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The Production of Boson Resonances in Quasi-two-body Processes and the Coherent Droplet Model*

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Recently, Byers and Yang(1) have proposed a "Coherent Droplet" model of high-energy exchange processes, and have showed that the model provides a good explanation of charge exchange $\pi^- p$ scattering at very high energy. In the present letter the Coherent Droplet Model is applied to the resonance production in quasi-two-body reactions at high energy. We discuss the t-dependence of the differential cross-section in the reactions $\pi + p \longrightarrow A + B$ where A is the pion or a pionic resonance, and B is the nucleon or a nucleonic resonance.

A Coherent Droplet Model of quasi-two-body interactions implies that the differential cross-section for all processes has a t-dependence similar to that of elastic scattering and essentially energy-independent. As reported in the preceding Letter, the experimental data seem in general to support this. Variations of the slope from that of elastic scattering can be qualitatively understood by considering the internal structure of the boson resonances produced.

In the Coherent Droplet Model reactions

$$X + Y \longrightarrow C + D \qquad (1)$$

are described as processes of coherent excitation of an absorptive medium, the particles involved being represented as droplets of "nuclear matter". By "coherent" is meant that the droplets X, Y, C and D interact as a whole, and therefore the range of the interaction is related to their radii. Obviously, the same radii of the droplets enter also in the determination of the range of the interaction in other coherent processes, in particular in elastic scattering, $X + Y \longrightarrow X + Y$ and $C + D \longrightarrow C + D$. Therefore, the Coherent Droplet Model relates the range of the interaction in the inelastic two-body scattering to that in the elastic scattering between the particles which appear in the initial and in the final states. Byers and Yang consider the case in which the elastic scattering of the particles in the initial and final states have essentially the same range. It is then clear that the range of the coherent inelastic process also has to be essentially the same. Let us remark that the range of the interaction in elastic scattering is related to the radii of the colliding droplets by the approximate formula

$$R^2 = r_1^2 + r_2^2 \qquad (2)$$

where R is the range of the interaction and r_1, r_2 are the radii of the colliding particles[2]. Assuming that most of the strongly interacting particles and resonances have essentially the same size (which is of the order of the range of the nuclear forces, i.e. $\frac{1}{m_\pi}$ where m_π is the pion mass), we come to the conclusion that the elastic scattering between all pairs of known strongly interacting particles has to have approximately the same range of interaction. From what we have said before, it follows that also all inelastic two-body processes involving these particles have to have essentially the same range of interaction as the elastic processes.

This implication of the Coherent Droplet Model has strong experimental consequences. In fact, the interaction range is simply related to the momentum transfer dependence of the corresponding differential cross-section. If one neglects the spin of the colliding particles, the relation at high energy is[3]

$$a \equiv \left\{ \frac{d\,\ell n\,(d\sigma/dt)}{dt} \right\}_{t=0} = \frac{R^2}{2} \qquad (3)$$

i.e., the range of the interaction can be calculated from the slope a on the plot of $\ell n\,(d\sigma/dt)$ versus t[4].

This relation between the t-dependence and the range of the interaction, as well as our preceding considerations imply that, at high energy, the logarithmic slope of the differential cross-section $d\sigma/dt$ should be, in the small t-region, essentially the same for all two-body processes.

This consequence of the Coherent Droplet Model is rather well confirmed by the experimental data. The t-dependence of the cross-sections in the inelastic two-body processes has been investigated recently by the Aachen-Berlin-CERN collaboration for π^+p interactions at 8 GeV/c primary momentum[5]. The differential cross-section of 6 out of 8 two-body inelastic processes studied in the experiment show a t-dependence which for $0.1 < t < 0.6$ is very similar to that of the elastic $\pi^+ p$ scattering. [The very small t region $|t| \lesssim 0.1\ (\mathrm{GeV}/c)^2$ must be disregarded as the effects of spin-flip and finite width of the resonances are expected to modify the shape of the cross-section.] There are, however, two exceptions : the slope of the differential cross-section for the reactions

$$\pi^+ p \longrightarrow \eta\, N^* \tag{4}$$

and

$$\pi^+ p \longrightarrow \omega\, N^* \tag{5}$$

is much smaller than that of the other reactions, suggesting therefore a smaller interaction range for these processes. It is a purpose of the present paper to discuss these results in terms of the Coherent Droplet Model.

From the point of view of the Coherent Droplet Model, such small range of the interaction in the reactions (4) and (5) can be explained in two ways. The first possibility is that these reactions are due to a completely different mechanism, which has little to do with the coherent excitation. If the interaction is not coherent, then there is no reason why it should have the same range as that of elastic scattering. The second possibility, which we will follow here, is to admit that the Coherent Droplet Model does apply also for the reactions (4) and (5). It follows then that the range of the coherent interaction of the η and ω mesons with nucleon and N^* has to be small. Since the range of the coherent interaction is related to the radii of the particles, it follows that the radii of η and ω are small compared to the typical radius r_o of a strongly interacting "Coherent Droplet". It is easy to see, from the formula (2) that the smallest range of the interaction is reached if the radius of η (ω) $\longrightarrow 0$[(6)]. The logarithmic slope of the cross-section is then two times smaller than that in πp elastic scattering. Indeed, according to the formula (2), we have

$$a \equiv \left\{ \frac{d}{dt} \left(\ln \frac{d\sigma}{dt} \right) \right\}_{t=0} = \frac{r_1^2 + r_2^2}{2} \tag{6}$$

The R.H.S. of Eq. (6) equals r_o^2 for elastic πp scattering. If the radius of the one of the interacting particles $\longrightarrow 0$, the R.H.S. becomes $\frac{r_o^2}{2}$.
It is also interesting to note that the limit is approached rather fast.
For $r_\eta / r_o = 1/2$ and $1/3$, the ratio of the slopes is 0.63 and 0.55, respectively.

The data of reference (5) provide definite evidence that the t-dependence in the reactions (4) and (5) differ remarkably from that in the elastic πp scattering. However, in our opinion, they are not accurate enough to give quantitative information on the range of the interaction, especially if one takes into account the possibility of spin dependence. For the case of the η, the range can be calculated from another recently measured reaction, namely $\pi^- p \to \eta n$ (7). The logarithmic slope of $d\sigma/dt$ is very close to half of that found in elastic scattering. We consider this result as an indication that the process is due to coherent excitation.

It remains to be explained why the radii of η and ω are small compared to the typical radius of strongly interacting particles. We would like now to speculate on that subject. If the resonance is a sort of "droplet", one is inclined to relate its size to its stability : the smaller the radius, the more stable is the droplet. This argument relates the size of the resonance to its life-time, i.e. to its width. One expects the narrow resonances to have smaller spatial dimensions than the broad ones.

To put this idea in a more explicit form, we assume that the forces which keep the different parts of the droplet together can be represented by a potential of the form given in the Figure 1 (8). We will show that, in this model, the two known effects leading to the reduction of the resonance width imply also a reduction of its spatial dimensions.

There are two well-known mechanisms which can reduce the width of a resonance. One of them is a reduction of the phase-space available to the decay by conservation laws which forbid the decay into a small number of particles. The second one is that, if there is a large orbital angular momentum, the width is reduced because of the strong centrifugal barrier. These two effects explain, as shown by Feld (9), the difference between the ω and ρ width. It seems quite plausible that the second effect can account also for the difference between the widths of the A_1 and A_2 resonances.

If the width of a resonance is reduced by any of these effects then, in our model, also its spatial dimensions are reduced. The diminution of the available phase-space is described in this model by the decrease of the distance of the resonant level from the value of the energy for $r \to \infty$. For any reasonable shape of the potential, this effect will move the resonant level to the region where the size of the potential well (which represents the size of the resonance) is smaller[10]. The presence of the large orbital angular momentum requires the existence of an additional potential of the form A/r^2 due to the strong centrifugal force. It is clear that this term has the effect of reducing the size of the potential well[11].

On the basis of this qualitative argument we suggest that there exists a correlation between the width of the resonance and its spatial dimensions. As a consequence, we expect that η, ω, η' and ϕ resonances have to be rather small objects[12]. A_2 is probably larger, but still smaller than ρ, f, A_1 and B. Correspondingly, the forward peak in the reactions

$$\pi + p \longrightarrow (\eta,\ \omega,\ \eta' \text{ or } \phi) + (N \text{ or } N^*) \qquad (7)$$

has to be approximately two times wider than that in the elastic πp scattering. The measurements of the t-distribution in πp interactions with two-body production of η' and ϕ seems therefore to be very interesting.

The simple relations between the slopes of different quasi-two-body reactions presented here have to be treated only as a first approximation. There are several reasons why they can be violated to some extent. The most important one seems to be the influence of the absorption which, as is well known, tends to increase the slope.

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References and Footnotes

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(1) N. Byers and C.N. Yang, "$\pi^- p$ charge exchange scattering and a "Coherent Droplet" Model of high-energy exchange processes", Preprint, Princeton, August 1965.

(2) The relation between the effective interaction radius and the radii of the droplets depends actually on the detailed structure of the droplets. The formula (2) is valid for the scattering of two objects with the opacity changing like e^{-b^2}. The experimental data, showing the t-dependence of the elastic scattering to be approximately e^{at}, strongly suggest the guassian form of the absorption and therefore the formula (2).

(3) See, for example, Ref. 1, Remark (c).

(4) Both spin dependence and the effect of the finite width of the resonance can change the t-distribution for very small $|t|$ values (as is, for example, the case of $\pi^- p$ charge exchange scattering).

(5) M. Deutschmann, D. Kropp, R. Schulte, H. Weber, W. Woischnig, C. Grote, J. Klugow, H. Meyer, A. Pose, S. Brandt, V.T. Cocconi, O. Czyżewski, P.F. Dalpiaz, E. Flaminio, H. Hromadnik, G. Kellner, D.R.O. Morrison and S. Nowak, "Slope of $d\sigma/dt$ distributions in quasi-two-body interactions of 8 GeV/c positive pions", CERN Preprint, October 1965.

(6) The formula (2) applies for the elastic scattering, e.g. $\eta N^* \rightarrow \eta N^*$. It seems natural, however, to assume the range of the coherent inelastic scattering $X + Y \rightarrow C + D$ is equal to the smaller of the ranges of the coherent elastic interactions in the initial and final states.

(7) O. Guisan, J. Kirz, P. Sonderegger, A.V. Stirling, P. Borgeaud, C. Bruneton, P. Falk-Vairant, B. Amblard, C. Caversasio, J.P. Guillaud, M. Yvert, Phys.Lett. 18, 200 (1965).

(8) This suggestion has been made to me by Professor V.F. Weisskopf.

(9) B.T. Feld, Phys.Rev.Letters 8, 181 (1962).

(10) Another model of the considered phenomenon, based on the thermodynamical considerations is also possible. Let us imagine that the particle is formed from a non-interacting bose-gas closed in the volume V. In the non-relativistic approximation the volume V is related to the total kinetic energy of the particles E and the pressure P by the relation $V = \frac{2}{3} \frac{E}{P}$. The diminution of the phase-space corresponds to the diminution of E, while forces, i.e. P, remain unchanged. This effect implies, of course, the diminution of V. One can show that the similar argument is valid also in the relativistic case.

(11) The argument can be equivalently stated in the following way. The presence of the orbital momentum l implies that the wave function behaves like p^l (for small p), and therefore an increase of l leads to an increase of the size of the wave packed in the momentum space. The size of the wave packet in the x-space then decreases.

(12) One can ask why the same argument does not apply for the π-meson. A possible explanation if that, because of the small pion mass m_π, the uncertainty principle gives a large lower limit for the pion radius r_π : $2r_\pi \gtrsim \frac{1}{m_\pi}$.

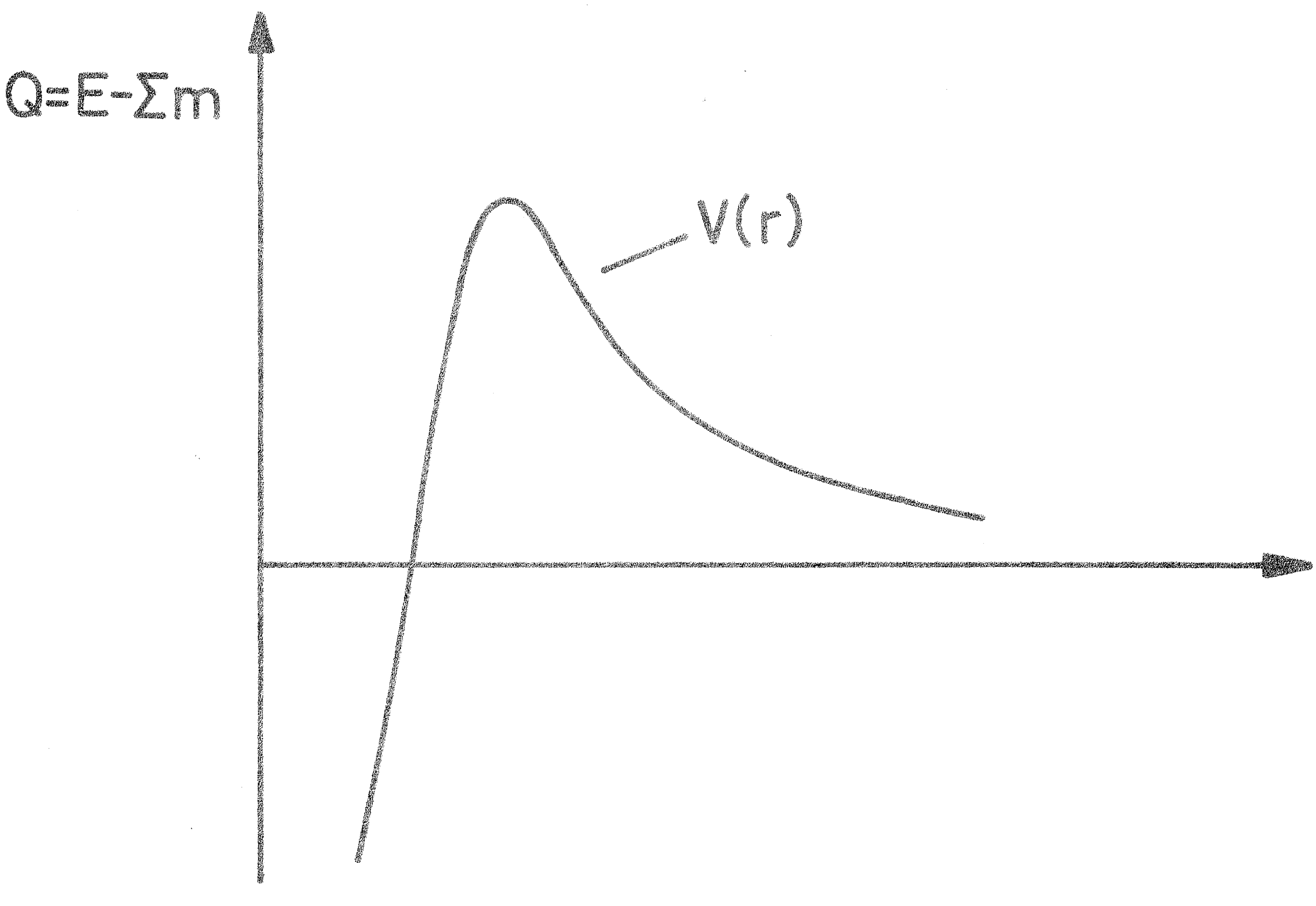

Fig. 1