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STABILITY OF HOLLOW BEAMS IN LONGITUDINAL PHASE SPACE

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Abstract. In synchrotrons, bunched beams which are hollow in longitudinal phase space are known to be unstable. We have also found this to be true in simulations which include only the space charge force. Mode-coupling theory does predict instability but gives thresholds which are much higher than those found from simulations.

THEORY

A common way to proceed is to use the Vlasov equation for the density function, Ψ, in longitudinal phase space (coordinates ϕ and $\dot{\phi}$ where ϕ is the rf phase). We neglect synchrotron tune spread due to rf wave nonlinearity and substitute for $\ddot{\phi}$ the equation of longitudinal motion containing the incoherent force, $\omega_i^2\phi$, and the collective effect by way of the fourier components of the beam's line density, λ_p, multiplied by the longitudinal impedance at that frequency, Z_p. Ψ is then broken down into an initial distribution, $\Psi_0(r)$ (r and θ are the polar coordinates in longitudinal phase space), and a harmonic perturbation of frequency ω and only terms linear in the perturbation or the driving term are retained. Zotter[1] and others have shown that this results in

$$\lambda_q = -j\xi \sum_{p=-\infty}^{\infty} \frac{Z_p}{p} H_{pq}\lambda_p \tag{1}$$

$$\text{where} \quad H_{pq} = \int_0^{\infty} \frac{d\Psi_0}{dr}\Big[\sum_{m=-\infty}^{\infty} \frac{m}{\omega/\omega_i - m} J_m(pr)J_m(qr)\Big]\,dr \quad \text{and} \quad \xi = \frac{2\pi h I}{V\cos\phi_s}\,.$$

The J_m are Bessel functions, and h, I, and V are, respectively, the harmonic number, the beam current, and the rf voltage. The cosine of the synchronous phase, ϕ_s, is negative below transition and positive above.

If the current is small enough, solutions of (1) cluster around integer multiples of ω_i. In that case only one of the terms in the sum in H_{pq} will dominate for each solution so the summation can be dropped and we get the usual (non-mode-coupling) expression for tune shifts as derived by Sacherer[2]. For larger current, modes will couple and instability can occur even for purely reactive impedance. Zotter[1] has converted the difficult sum in (1) into a double integral. Moreover, with the choice of the following hollow distribution,

$$\Psi_0(r) = \frac{r^2}{4\pi\sigma^4}\exp\Big(\frac{-r^2}{2\sigma^2}\Big)\ , \tag{2}$$

one of the integrals can be solved analytically.

To find the modes, ω was stepped through a series of values: for each value, the eigenvalues of H_{pq} (Z_p/p is a constant here since we are considering only space charge forces) were found and these were inverted to find the intensities at which modes of that frequency occur. The matrix was truncated at some value $|p| = \hat{p}$ and values of 5, 10, 20, and 50 were used for $\hat{p}$ to be sure that the results had converged. It may be remarked that a finite $\hat{p}$ is not non-physical but corresponds to the case where the impedance falls abruptly to zero at the frequency $\hat{p}\omega_{rf}$. For the sake of comparison this calculation was done both for a gaussian and for the hollow distribution (2). The results are shown in Figs. 1 and 2, respectively. In these figures, the ordinate is frequency in units of the zero intensity synchrotron frequency (ω_0) and the abscissa is related to intensity; negative intensities are for spacecharge impedance below transition (or inductive impedance above transition) and positive intensities are for spacecharge impedance above transition. Modes can be characterized by two indices: their frequencies at zero intensity give the azimuthal index (m =1 for dipole, 2 for quadrupole, etc.) and different frequencies for a given m correspond to different radial modes[3].

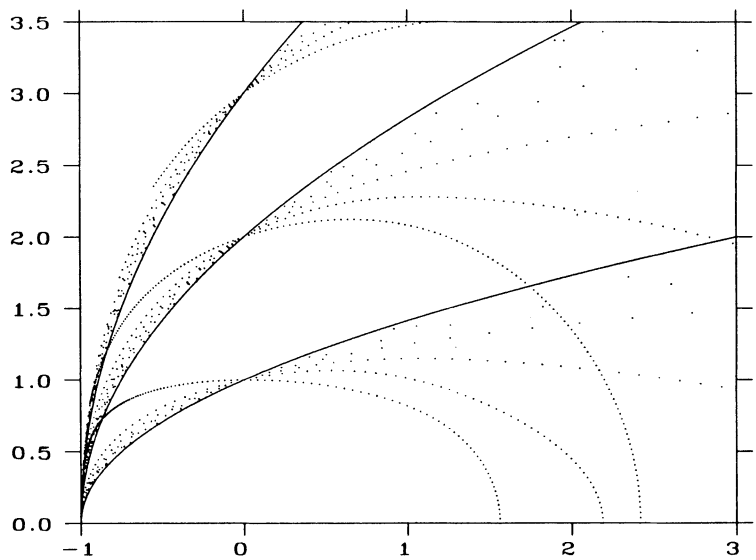

FIGURE 1 Coherent mode frequencies (dots) as a function of fractional shift in the square of the incoherent frequency (averaged over the bunch) for the case of a gaussian bunch. Solid lines are 1, 2, and 3 times the incoherent frequency. Matrix size: $\hat{p} = 10$.

In the case of the gaussian (Fig.1), higher radial modes are shifted down (below transition) compared with the most rigid mode and converge to m times the incoherent frequency. For this case, the x-axis is the fractional shift in the square of the incoherent tune averaged over the bunch. Hence, the equation of the incoherent frequency is simply $y = \sqrt{x+1}$. As one would expect, the rigid dipole mode frequency is stationary at low enough intensity. All modes disappear at $x = -1$: this is simply the case where the focusing force is cancelled by space charge. Above transition, there is a dipole mode-coupling ($m = 1, -1$) instability threshold at $x \simeq 1.5$ (actually converges to 1.41 at high enough $\hat{p}$). This can be considered as the 'negative mass' instability for bunched beams; it is not generally observed because high intensity machines tend to be inductive wall dominated above transition and space charge dominated below.

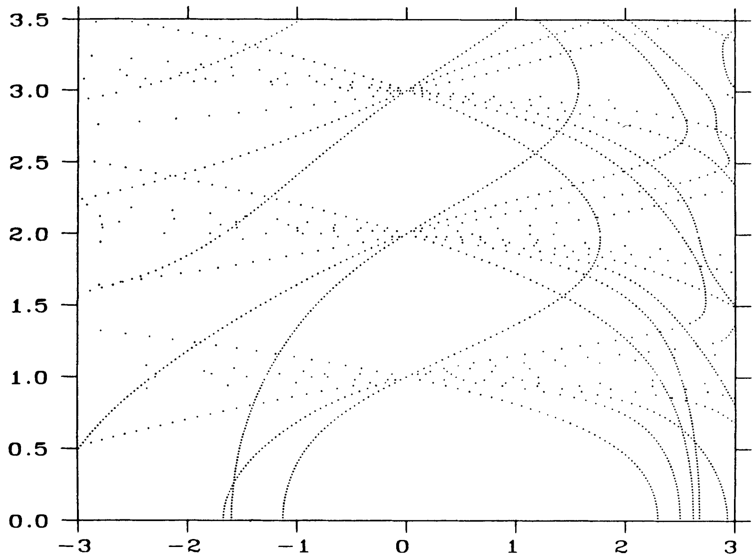

FIGURE 2 Coherent mode frequencies as a function of the fractional shift in the square of the incoherent frequency at zero synchrotron amplitude for the case of the hollow beam of equation (2). Matrix size: $\hat{p} = 10$.

In the hollow beam case (Fig.2), the x-axis is the fractional shift in the square of the incoherent tune of particles with vanishing amplitude (these are shifted **up** by space charge in a hollow beam). The x-axis cannot be normalized as in the gaussian case since the incoherent space charge tune shift practically vanishes when averaged over the beam. The main difference with Fig.1 is that here tune shifts of both signs appear for a given m. Below transition a dipole mode-coupling threshold occurs at $x \simeq 1.1$ ($x = 1.0$ at $\hat{p} = \infty$) which looks like the 'negative mass' instability in the gaussian case. This is understandable since the hole will have a negative mass character. Above transition, the dipole mode is not the first to go unstable: the lowest threshold appears to be given by $m = \hat{p} - 1, \hat{p} + 1$ coupling and converges to $x = 1$ for $\hat{p} = \infty$.

SIMULATIONS

The space-charge dynamics simulations were carried out with the LONG1D[4] tracking code. A phase-space ensemble of 6×10^4 macro-particles is 'binned' to give the bunch shape and this Fourier analysed and the harmonics processed to find the space-charge voltage, at each integration step. There are 5 such steps between each of 3 accelerating cavities.

The starting ensemble was prepared using a random number generator to produce the $r^2 \exp(-r^2)$ density distribution. No attempt was made to adjust the ensembles to provide space-charge matching. Machine parameters were taken for the KAON[5] Accumulator ring ; but the restoring force is linearised, with a voltage gradient of 565 kV per radian per turn. As a consequence, the separatrix is circular. The nominal Accumulator ring beam-current is 2 Amps circulating, equivalent to 10^{13} protons. The ring operates below transition.

A number of trials were performed with the beam current as variable. In addition, tests at fixed current, showed the results to be fairly insensitive to both the number of harmonics and the number of integration sub-steps. The progress of the instability is monitored by forming the dipole moment (D) for the ensemble.

$$D = \left[(\bar{\phi})^2 + \left(\frac{2\pi Q_s}{eV}\right)^2 (\bar{\Delta E})^2\right]^{1/2} \quad \text{where } \bar{\phi} = \frac{1}{N}\sum \phi_i \text{ and } \bar{\Delta E} = \frac{1}{N}\sum \Delta E_i \, .$$

The particle phase ϕ_i and ensemble moment D are measured in radians. The energy gain per turn eV and deviation from synchronous energy ΔE_i are measured in MeV. The synchrotron tune is Q_s, and number of macro-particles N.

Results

The evolution of dipole moment with turns is shown in figure 3. One turn is $\sim 1\ \mu$s. The cases shown are 1, 2, 4, 8 times the nominal current (I_0). Note how the dipole amplitudes increase with no sign of saturation for the cases $2I_0$ and above. This is because of the linear restoring force.

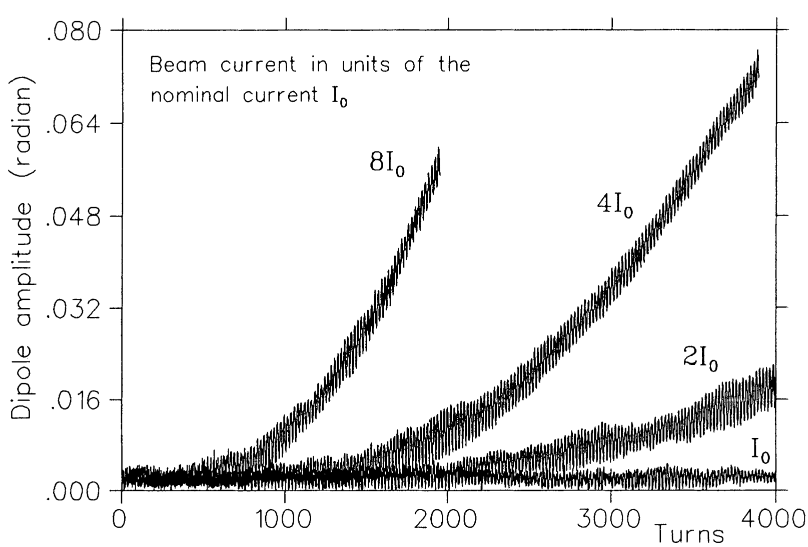

FIGURE 3 Dipole moments versus turns.

For beam currents below the nominal current (not shown), there is no detectable growth of the dipole amplitude within the 4 ms period, or 200 synchrotron oscillations. Consequently, the threshold is recognised as $\sim$ 4 Amps circulating current. This limit occurs when x, the fractional shift in the square of the synchrotron frequency for small amplitude particles, is only 13%. This is much smaller than the prediction from theory, namely $x = 1$.

Above transition

In one final test-case, the beam behaviour was compared above and below transition energy. 'Snap-shots' of the phase-space ensembles are presented in figures 4a,b. The beam current is 4 A in each case.

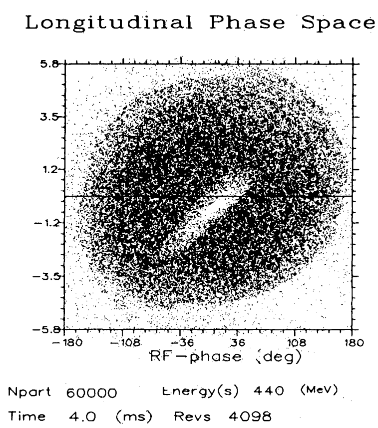

FIGURE 4a Above Transition.

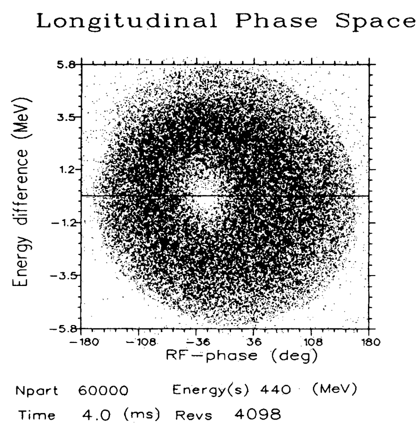

FIGURE 4b Below Transition.

As anticipated from theory, the motions are qualitatively different, with the below transition case being much more clearly a dipole instability. The difference occurs because space-charge is 'focusing' above transition energy. Unfortunately, the (marked) differences between the evolution of dipole (and quadrupole) moments in the two cases are difficult to interpret or understand ; and further (or more refined tools) for analysis are required.

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