



DFTT-29/89
CERN-TH-5635/90

**SPECIAL KÄHLER GEOMETRY : AN INTRINSIC FORMULATION
FROM N=2 SPACE-TIME SUPERSYMMETRY ***

Leonardo Castellani

*Istituto Nazionale di Fisica Nucleare, Sezione di Torino
Via P. Giuria 1, I-10125 Turin, Italy*

Riccardo D' Auria

*Dipartimento di Fisica, Università di Padova and
Istituto Nazionale di Fisica Nucleare, Via Marzolo 8,
I-35131 Padua, Italy*

and

Sergio Ferrara

CERN Theoretical Division, 1211 Geneva 23, Switzerland

Abstract

N=2, 4D supergravity coupled to vector multiplets (1, 1/2, 0) is reanalysed in a geometrical setting. By requiring the closure of the supersymmetry transformation laws, we find a coordinate-free characterization of the manifold \mathcal{M} spanned by the scalar fields. The geometry of \mathcal{M} , called "special geometry", is relevant to compactified string theories, since it is common to the moduli spaces of Calabi-Yau threefolds and $c=9$ (2,2) superconformal field theories.

* This work is supported in part by the Ministero della Pubblica Istruzione, and by the Dept of Energy of U.S.A. under contract DOE-AT03-88ER40 384, TASK E.

In this letter we address the problem of formulating in an intrinsic way the geometry of scalar field configurations which occur in coupling N=2 vector multiplets to four-dimensional N=2 supergravity.

In supersymmetric theories, the manifold \mathcal{M} spanned by the scalar fields, viewed as coordinates on \mathcal{M} , is known to be constrained by supersymmetry. Thus, in N=1 4D supergravity coupled to chiral multiplets, \mathcal{M} is a restricted Kähler manifold, while in N=2 4D supergravity coupled to hypermultiplets \mathcal{M} is a quaternionic manifold [1,2].

Here we are concerned with N=2 supergravity coupled to vector (0, 1/2, 1) multiplets. The geometry of \mathcal{M} , referred to as "special geometry", has recently become of interest because of its relation to the moduli space of Calabi-Yau compactifications of superstring theories [3,4,5], or in more general terms to the so-called Zamolodchikov metric [6] for the moduli of (2,2) superconformal field theories.

A number of authors [3-10] have emphasized, in the recent past, that the metric of the moduli space of Calabi-Yau manifolds should not only be Kähler, as implied by N=1 space-time supersymmetry, but also "special Kähler", as implied by N=2 space-time supersymmetry when one regards (2,2) theories as possible vacua for type II superstrings.

The constraints on \mathcal{M} due to N=2 supersymmetry were found in [1,2], and were formulated in a particular coordinate system of \mathcal{M} , the "special gauge". More precisely, the metric of \mathcal{M} is constrained to be generated by a Kähler potential

$$G(z, \bar{z}) = -ln z^I N_{IJ} \bar{z}^J \quad I = 0, 1, \dots, n \quad (1)$$

with

$$N_{IJ}(z, \bar{z}) = \frac{1}{4} \frac{\partial}{\partial X^I} \frac{\partial}{\partial X^J} F(X) + h.c. \quad (2)$$

$$z^I = \frac{X^I}{X^0} = (1, z^i) \quad (3)$$

and where $F(X^I)$ is a homogeneous holomorphic function of degree 2 in n+1 variables X^I . This implies

$$F(X) = (X^0)^2 f(z), \quad f(z) \text{ analytic} \quad (4)$$

and the only freedom in the geometry of \mathcal{M} resides in the arbitrary analytic function $f(z)$.

The Riemann curvature of \mathcal{M} is computed from (1) to be [2]:

$$R_{i,j^*k^*l^*} = -2g_{j^*(i^*gk^*)l^*} - Y^{-2} Q_{ikm} \bar{Q}_{j^*l^*n^*} g^{mn^*} \quad (5)$$

with

$$Y = e^{-G} = z^I N_{IJ} \bar{z}^J = 2f + 2\bar{f} - (f_i - \bar{f}_{i^*})(z^i - \bar{z}^{i^*}) \quad (6)$$

$$Q_{ij^*k^*} = \frac{1}{4} \partial_i \partial_j \partial_k f(z) \quad (7)$$

$$(f_i \equiv \frac{\partial}{\partial z^i} f, \quad \partial_i \equiv \frac{\partial}{\partial z^i})$$

In the case of rigid N=2 supersymmetry, i.e. when one considers a system of n vector multiplets in a flat supergravity background, the Kähler potential of the scalar manifold \mathcal{M} takes the form

$$G = z^i \bar{f}_{i^*} + \bar{z}^{i^*} f_i \quad i = 1, \dots, n \quad (8)$$

and the corresponding Riemann curvature is:

$$R_{i,j^*k^*l^*} = -Q_{ikm}(z) \bar{Q}_{j^*l^*n^*}(\bar{z}) g^{mn^*} \quad (9)$$

Formulas (5), (6) and (7) hold only in a particular coordinate system of \mathcal{M} : the metric and the curvature computed through eq. (1) are not covariant under reparametrizations of the Kähler manifold \mathcal{M} ; they are only covariant under rigid $Sp(2(n+1), R)$ symplectic transformations [4] acting on the $2(n+1)$ vectors $(X^I, \frac{\partial}{\partial X^I} F(X))$. Therefore g_{ij^*} and $R_{i,j^*k^*l^*}$ can be expressed in terms of an analytic function $f(z)$ only in some parametrizations of \mathcal{M} .

In this letter, we present the covariant generalization of the above formulas. By requiring the closure of the N=2 supersymmetry algebra, we arrive at the following expression for the curvature of \mathcal{M} :

$$R_{i,j^*k^*l^*} = -2g_{j^*(i^*gk^*)l^*} - e^{2G} T_{ikm} \bar{T}_{j^*l^*n^*} g^{mn^*} \quad (10)$$

where T_{ikm} is a "good" covariant tensor on \mathcal{M} , totally symmetric and analytic, given in terms of covariantly holomorphic scalar functions $L^I(z)$ appearing in the supersymmetry transformation laws of the spin 1 fields.

In heterotic string theory (or actually in a compactification of 10D anomaly-free supergravity [11]), the tensor T_{ij^*k} is related to the Yukawa couplings of the E_6 families [7,10,12,13,14]. A derivation of formula (8) for the moduli space of an abstract (2,2) superconformal field theory has recently been given in ref. [10]. In the large radius limit of Calabi-Yau manifolds, explicit formulas for the holomorphic function $f(z)$ have been found as integrals over Calabi-Yau spaces in [4,8,9,13,14]. In this case the moduli space is a direct product $\mathcal{M} = \mathcal{M}_A \times \mathcal{M}_B$, where \mathcal{M}_A and \mathcal{M}_B are two special Kähler manifolds as defined by formulas (1)-(7), with Y functions respectively given by [8,9]

$$Y_A = \int_{C_3} J \wedge J \wedge J, \quad Y_B = i \int_{C_3} \Omega \wedge \bar{\Omega} \quad (11)$$

where J and Ω are (1,1) and (3,0) forms parametrized by the real (1,1) and complex (2,1) moduli respectively. The corresponding holomorphic functions $f(z)$ are given by

$$f^A = \int_{C_3} J \wedge J \wedge J, \quad (12)$$

$$f^B = -\frac{i}{2} \int_{C_3} \hat{\Omega} \wedge (\alpha_0 + z^i \alpha_i) \quad (13)$$

where J is a complexified Kähler form

$$J = B + iJ = \sum_{i=1}^{h(1,1)} z^i V_i \quad (14)$$

(V_i is a basis in $H^{(1,1)}$ Dolbeault cohomology), and $\hat{\Omega}$ is a holomorphic three-form in projective coordinates

$$\hat{\Omega}(z) = (X^0)^{-1} \Omega(X^i), \quad z^I = \frac{X^i}{X^0} \quad (15)$$

α_0, α_i are (with β^0, β^i) a cohomology basis in H^3 dual to the homology cycles A_a, B^a . If one introduces homogeneous coordinates for the moduli spaces $\mathcal{M}_A, \mathcal{M}_B$, the Y functions can be rewritten as

$$Y_{A(B)} = \bar{X}^I F_I^{A(B)} + X^I \bar{F}_I^{A(B)} \quad (16)$$

where

$$F^A(B)(X) = (X^0)^2 f^A(B) \quad (17)$$

String arguments and space-time supersymmetry imply that eq. (12) is only valid in σ -model perturbation theory, while eq. (13) is an exact formula. On the other hand all the equations (1)-(17) are general, since they are merely a consequence of $N=2$ space-time supersymmetry or alternatively of world-sheet superconformal invariance.

Although we only discuss the local properties of special geometry, global properties can be discussed as well. Actually these properties, studied for particularly simple systems, such as superstrings propagating on orbifolds, indicate that the moduli space is a special Kähler orbifold [15] rather than a manifold, the fixed points corresponding to the enhanced gauge symmetry of heterotic string compactifications. The Kähler potential, because of its relation to the superpotential, is related to the theory of modular forms of the duality group of the background string parameters.

Recently some attempts have been made [10,16] to understand in an intrinsic way the special geometry of Calabi-Yau moduli space, as well as its relation to general properties of superconformal field theories. In all these approaches the special geometry

is connected to some underlying properties of the internal conformal field theory (or Calabi-Yau space).

Our point of view is somewhat orthogonal, in the sense that we only require the closure of the supersymmetry transformations of the $N=2$ vector multiplets, in a geometrical framework that ensures from the start the covariance under reparametrizations of \mathcal{M} .

Our method allows us to define special geometries both for $N=2$ rigid and local supersymmetry. These geometries are actually very different: for example a flat metric is special only in the rigid case, in the sense that it satisfies eq. (9) but not eq. (5). It corresponds in a special gauge to a choice of $f(z)$ quadratic in the coordinates.

We recall that also for the $N=1$ chiral multiplets, and $N=2$ hypermultiplets, rigid and local supersymmetry yield different geometries for the scalar manifold. Indeed for $N=1$ chiral multiplets rigid supersymmetry requires a Kähler structure, while local supersymmetry requires \mathcal{M} to be a Hodge manifold. For hypermultiplets the rigid case corresponds to hyper-Kähler geometry, while the local case corresponds to quaternionic manifolds [17,18,19]

In order to find an intrinsic definition of special $N=2$ geometry, covariant under holomorphic reparametrizations of \mathcal{M} , it is convenient to adopt the geometrical methods of refs. [20,21]. We introduce the basic fields of $N=2$ supergravity, and their associated curvatures, as differential forms in superspace. We take the one-forms V^a, ψ_A as a basis for $N=2$ superspace. Their spacetime restrictions $V_\mu^a(x), \psi_{A\mu}(x)$ are the ordinary vielbein and gravitino fields. The curvatures of $N=2$ supergravity coupled to the vector multiplets (F^i, λ^{iA}, z^i) are

$$R^{ab} = d\omega^{ab} - \omega^a c \omega^{cb} \quad (18)$$

$$R^a = \mathcal{D}V^a - i\psi_A \gamma^a \psi^A, \quad \mathcal{D}V^a \equiv dV^a - \omega^{ab} V^b \quad (19)$$

$$\rho_A = d\psi_A - \frac{1}{4} \omega^{ab} \gamma_{ab} \psi_A + \frac{i}{2} Q \psi_A \quad (20)$$

$$F^I = dA^I + \varepsilon_{AB} L^I \bar{\psi}^A \psi^B + \varepsilon^{AB} L^{*I} \bar{\psi}_A \psi_B \quad (21)$$

$$\nabla \lambda^{iA} = d\lambda^{iA} - \frac{1}{4} \omega^{ab} \gamma_{ab} \lambda^{iA} - \frac{i}{2} Q \lambda^{iA} + \left\{ \begin{matrix} i \\ j \end{matrix} \right\} \lambda^{jA} \quad (22)$$

$$dz^i = dz^i \quad (23)$$

$$d\bar{z}^{i*} = d\bar{z}^{i*} \quad (24)$$

The index I runs on $0, 1, \dots, n$ values (1, ..., n in rigid supersymmetry), A runs on 1, 2 and its position indicates the positive or negative chirality:

$$\begin{aligned} \gamma_5 \psi_A &= \psi_A & \gamma_5 \lambda^{iA} &= \lambda^{iA} \\ \gamma_5 \bar{\psi}^A &= -\bar{\psi}^A & \gamma_5 \lambda^{i*} &= -\lambda^{i*} \end{aligned} \quad (25)$$

The covariant derivative ∇ includes the $U(1)$ Kähler connection Q :

$$Q = \frac{1}{2i}(\partial_i G dz^i - \bar{\partial}_i G d\bar{z}^i) \quad (26)$$

Q is a $U(1)$ connection, defining a $U(1)$ bundle on the Kähler space \mathcal{M} , absent in the rigid case. The fields have the following $U(1)$ charges :

$$\begin{aligned} [V^a] &= 0 & [F^I] &= 0 & [L^I] &= 1 \\ [\psi_A] &= \frac{1}{2} & [\lambda^{iA}] &= -\frac{1}{2} & [L^{*I}] &= -1 \\ [\psi^A] &= -\frac{1}{2} & [\lambda_A^i] &= \frac{1}{2} & [z^i] &= 0 \end{aligned} \quad (27)$$

Eqs. (18)-(21) define curvature two-forms, while (22)-(24) are curvature one-forms for the z^i, λ^{iA} zero-forms.

The definitions (18)-(24) imply the Bianchi identities:

$$\mathcal{D}R^{ab} = 0 \quad (28)$$

$$\mathcal{D}R^a + i\bar{\rho}_A \gamma^a \psi^A - i\bar{\psi}_A \gamma^a \rho^A + R^{ab} \gamma^b = 0 \quad (29)$$

$$\mathcal{D}\rho_A + \frac{1}{4} R^{ab} \gamma_{ab} \psi_A - \frac{i}{2} R[Q] \psi_A + \frac{i}{2} Q \rho_A = 0 \quad (30)$$

$$dF^I - 2\bar{\rho}_A \psi_B \varepsilon^{AB} L^I - 2\varepsilon_{AB} \bar{\rho}^A \psi^B L^I - \varepsilon^{AB} \bar{\psi}^A \psi^B \nabla L^I - \varepsilon_{AB} \bar{\psi}^A \psi^B \nabla L^I = 0 \quad (31)$$

$$\nabla^2 \lambda^{iA} + \frac{1}{4} R^{ab} \gamma_{ab} \lambda^{iA} + \frac{i}{2} R[Q] \lambda^{iA} + R[\Gamma]^i{}_j \lambda^{jA} = 0 \quad (32)$$

$$d^2 z^i = d^2 \bar{z}^i = 0 \quad (33)$$

where $R[Q] = dQ$ and $R[\Gamma]^i{}_j$ is the Riemann curvature of \mathcal{M} .

When the curvatures are expanded in a V, ψ basis, the compatibility of their parametrizations with the Bianchi identities yields the dynamics of the theory, i.e. the field equations, and fixes the geometry of the scalar manifold in an intrinsic (coordinate independent) way.

The parametrization of the curvatures of the matter multiplets is obtained by expanding the forms defined in (21)-(24) on the V, ψ basis as follows:

$$F^I = F_{ab}^I V^a V^b + (f_i^I \bar{\lambda}^{ia} \gamma^a \psi^B \varepsilon_{AB} - f_i^I \bar{\lambda}^i{}_A \gamma^a \psi^B \varepsilon^{AB}) V^a \quad (34)$$

$$\nabla \lambda^{iA} = (\nabla_a \lambda^{iA}) V^a + i Z_a^i \gamma^a \psi^A + (G^+)^i{}_{ab} \gamma^{ab} \psi^B \varepsilon^{AB} + Y^{iAB} \psi_B \quad (35)$$

$$dz^i = Z_a^i V^a + \bar{\psi}_A \lambda^{iA} \quad (36)$$

$$d\bar{z}^i = Z_a^i V^a + \psi^A \lambda_A^i \quad (37)$$

where the new fields appearing as coefficients in these expansions, i.e. $f_i^I, f_i^*, (G^+)^i{}_{ab}$ and Y^{iAB} are determined below as functions of the matter fields.

We recall that the curvature parametrizations encode the transformation rules of the fields $*$, whereas the Bianchi identities ensure the closure of these transformations.

The constraints due to the closure of the $N=2$ supersymmetry algebra are found simply by substituting (34)-(37) into the Bianchi identities (31)-(33).

The spin-1 Bianchi identity (32) yields:

$$f_i^I(z) = i \nabla_i L^I(z) \equiv i \partial_i L^I + i Q_i L^I, \quad I = 0, 1, \dots, n \quad (38)$$

$$\nabla_{i^*} L^I = 0 \Rightarrow L^I \text{ is covariantly holomorphic} \quad (39)$$

$$\nabla_{j^*} f_i^I = g_{ij^*} L^I \quad (40)$$

$$\frac{1}{2} \nabla_{(j} f_{i^*)}^I - f_{i^*}^I C_{j^*}^k - f_{i^*}^I C_{j^*}^k = 0 \quad (41)$$

$$16 f_i^I (G^+)^i{}_{ab} = 4i (F_{ab}^I + \frac{1}{2i} \varepsilon_{abcd} F_{cd}^I) + \nabla_{(j} f_{i^*)}^I \bar{\lambda}^{iA} \gamma^{ab} \lambda^{jB} \varepsilon_{AB} \quad (42)$$

where $C_{j^*}^k, C_{j^*}^k$ are three-index tensors, symmetric in j, i , entering the definition of Y^{iAB} in eq. (33):

$$Y^{iAB} = C_{j^*}^i \lambda_C^{j^*} \lambda_D^{k^*} \varepsilon^{AC} \varepsilon^{BD} + C_{j^*}^i \bar{\lambda}^{jA} \lambda^{kB} \quad (43)$$

The interesting constraints come from the compatibility conditions for the gaugino curvature, i.e. from the Bianchi identity (30). One obtains:

$$C_{j^*}^i = 0 \quad (44)$$

$$\nabla_{i^*} C_{j^*}^k = 0 \quad (45)$$

$$\nabla_{[i^*} C_{j^*}^k = 0 \quad (46)$$

From eqs. (41) and (38) we deduce

$$C_{j^*}^i = \frac{1}{2} f_A^{i^*} \nabla_j f_k^A, \quad f_A^{i^*} = \text{inverse of } f_{i^*}^A, \quad A = 1, \dots, n \quad (47)$$

Eq. (45) yields, after use of (47), the constraint on the curvature of \mathcal{M} :

$$R_{ij^*k^*l^*} = -2g_{j^*(i} g_{k^*)l^*} - C_{ikm} \bar{C}_{j^*i^*n^*} g^{mn^*} \quad (48)$$

where $C_{ikm} \equiv g_{i^*l^*} C_{km}^{l^*}$ is totally symmetric. From (46) we see that

$$\nabla_{[i^*} C_{j^*}^k = 0 \quad (49)$$

implying that $C_{ij^*k^*}$ is the third covariant derivative of a function S :

* The supersymmetry transformations are obtained after replacing ψ^A by the supersymmetry parameter ε^A . For example: $\delta_{sup} z^i = \bar{\varepsilon}_A \lambda^{iA}$.

$$C_{ijk} = \nabla_i \nabla_j \nabla_k S \quad (50)$$

No other restrictions are obtained from Bianchi identities.

Note that C_{ikm} has $U(1)$ weight $=2$, and is covariantly holomorphic because of eq. (15). It is easy to define a holomorphic tensor T_{ikm} :

$$\begin{aligned} T_{ikm}(z) &\equiv e^{-G} C_{ikm} \\ \partial_i T_{ikm} &= 0 \end{aligned} \quad (51)$$

so that the Riemann curvature can finally be expressed as in eq. (10). Thus we have achieved the covariant generalization of the formula (5).

What are the metrics satisfying eq. (48) ?

Let us address this question in the rigid case, i.e. when the supergravity curvatures and the $U(1)$ connection Q_i are vanishing. Almost all the consistency conditions due to Bianchi identities retain the same form as in the local case. The range of the index I is limited to $1, \dots, n$. The only modifications are in formula (40), which becomes:

$$\partial_i f_j^i = 0 \quad (40')$$

(f_j^i is holomorphic) and in equation (48) for the curvature of \mathcal{M} , which loses the $g_{j^*(i)k^*}$ term:

$$R_{ij^*k^*l^*} = -C_{ikm}(z) \bar{C}_{j^*l^*n^*}(\bar{z}) g^{mn^*} \quad (48')$$

It is easy to show that the Kähler potential

$$Y = L^I \bar{F}_I + \bar{L}^I F_I \quad (52)$$

with

$$F_I \equiv \frac{\partial}{\partial L^I} F(L), \quad F : \text{analytic function of } L^I \quad (53)$$

yields the metric

$$g_{ij^*} = f_i^I f_j^{J^*} (F_{IJ} + \bar{F}_{IJ}) \quad (54)$$

which indeed satisfies the covariant generalization of eq. (9) given in eq. (48').

The three-index symmetric tensor C_{ijk} takes the form:

$$C_{ijk} = -f_i^I f_j^K f_k^J F_{IJK} \quad (55)$$

In the rigid case C_{ijk} is indeed holomorphic. Moreover, the function S of formula (50) is found to be :

$$S = L^A F_A - F + \frac{1}{2} L^A L^B \bar{F}_{AB} + H(\bar{z}) \quad (56)$$

with an $H(\bar{z})$ arbitrary antiholomorphic function.

The special gauge (8)-(9) corresponds to choosing

$$L^I(z) = z^I \Rightarrow f_k^I = \delta_k^I \quad (57)$$

so that from eq. (55):

$$C_{ijk} = -F_{ijk} = -\partial_i \partial_j \partial_k F \quad (58)$$

A detailed discussion in the case of local $N=2$ supersymmetry is given in ref. [22]. Here we summarize the results.

The metric that solves eq. (48) is :

$$g_{ij^*} = -\frac{1}{4} f_i^I f_j^{J^*} (F_{IJ} + \bar{F}_{IJ}) \quad (59)$$

where

$$F_{IJ} = \frac{\partial}{\partial L^I} \frac{\partial}{\partial L^J} F(L) \quad (60)$$

and $F(L)$ is a homogeneous analytic function of degree 2 in the $n+1$ variables L^I . These variables satisfy the constraint:

$$\frac{1}{4} L^I \bar{L}^J (F_{IJ} + \bar{F}_{IJ}) = 1 \quad (61)$$

The metric (59) is a Kähler metric with potential given by

$$G = -ln \frac{1}{4} L^I \bar{L}^J (F_{IJ} + \bar{F}_{IJ}) \quad (62)$$

The special gauge (1) used in the tensor calculus approach corresponds to the particular choice of $L^I(z)$:

$$\frac{L^I(z)}{L^0(z)} = z^I, \quad I = 0, 1, \dots, n \quad (63)$$

Indeed in this gauge the Kähler potential (62) reduces to the form given in (1), and we recover the results of refs. [1,2].

Acknowledgements

It is a pleasure to thank D. Franco, P. Fré and F. Gliozzi for interesting discussions.

References

- [1] B. de Wit, P.C. Lowers, R. Philippe, S. Q. Su and A. Van Proeyen, Phys. Lett. B134 (1984) 37; B. de Wit and A. Van Proeyen, Nucl. Phys. B245 (1984); B. de Wit, P. G. Lowers and A. Van Proeyen, Nucl. Phys. B255 (1985) 569; J. P. Derendinger, S. Ferrara, A. Masiero and A. Van Proeyen, Nucl. Phys. B140 (1984) 307.
- [2] E. Cremmer, C. Kounnas, A. Van Proeyen, J. P. Derendinger, S. Ferrara, B. de Wit and L. Girardello, Nucl. Phys. B250 (1985) 385.
- [3] N. Seiberg, Nucl. Phys. B303 (1988) 286.
- [4] S. Ferrara, S. Cecotti and L. Girardello, Int. Journal of Modern Physics A, vol. 4 (1989) 2475; Phys. Lett. B213 (1988) 443.
- [5] P. Candelas, G. Horowitz, A. Strominger and E. Witten, Nucl. Phys. B258 (1985) 46.
- [6] A. B. Zamolodchikov, JEPT Lett. 43 (1986) 731.
- [7] L. Dixon, V. S. Kaplunovsky and J. Louis, Nucl. Phys. B239 (1990) 27.
- [8] S. Ferrara and A. Strominger, CERN-TH-5291/89, UCLA-89-TEP/6, (1989)
- [9] P. Candelas, P. S. Green and T. Hubsch, UTTG-17-89 (Austin report) (1989).
- [10] V. Periwal and A. Strominger, UCSB preprint NSF-ITP-89-144, (1989)
- [11] R. D' Auria and P. Fré, Phys. Lett. B200 (1988) 63; R. D' Auria, P. Fré, M. Raciti and F. Riva, Int. Jour. of Mod. Phys. A3 (1988) 953.
- [12] M. Dine, P. Huet and N. Seiberg, Nucl. Phys. B322 (1989) 301.
- [13] A. Strominger, Phys. Rev. Lett. 55 (1985) 2547; A. Strominger and E. Witten, Comm. Math. Phys. 101 (1985) 341; A. Strominger, in Proceedings of the Santa Barbara Workshop, "Unified String Theory", World Scientific (edited by M. Green and D. Gross), (1985).
- [14] P. Candelas, Nucl. Phys. B298 (1988) 458.
- [15] R. Dijkgraaf, E. Verlinde and H. Verlinde, preprint THU-87/30 (Princeton report); V. P. Nair, A. Shapere, A. Strominger and F. Wilczek, Nucl. Phys. B287 (1987) 402; A. Shapere and F. Wilczek, Nucl. Phys. B320 (1989) 669; A. Giveon, E. Rabinovici and G. Veneziano, Nucl. Phys. B322 (1989) 167; S. Ferrara, D. Lüst, A. Shapere and S. Theisen, Phys. Lett. 225B (1989) 363; J. Lauer, J. Mas, H. P. Nilles, Phys. Lett. B226 (1989) 251; W. Lerche, D. Lüst and N. P. Warner, Phys. Lett. B231 (1989) 417; E. J. Chun, J. Mas, J. Lauer, and H. P. Nilles, Phys. Lett. B233 (1989) 141; C. Vafa, Harvard preprint HUTP-89/A021(1989); S. Ferrara, D. Lüst, and S. Theisen, Phys. Lett. B233 (1989) 147; B. Greene, A. Shapere, C. Vafa and S.T. Yau, Harvard Preprint HUTP-89/A047 and IASSNS-HEP-89/47; J. H. Schwarz, Caltech preprint CALT-68-1581/19.
- [16] A. Strominger, in preparation.
- [17] J. Bagger and E. Witten, Phys. Lett. 115B (1982) 202.
- [18] J. Bagger and E. Witten, Nucl. Phys. B222 (1983) 1.
- [19] B. Zumino, Phys. Lett. 87B (1979) 203.
- [20] L. Castellani, R. D' Auria and P. Fré, Seven Lectures on the group manifold approach to supergravity and the spontaneous compactification of extra dimensions, Proc. XIX Winter School, Karpacz 1983, ed. B. Milewski (World Scientific, Singapore);
- L. Castellani, R. D' Auria and P. Fré, "Supergravity and Superstrings : a geometric perspective", World Scientific (1989), in press.
- [21] R. D' Auria, P. Fré, E. Maina and T. Regge, Ann. of Physics 135 (1981) 237.
- [22] L. Castellani, R. D' Auria and S. Ferrara, in preparation