with $\chi^2/NDF = 14.94/18$, the curves representing the fit are shown in Fig. 4. If hadronization effects are ignored $\Lambda_{\overline{MS}}$ is reduced to 77 MeV. Similar results have been obtained by the OPAL [6] Collaboration. Table 4 contains the results of fits to the same data with different scales μ^2.

Since the resultant small scale factor f is mainly dominated by the 4-jet rate, where only the leading order contribution has been calculated, we expect a fit to yield a larger scale factor once the $O(\alpha_S^3)$ contribution to σ_4-jet is available. We take the fact that for large scales the $O(\alpha_S^2)$ 4-jet cross-section is below the data (Fig. 3b) as indicative of a positive $O(\alpha_S^3)$ contribution to the 4-jet cross-section.

The fits presented above have been performed using one of the $O(\alpha_S^2)$ calculations of the 3-jet rate of Kramer and Lampe. An independent complete $O(\alpha_S^2)$ calculation has been performed by Ellis, Ross and Terrano [22]. The computation of the second order coefficients $K_2^{(2)}$ and $K_2^{(3)}$ contains ambiguities in so far as several techniques exist of how to combine unresolved partons into resolved on-shell quarks or gluons. The theoretical ambiguities introduced due to the various parton recombination schemes in the calculation of the 2- and 3-jet rate have been studied in [7,23]. We refer to [7,20] for a detailed discussion of this subject. In addition to the first series of fits to the KL' calculations we have used the coefficients $K_2^{(3)}(y)$ for the so called p-, E- and E0 recombination scheme as given in [7] for fits after corrections for hadronization in the range $0.05 \leq y_c < 0.25$ with $\mu^2 = Q^2$ (Table 5). In this region of y_c hadronization effects are found to be small. The resulting values of $\Lambda_{\overline{MS}}$ vary between 89 and 344 MeV which reflects the theoretical uncertainty of next-to-leading order calculations of jet rates and provides a conservative estimate of the systematic uncertainty in the measurement of $\Lambda_{\overline{MS}}$ in this process.

6. Conclusion

The relative rates of 2-, 3-, 4- and 5-jet events and the differential jet multiplicities have been measured with the DELPHI detector at an average centre of mass energy of 91.5 GeV. The data, based on a y-cluster analysis of charged tracks, are fully corrected for all detector effects and for initial state radiation. The predictions of second order QCD as evaluated by Kramer and Lampe have been fitted to the data after inclusion of hadronization corrections in order to determine the scale parameter $\Lambda_{\overline{MS}}$ and to obtain information on the renormalization scale μ^2.

Limiting the fit to a region of the cut resolution $0.05 \leq y_c < 0.25$ where the 4-jet rate is negligible and hadronization corrections are small and further assuming $\mu^2 = Q^2$ results in $\Lambda_{\overline{MS}} = 180^{+33+43}_{-30-38}$ MeV. Here the first error is due to statistics and the second contains the systematic uncertainty due to the data and due to hadronization effects. Changing the renormalization scale from $\mu^2 = Q^2$ to $\mu^2 = 0.001\,Q^2$ yields $\Lambda_{\overline{MS}} = 90^{+16+8}_{-14-8}$ MeV without significantly changing the quality of the fit confirming the renormalization scale invariance for R_2 and R_3 as given in (5.2). In the measurement of R_2 and R_3, $\Lambda_{\overline{MS}}$ can thus be determined to be between 90 and 180 MeV. From fits to the parton rates determined in different parton recombination schemes on the basis of the calculation by Ellis, Ross and Terrano [22] we estimate the theoretical recombination uncertainty to be about a factor of two in $\Lambda_{\overline{MS}}$. Including all uncertainties our final result for the strong coupling is

$$\alpha_S = 0.114 \pm 0.003[stat.] \pm 0.004[syst.] \pm 0.012[theor.]$$

Extending the region of y_c values down to 0.02 where the 4-jet rate becomes important and hadronization corrections can no longer be ignored leads to an apparent sensitivity with respect to the renormalization scale μ^2. Simultaneous fits of $\Lambda_{\overline{MS}}$ and of μ^2/Q^2 to the differential jet multiplicities D_{2-jet} and D_{3-jet} result in values $\Lambda_{\overline{MS}}$ of about 90 MeV and in small values of μ^2/Q^2 of the order of 0.001.

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y_c	R_2 [%]	R_3 [%]	R_4 [%]	R_5 [%]
0.01	25.1 ± 0.7	51.0 ± 0.9	20.2 ± 0.7	3.7 ± 0.4
0.02	44.4 ± 0.9	46.3 ± 0.9	9.0 ± 0.5	0.25 ± 0.08
0.03	57.1 ± 0.9	38.9 ± 0.9	4.0 ± 0.4	
0.04	65.2 ± 0.9	32.9 ± 0.8	1.9 ± 0.3	
0.05	71.1 ± 0.8	28.0 ± 0.8	0.9 ± 0.2	
0.06	75.3 ± 0.8	24.2 ± 0.8	0.6 ± 0.1	
0.08	82.9 ± 0.7	17.1 ± 0.7		
0.10	87.1 ± 0.6	12.9 ± 0.6		
0.12	90.7 ± 0.5	9.3 ± 0.5		
0.14	93.0 ± 0.4	7.0 ± 0.5		
0.16	95.0 ± 0.4	5.0 ± 0.4		
0.18	96.5 ± 0.3	3.5 ± 0.3		
0.20	97.5 ± 0.3	2.5 ± 0.3		
0.22	98.3 ± 0.2	1.7 ± 0.3		
0.25	99.4 ± 0.1	0.6 ± 0.1		

Table 1: Corrected jet-rates

y_c	D_2	D_3
0.01	19.2 ± 0.8	14.5 ± 0.7
0.02	12.5 ± 0.7	5.1 ± 0.4
0.03	8.1 ± 0.5	2.1 ± 0.3
0.04	6.0 ± 0.4	1.0 ± 0.2
0.05	4.1 ± 0.3	0.32 ± 0.09
0.06	3.8 ± 0.3	0.25 ± 0.07
0.08	2.1 ± 0.2	0.03 ± 0.02
0.10	1.8 ± 0.2	
0.12	1.2 ± 0.1	
0.14	1.0 ± 0.1	
0.16	0.7 ± 0.1	
0.18	0.5 ± 0.08	
0.20	0.39 ± 0.08	
0.22	0.37 ± 0.07	

Table 2: Differential distributions

μ^2	$\Lambda_{\overline{MS}}$ [MeV]	χ^2/NDF
$1.0 \cdot Q^2$	180^{+33}_{-30}	10.9/10
$0.5 \cdot Q^2$	155^{+28}_{-25}	10.8/10
$0.1 \cdot Q^2$	113^{+20}_{-18}	10.7/10
$0.05 \cdot Q^2$	101^{+18}_{-16}	10.7/10
$0.01 \cdot Q^2$	83^{+14}_{-13}	10.5/10
$0.005 \cdot Q^2$	80^{+13}_{-12}	10.4/10
$0.001 \cdot Q^2$	90^{+16}_{-14}	9.9/10
$1.0 \cdot y\,Q^2$	101^{+17}_{-16}	10.5/10
$0.5 \cdot y\,Q^2$	91^{+16}_{-14}	10.4/10
$0.1 \cdot y\,Q^2$	80^{+13}_{-12}	10.3/10

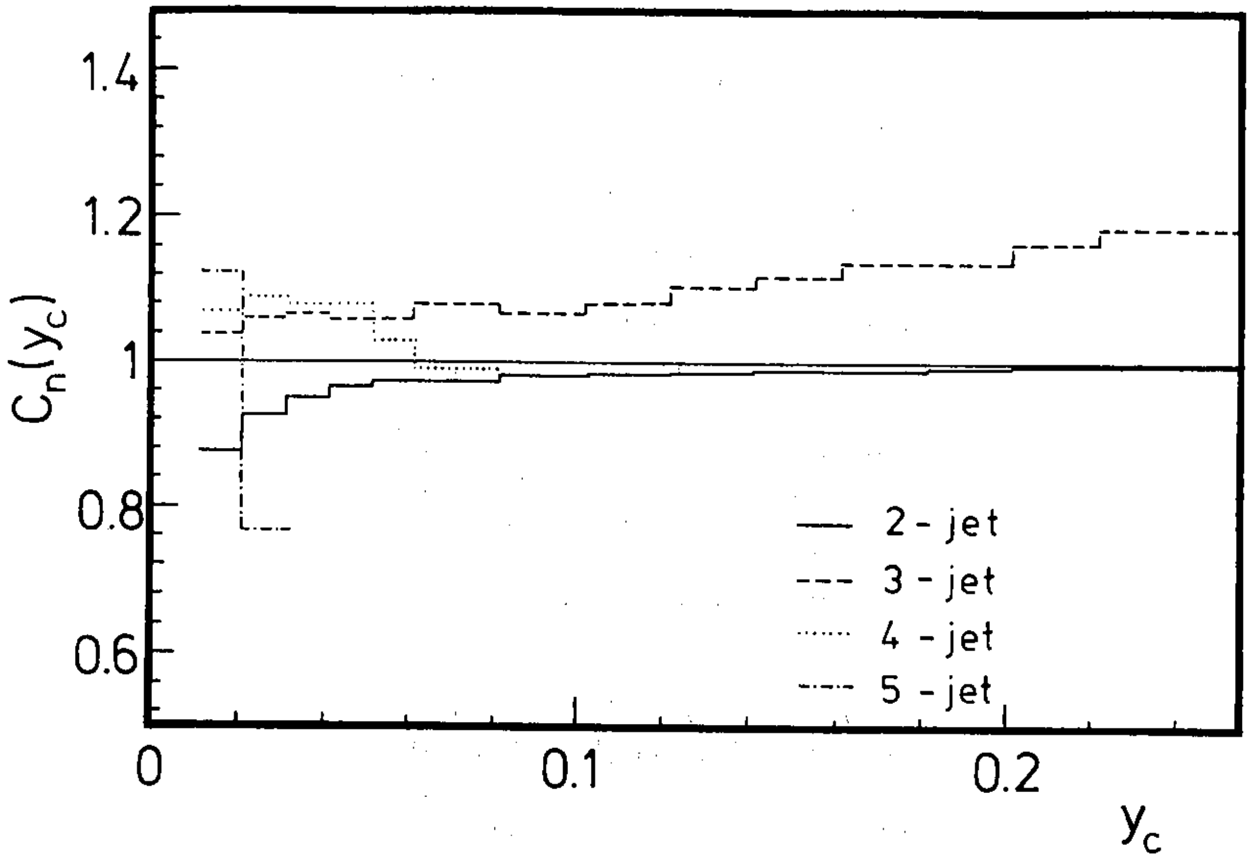
Table 3: Fits to D_{2-jet}, (R_{2-jet})

μ^2	$\Lambda_{\overline{MS}}$ [MeV]	χ^2/NDF
$1.0 \cdot Q^2$	203^{+24}_{-23}	47.31/19
$0.5 \cdot Q^2$	175^{+21}_{-19}	45.22/19
$0.1 \cdot Q^2$	128^{+15}_{-14}	39.33/19
$0.05 \cdot Q^2$	114^{+13}_{-12}	36.29/19
$0.01 \cdot Q^2$	94^{+10}_{-9}	27.74/19
$0.005 \cdot Q^2$	90^{+9}_{-9}	23.18/19
$0.001 \cdot Q^2$	92^{+9}_{-9}	15.01/19
$0.0005 \cdot Q^2$	96^{+10}_{-9}	23.75/19
$1.0 \cdot y\,Q^2$	101^{+11}_{-10}	32.11/19
$0.5 \cdot y\,Q^2$	94^{+10}_{-10}	28.12/19
$0.1 \cdot y\,Q^2$	88^{+9}_{-9}	18.43/19
$0.05 \cdot y\,Q^2$	90^{+9}_{-9}	17.67/19
$0.025 \cdot y\,Q^2$	90^{+9}_{-9}	30.40/19

Table 4: Fits to D_{2-jet}, (R_{2-jet}), D_{3-jet}, (R_{3-jet})

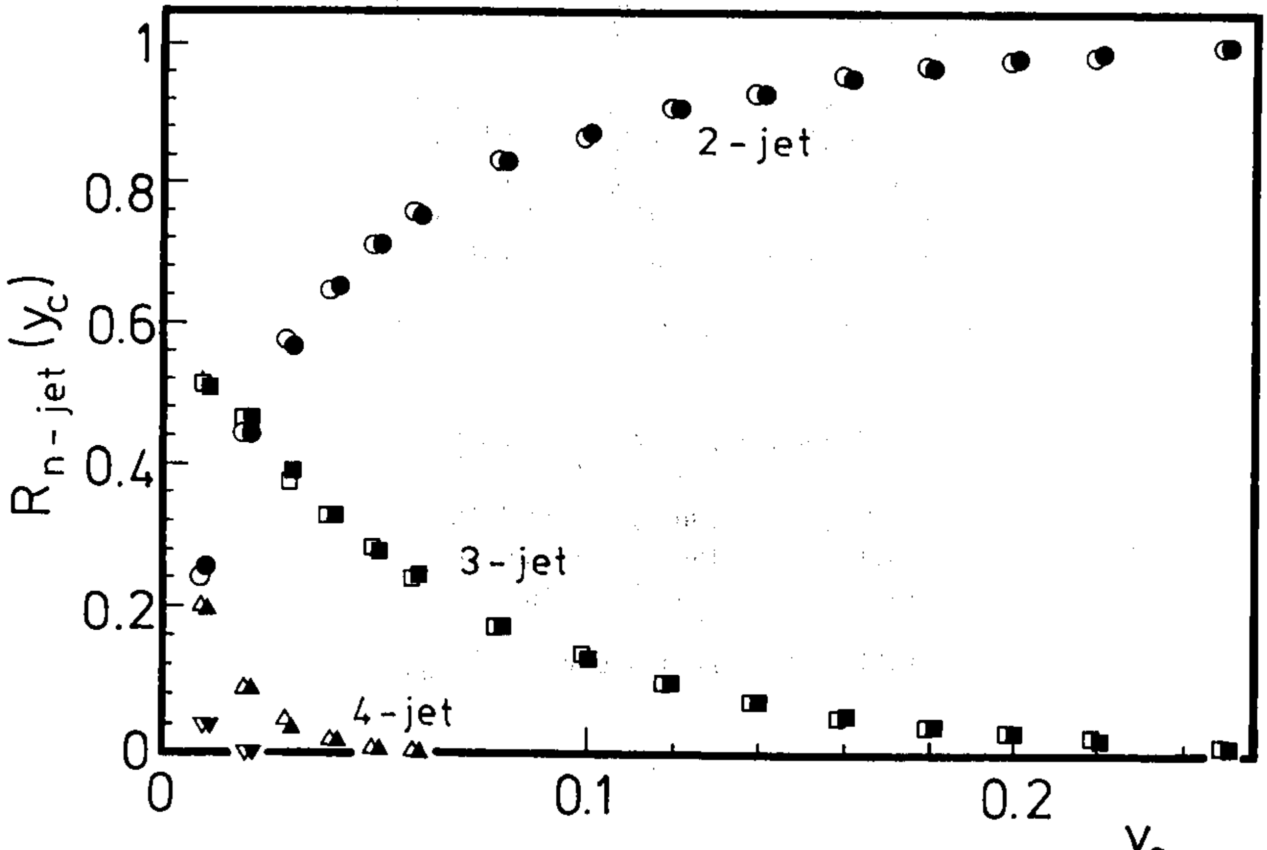
Scheme	$\Lambda_{\overline{MS}}$ [MeV]	χ^2/NDF
E	89^{+17}_{-15}	11.6/10
E0	190^{+35}_{-32}	11.7/10
KL'	180^{+33}_{-30}	10.9/10
p	344^{+61}_{-55}	10.8/10

Table 5: Scheme dependence of $\Lambda_{\overline{MS}}$



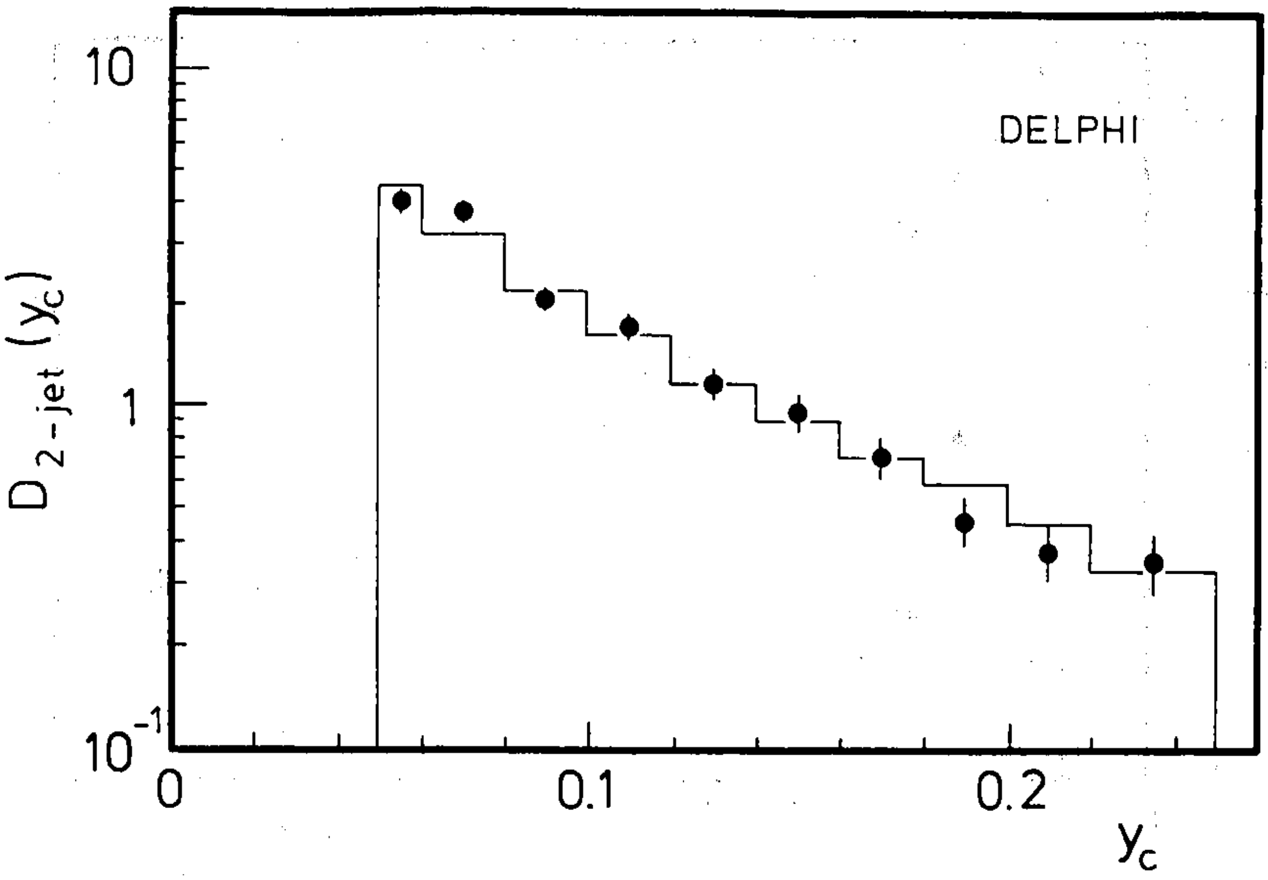
1.4
1.2
1
0.8
0.6
$C_n(y_c)$
0
0.1
0.2
y_c
2 - jet
3 - jet
4 - jet
5 - jet

Fig. 1



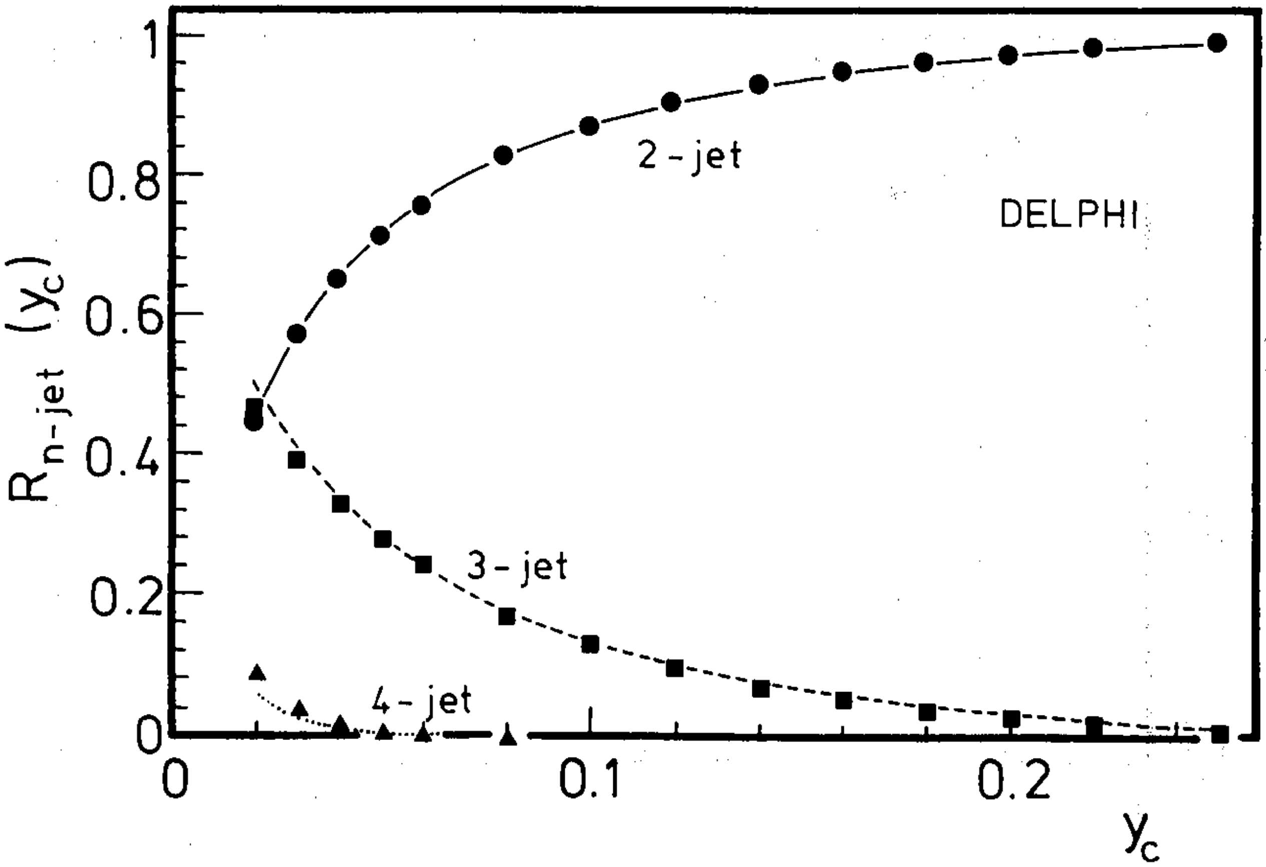
1
0.8
0.6
0.4
0.2
0
$R_{n-jet}(y_c)$
2 - jet
3 - jet
4 - jet
0.1
0.2
y_c

Fig. 2



DELPHI
10
1
10^{-1}
0
0.1
0.2
$D_{2-jet}(y_c)$
y_c

Fig. 3a



DELPHI
2 - jet
3 - jet
4 - jet
1
0.8
0.6
0.4
0.2
0
0.1
0.2
$R_{n-jet}(y_c)$
y_c

Fig. 3b

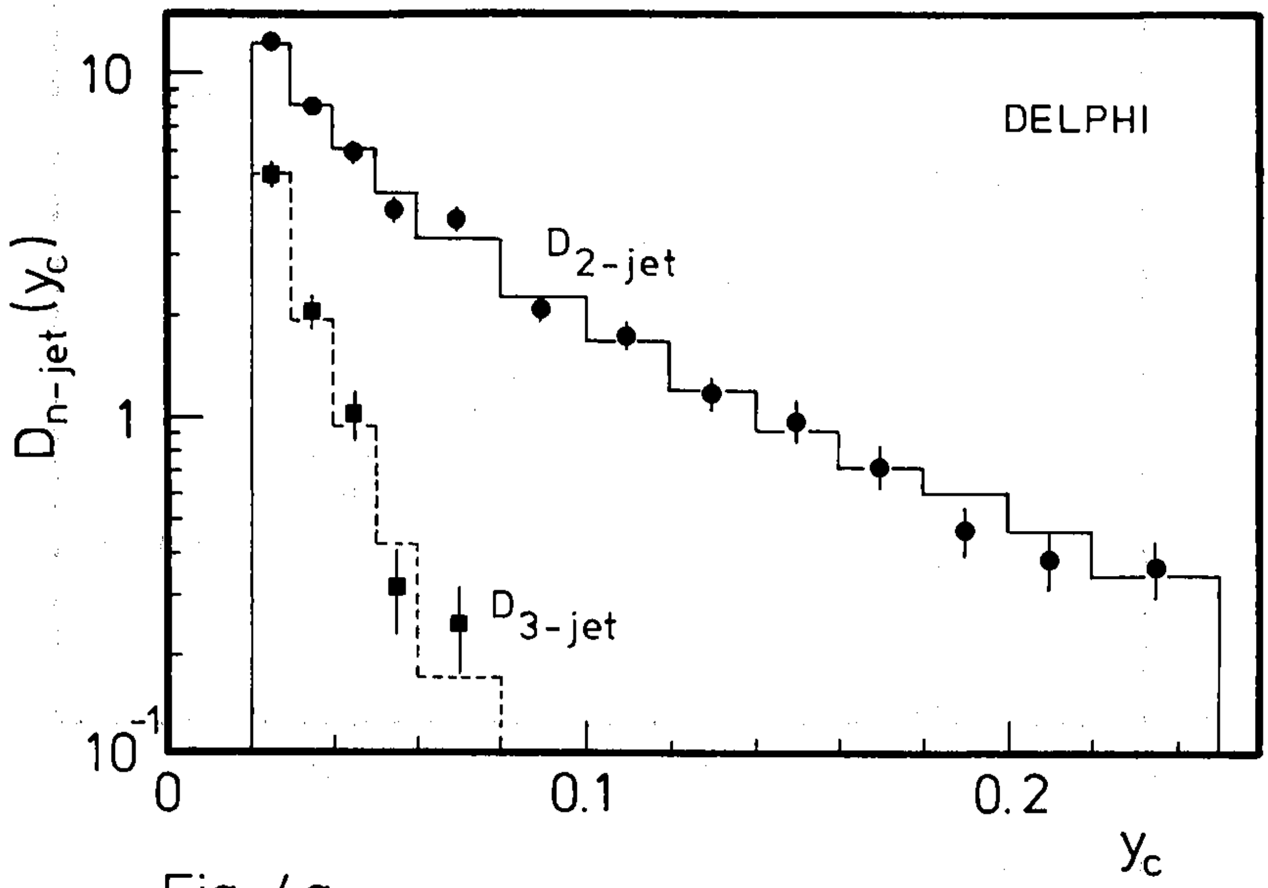


Fig. 4a

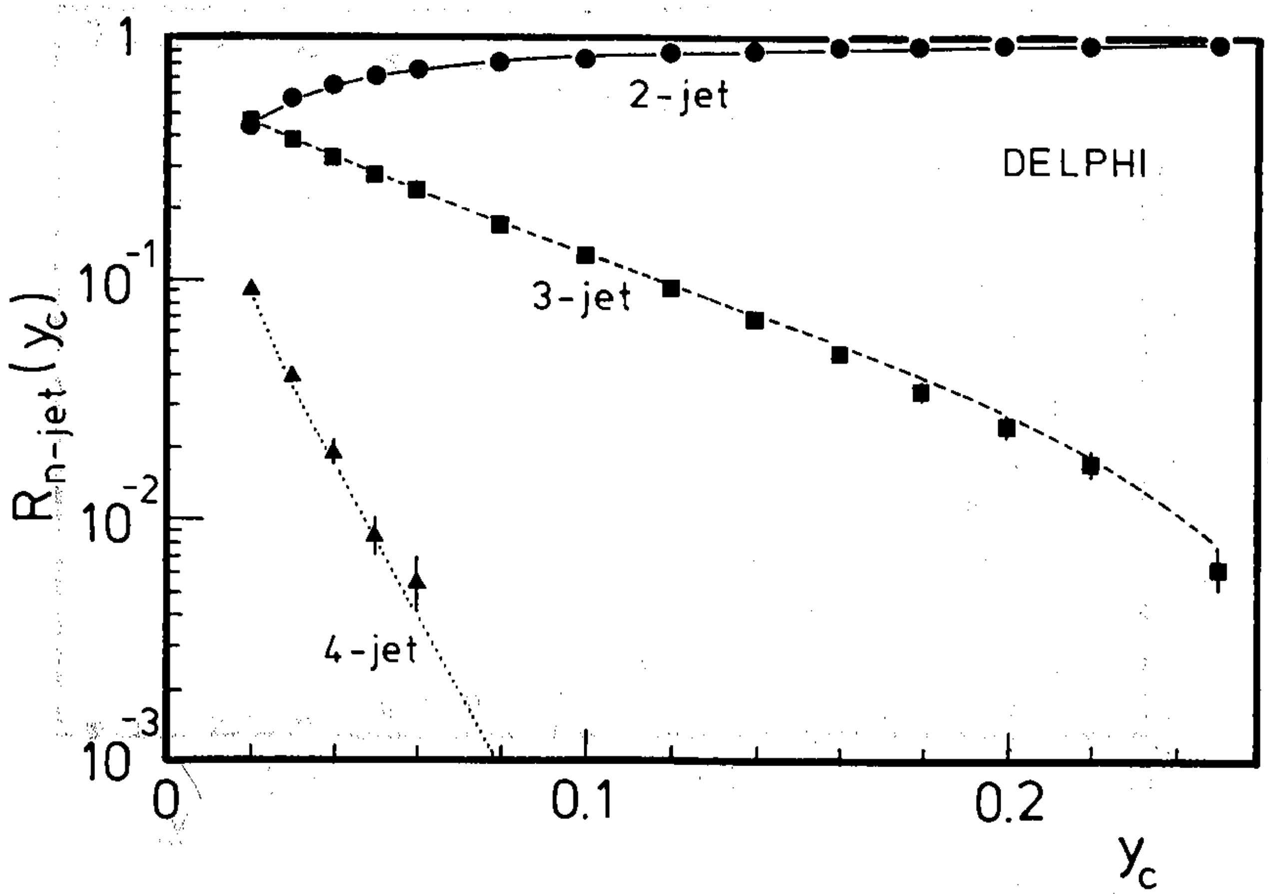


Fig. 4b

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