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Flux of atmospheric neutrinos

Giles Barr
CERN, 1211 Geneva 23, Switzerland

T. K. Gaisser and Todor Stanev
Bartol Research Institute, University of Delaware, Newark, Delaware 19716

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We have repeated a one-dimensional calculation of neutrino fluxes, including the effect of muon polarization on neutrinos from the decay of muons. Fluxes have been calculated as a function of energy and angle with cutoffs appropriate for locations of five deep nucleon-decay experiments. Overall, the ν_e/ν_μ ratio increases by 5% (20%) for neutrinos with energies around 200 MeV (2–3 GeV), in agreement with a previous analytic estimate of the effect.

I. INTRODUCTION

Motivated by the recent measurement of the ratio of atmospheric ν_e to ν_μ reported by the Kamiokande Collaboration,[1] we have repeated our earlier Monte Carlo calculation[2] of the flux of atmospheric neutrinos, now including muon polarization. In view of the reported[1] relative deficit of muonlike events, the muon polarization becomes an important detail because it causes the ν_e's from muon decay to be thrown forward relative to the ν_μ's from muon decay. This in turn increases the ratio

$$R_\nu \equiv (\nu_e+\bar{\nu}_e)/(\nu_\mu+\bar{\nu}_\mu)$$

at a fixed neutrino energy.[3] The direction of the effect is the same for neutrinos and antineutrinos.[4]

An analytic calculation of neutrino ratios,[4] including muon polarization, showed that for a differential pion spectrum

$$\phi(E_\pi) \propto E^{-\alpha} .$$

R_ν increases by 20% at high energy for $\alpha=2.65$ when muon polarization is included. This value of α corresponds to the primary spectrum of highly relativistic cosmic rays at the top of the atmosphere. At a fixed neutrino energy, the increase in R_ν due to polarization is smaller for smaller values of α. This is because the forward kinematic region is emphasized less in the convolution of the parent spectrum with the pion and muon decay distributions. The production spectrum of pions has the same spectral index as the primary nucleon spectrum for pion energies high enough to be in the forward c.m. hemisphere of the hadron-nucleon collisions in which they are produced. For neutrinos with low energies the parent pion spectrum is flatter, reflecting both the rollover of the primary nucleon spectrum below ~ 10 GeV and the importance of pions produced backward in the c.m. frame of an interaction. Thus for low-energy neutrinos the effect of polarization is smaller, and at very low energies ($<m_\mu/2$) the effect on R_ν is reversed. Because of the importance of cascading and energy loss in the atmosphere the Monte Carlo simulation is required to evaluate the fluxes quantiatively.

In this Rapid Communication we first describe how the muon polarization is added to the Monte Carlo program, and we then summarize the results. The simulation has been described in detail in Ref. 2, and many of the results remain essentially unchanged. We have calculated the neutrino fluxes in such a way that the same simulated cascades are used for each detector location. The only source of difference between the fluxes at different sites is that the neutrino yields are folded with different geomagnetic cutoffs on the primary spectrum as appropriate for each geographical site. The calculated fluxes can therefore be used self-consistently as the basis for comparing neutrino interaction rates and neutrino ratios in different experiments.

II. MUON POLARIZATION

For decay in flight of a pion, the resulting ν_μ and μ are produced isotrpically in the pion rest frame, which transforms into a flat distribution in the laboratory frame. The limits are

$$\begin{gathered}\frac{1+r}{2}-\beta\frac{1-r}{2} \le E_\mu/E_\pi \le \frac{1+r}{2}+\beta\frac{1-r}{2} ,\\ \frac{1-r}{2}(1-\beta) \le E_\nu/E_\pi \le \frac{1-r}{2}(1+\beta) ,\end{gathered} \tag{1}$$

where $r=m_\mu^2/m_\pi^2$. These distributions are of course unaffected by the fact that the muon is produced fully polarized along its direction of motion in the pion rest frame for decay of π^- and against it for decay of π^+. There is, however, a correlation between the laboratory energy of the muon and its polarization, which affects the distribution of the decay products of the muon.

In the muon rest frame, the distribution of the decay products is

$$g(x)=f_0(x) \mp f_1(x)\cos\theta , \tag{2}$$

for $\mu^\pm \rightarrow e^\pm + \nu_e(\bar{\nu}_e) + \bar{\nu}_\mu(\nu_\mu)$, where θ is the angle between the muon polarization and the direction of the lepton in the muon rest frame and $x \equiv 2E_\nu^*/m_\mu$ is the fractional energy of the decay product. The functions $f_i(x)$ are given[4] in Table I. In Ref. 4 the distribution (2) is transformed first to the pion rest frame and then to the

TABLE I. Functions for muon decay.

	$f_0(x)$	$f_1(x)$
ν_μ, e	$2x^2(3-2x)$	$2x^2(1-2x)$
ν_e	$12x^2(1-x)$	$12x^2(1-x)$

laboratory frame, integrating over the muon energies and angles. For the complete Monte Carlo calculation it is necessary to compute the decay in the muon rest frame so that muon energy-loss processes can be accounted for.

To do this, we project the muon polarization along a fixed z axis which is the direction of the muon in the laboratory (see Fig. 1). If the projection is P_μ then

$$g(x) = f_0(x) \mp f_1(x) P_\mu \cos\theta_l \,, \tag{3}$$

where θ_l is the angle between the lepton and the fixed z axis. We need to evaluate P_μ in terms of E_μ and E_π, the laboratory energies of the pion and muon. This is done by noting that the muon polarization is parallel (antiparallel) to the direction of a π^+ (π^-) in the muon rest frame. Then one can find the relation between quantities in the muon rest frame and quantities in the laboratory by evaluating the four-vector scalar product $p_\pi \cdot u_{\text{lab}}$ in both frames. Here

$$u_{\text{lab}} = (E_\mu, -\mathbf{p}_\mu)/m_\mu \tag{4}$$

is the four-velocity of the laboratory. The result is

$$\pm P_\mu = \frac{1}{\beta_\mu}\left[\frac{E_\pi}{E_\mu}\frac{2r}{1-r} - \frac{1+r}{1-r}\right] = \cos\theta_\pi \,, \tag{5}$$

where θ_π is the angle in the muon rest frame between the pion and the fixed z axis. The final expression for the distributions of the decay products of the muon in the muon rest frame is

$$\frac{dn}{dx\, d\cos\theta_l} = \tfrac{1}{2}\,[f_0(x) - f_1(x)\cos\theta_l \cos\theta_\pi] \,. \tag{6}$$

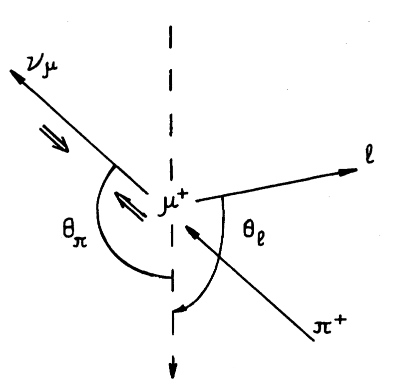

FIG. 1. The decay $\pi^+ \to \mu^+ \nu_\mu$ as viewed from the muon rest frame. Also shown is one of the leptons (l) from the subsequent muon decay. The dashed arrow is the direction of the muon in the laboratory system. This direction and the $\pi^+ \to \nu_\mu$ direction lie in the plane of the page. The lepton l in general is out of this plane.

Note that, because of the opposite relation between spin and kinematics for the two charges, the final distribution is the same for particles and antiparticles.

The simulation of the neutrino yields proceeds by calculating $\cos\theta_\pi$ for each muon in a cascade from Eq. (5) with E_μ selected randomly from the step-function distribution of Eq. (1). The value of E_μ at the point of decay of a parent pion of energy E_π is used to evaluate $\cos\theta_\pi$. Next the lifetime of the muon in its rest frame is chosen from the appropriate exponential distribution. The muon propagates, losing energy, until it decays. At decay the energies (x) and angles (θ_l) of the neutrinos from the muon are chosen randomly from Eq. (6). The reduced energy of the muon at decay is then used to transform the energies of its decay products to the laboratory system. It is assumed that the energies of interest for current measurements of R_ν are high enough ($p_\mu > 100$ MeV/c) so that physical depolarization can be neglected.

As in our earlier paper[2] a linear approximation is made in which all particles are propagated along the direction of the incident cosmic rays. Thus the only change in the calculation is the addition of polarization. The linear aproximation is satisfactory[5] for lepton momenta above 100 MeV/c.

III. RESULTS

Many of the results of Ref. 2 remain essentially unchanged. The flux of atmospheric muons and of those ν_μ which come directly from decay of pions and kaons is of course unaffected by polarization. The ratios of neutrino fluxes at different parts of the solar cycle, as well as the relative angular distributions of neutrinos at the various sites, are virtually unchanged. We show in Fig. 2 the ratio R_ν with and without polarization. The average fluxes per bin are shown for $E_\nu > 60$ MeV in Table II. In comparing to experiment, the ratio should be emphasized because many systematic effects largely cancel in the ratio. These

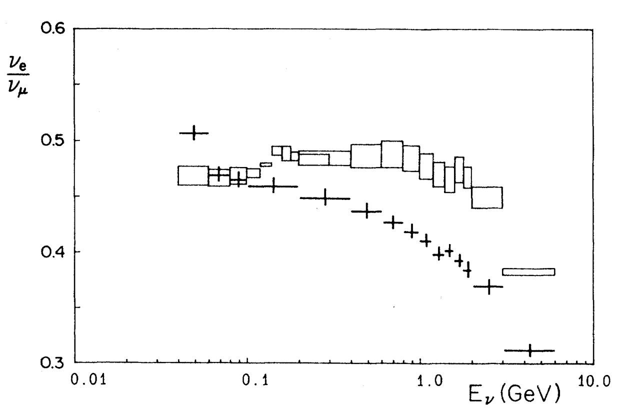

FIG. 2. R_ν as a function of the neutrino energy. Boxes show the result of this calculation, crosses are from Ref. 2. The error bars and heights of the boxes include not only statistical errors from the simulation, but also variations of R_ν with the solar epoch and variations among the different locations. For each location, variations of R_ν are significantly smaller.

TABLE II. Angular-averaged ν fluxes at solar maximum in $(\mathrm{m}^2\mathrm{sr\,s\,bin})^{-1}$.

Site	Northern U.S.				Kamiokande				Mt. Blanc/Frejus				Kolar Gold Fields (India)			
E (GeV)	ν_e	$\bar{\nu}_e$	ν_μ	$\bar{\nu}_\mu$	ν_e	$\bar{\nu}_e$	ν_μ	$\bar{\nu}_\mu$	ν_e	$\bar{\nu}_e$	ν_μ	$\bar{\nu}_\mu$	ν_e	$\bar{\nu}_e$	ν_μ	ν_μ
0.06–0.08	165	150	340	338	92	87	189	190	141	130	290	290	75	70	153	153
0.08–0.10	133	120	272	271	76	72	156	156	115	107	236	235	62	58	126	126
0.10–0.14	194	175	390	389	116	109	236	235	170	158	346	345	94	88	191	191
0.14–0.20	185	167	360	360	118	111	235	236	167	154	329	329	96	90	192	192
0.20–0.30	172	153	337	336	117	110	238	237	160	145	317	316	96	90	194	194
0.30–0.40	96	85	184	184	70	64	138	139	91	82	176	176	57	52	115	114
0.40–0.60	98	85	188	187	75	68	150	149	95	83	184	182	62	57	126	125
0.60–0.80	47	40	89	88	38	34	76	75	46	40	88	87	32	29	64	64
0.80–1.00	26	22	49	49	22	19	43	43	26	22	49	48	19	16.5	38	37
1.00–1.20	16	13	30	30	14	12	27	27	16	13	30	30	12	10	24	23
1.20–1.40	10	8.3	20	19	9.1	7.6	18	18	10	8.3	20	19	8.0	6.8	16	16
1.40–1.60	6.9	5.6	13	13	6.2	5.2	13	12	6.8	5.6	13	13	5.5	4.7	11	11
1.60–1.80	5.0	4.1	9.5	9.3	4.5	3.8	9.0	8.8	4.9	4.1	9.5	9.3	4.0	3.5	8.2	8.0
1.80–2.00	3.6	3.0	7.0	6.9	3.4	2.8	6.7	6.6	3.6	3.0	7.0	6.9	3.0	2.6	6.2	6.0
2.00–3.00	8.7	7.0	18	17	8.3	6.8	17	17	8.6	7.0	18	17	7.5	6.3	16	15.5
> 3.00	6.4	5.0	15	14	6.3	5.0	15	14	6.4	5.0	15	14	6.0	4.8	15	13.5

include effects related to the primary spectrum, to the linear nature of the present calculation, and to the interaction model.

The ratio of electronlike to muonlike events of Ref. 1 is 1.09 ± 0.16, which is 2.9σ larger than the expected value of 0.62 that the Kamioka group obtained from their detector Monte Carlo simulation with the fluxes of Ref. 2 as input. Because of the complexity of the analysis of the data, and because the change in R_ν due to polarization is energy dependent, the effect on the interpretation of the data of including muon polarization must await the full analysis with the detector Monte Carlo simulation. If, however, the ratio of electron to muon events is proportional to R_ν, we can use the new calculation of R_ν around the maximum of the flux times cross section to estimate an upper limit to the size of the effect. In the 1-2-GeV region of neutrino energy R_ν is increased by about 18%. This leads to an estimate of ~ 0.73 for the ratio of electron to muon events, a 2.2σ difference from the measured value.

In summary, when polarization is included, the calculated fluxes do not change by large amounts. Most of the change in the ratio is due to the change in the calculated flux of ν_e. The change in the calculated ratio does not appear sufficient to remove the discrepancy with the Kamioka data. At the same time, preliminary indications are that the revised fluxes give predictions in good agreement with measurements by the Irvine-Michigan-Brookhaven and Frejus Collaborations.

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