BEAMSTRAHLUNG IN THE DEEP QUANTUM REGIME

M. Jacob and Tai Tsun Wu

Abstract

Electron–positron linear colliders working in the multi–TeV range cannot achieve the very high luminosities which are required, without bunch densities which imply an abundant bremsstrahlung. We discuss this radiation in the deep quantum regime, where such machines are likely to operate. Bringing radiation losses to a tolerable level is an interesting challenge to the design of such linear colliders.

At present there is much interest in electron–positron linear colliders which could operate in the TeV and multi–TeV ranges. While no design can yet be proposed for such machines, extensive studies being carried out at CERN and at SLAC in particular should lead to interesting developments [1]. The energy range covered by such machines should open up fascinating perspectives [2]. While a great deal of physics should probably be learned first with proton–proton colliders, the relative clarity of the event structure in e^+e^- collisions should be an invaluable asset for detailed studies [2].

However, there is no point in reaching very high energy if the luminosity is not high enough. The annihilation cross section into muon pairs which provides a bench mark falls as E^{-2}, where E is the centre–of–mass energy, and an electron–positron collider of $E = 1$ TeV, say, should have at least a luminosity of $10^{33}\mathrm{cm}^{-2}\mathrm{s}^{-1}$. The luminosity should increase as E^2.

The luminosity achieved through bunch–bunch crossing is

$$L \sim \frac{N^2 fH}{R^2} \qquad (1)$$

Here N is the number of particles per bunch, f is the bunch–crossing frequency, H a pinch parameter depending on the mutual focusing property of a bunch–bunch collision (a number which could vary between 1 and 10, say) and R is the radius of the bunch. Increasing N and f costs power, machine complications notwithstanding. A high luminosity imposes a small radius. One is talking about values of R of the order of 10^{-9} to 10^{-8} m (10^{-6} m at the Stanford Linear Collider (SLC)).

A high luminosity thus imposes very high fields (N/R very large) and the electrons (positrons) crossing a positron (electron) bunch will therefore experience much acceleration, and radiate. Such a bremsstrahlung radiation, involving the field due to the coherent effect of a bunch, is called "beamstrahlung". The word has stuck, despite its etymological weakness.

In a series of papers, we have calculated the properties of such a beamstrahlung [3–6]. The key features of the approach followed, and the main results obtained are presented here.

The question of electron radiation in a strong field has a long history. One may start with the seminal work of Schwinger [7], followed by extensive studies of synchrotron radiation and more recent calculations [8,9]. The topical nature of the problem stems from the fact that the parameter Υ, well known in synchrotron radiation, and which gives the ratio between the typical radiation energy and the beam energy in the classical case, may become much larger than 1. One has

$$\Upsilon = \frac{\gamma^2}{m\rho_c} \quad , \qquad (2)$$

where γ is the Lorentz contraction factor and ρ_c is the radius of curvature of the electron trajectory. The condition $\Upsilon >> 1$ defines the deep quantum regime in which the classical results are no longer valid.

The radiation spectrum and the fractional energy loss δ had been calculated for large values of Υ, extending previous treatments of synchrotron radiation to this new regime [9,10]. It was however deemed appropriate to have a new full quantum treatment of radiation based on Feynman graphs. This is what we have done. We came upon new effects.

In our approach we assume from the start that conditions are such that radiation occurs in the deep quantum regime and exploit the corresponding new simplifications. This is of course not necessary but it is interesting to use these simplifications which are different from those which can be exploited in the classical regime.

The deep quantum regime corresponds to the fact that the radiative length $L_e = \gamma/m$, over which an electron can freely emit and reabsorb a photon, is much larger than the classical coherent radiation length $L_c = \rho_c/\gamma$. Whereas at lower energies (even for the SLC), one has $L_c >> L_e$, a multi–TeV electron–positron linear collider is likely to operate in a regime where $L_e >> L_c$. This is due to the fact that gaining an order of magnitude in energy is meaningless unless one also gains two orders of magnitude in luminosity. One finds that one is thus driven into the deep quantum regime, unless the length of the bunch becomes prohibitively large.

Particle World, Vol. 1, No. 1, p. 20–23, 1989.

In that regime, a natural procedure is to calculate the radiation amplitude associated with the Feynman graph of the fig. 1. The crosses correspond to the interaction of the radiating electron with the bunch.

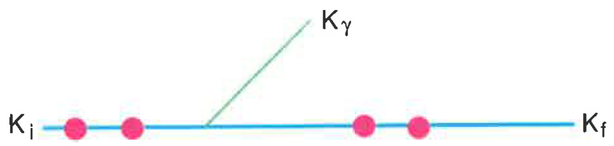

FIGURE 1

Feynman graph for the emission of a photon by an electron in the presence of a bunch.

To order α, this is a distorted wave Born approximation method. We merely outline here the successive steps followed in ref. [4]. The initial and final electron wave functions are calculated in a high E approximation [11,12], which however turns out to be a low D approximation, D being the disruption parameter. D, when small, is merely the ratio between the bunch length L_b and the focal length, associated with the lens effect which a positron bunch has on an incoming electron. A low D regime is particularly interesting for a machine. In that regime only the problem of bunch–bunch collision can be reduced to that of particle–bunch collision and solved analytically.

Once the phase of the radiation amplitude is known, the stationary conditions are obtained and the radiation matrix elements calculated in the neighbourhood of the stationary point. The full amplitude is then calculated, and the radiation rate $I(x)$ is obtained. This is the radiation probability for a photon taking a fraction x of the incident electron energy. The relative energy loss is

$$\delta = \int_0^1 x\, I(x)\, dx \quad . \tag{3}$$

With cylindrical bunches, it easily reaches values at the 20% level or more for a collider working in the multi–TeV range. Such radiation losses are a very serious challenge and the more so that slown down electrons and pair creation associated with such photons also become a serious problem. One can quench the photon radiation by making the bunches flat, thus reducing the value of the field inside the bunch [13]. This however adds to the technical difficulties in achieving good collisions between the bunches, considering their very small size.

The principle of the calculation procedure is given in ref. [3]. The calculation of the beamstrahlung in bunch–bunch collisions is presented in detail in ref. [4], for the case of uniform cylindrical bunches. We considered in turn the spinless (Klein–Gordon) case and the Dirac case. Reference [5] is a general review of that work. In ref. [6] we considered the effect of a varying bunch density. This includes a full study of edge effects.

Considering first the simplest case of a uniform bunch density, one finds a rather hard spectrum. It has the form

$$\left(\frac{1-x}{x}\right)^{2/3}$$

in the Klein–Gordon case, with simply an extra factor

$$\frac{1}{2}\left[(1+\{1-x\}^2)/\{1-x\}\right]$$

in the Dirac case, neglecting helicity flip, as is justified in the deep quantum regime. The explicit form of the helicity flip contribution, with its particular spectrum, was also calculated [4]. There are two leading terms in δ, namely

$$\delta = \frac{\alpha}{\pi}\left(K_e\, ln\, \frac{L_e}{l_c} + K_i\, \frac{L_b}{l_c}\right) \quad , \tag{4}$$

where K_e (external) and K_i (internal) are two numerical factors of order one. In the regime considered, both $ln\ (L_e/l_c)$ and L_b/l_c are relatively large quantities. Hence a value of 20% for δ can be deemed typical.

Relation (4) owes its simplicity to the introduction of a new coherent radiation length l_c, specific to that regime [4]. It is defined by the stationary phase condition in the deep quantum regime. It is

$$l_c = \left(L_c^2\, L_e\right)^{1/3} \quad . \tag{5}$$

Relation (4) is accurate provided that $L_e >> L_c$ or $l_e >> L_c$. This is the simple relation proper to the deep quantum regime. Indeed, one finds that

$$\Upsilon = \left(\frac{l_e}{L_c}\right)^3 \quad . \tag{6}$$

Relation (4) applies to a cylindical bunch with uniform density and sharp boundaries. The separation of an internal and an external contribution is somewhat arbitrary since the radiation amplitude is an integral over all space. However, the stationary phase condition gives some meaning to this intuitive separation. We can always call internal that part of the contribution which is proportional to L_b, and external that part which is proportional to $ln\, L_e$.

One remarks [4] that, while both L_c and L_e depend on the electron mass (the classical radius of the electron is a canonical parameter in the classical treatment of synchrotron radiation), this latter disappears in l_c. This is natural, since we are in a regime where the transverse momentum gained in bunch–crossing is now much greater than the mass. It may indeed be of the order of several hundreds of MeV! The presence of the first term in relation (4) and the very simple form in terms of l_c came out first and most naturally in our approach.

These results agree with those obtained through other approaches, not using Feynman graphs, earlier for the second term, later for the first one [9,10].

The question of varying bunch density has been the object of recent investigations by others [14–16] and by us [6]. Our Feynman graph method provides a full treatment of edge effects. They turn out to be relatively small.

One introduces in that case a normalized local density along the bunch axis

$$\frac{1}{L_b}\int \bar{\rho}(z)\,dz = 1 \quad . \tag{7}$$

One finds that the leading correction consists of replacing the factor K_i in relation (4), as calculated in a uniform cylindrical bunch, which is 1.94 in the Dirac case [4], by [14,5]

$$\bar{K}_i = K_i \frac{\int \bar{\rho}^{\,2/3}(z)\,dz}{\int \bar{\rho}(z)\,dz} \quad . \tag{8}$$

Taking an analytic and yet realistic bunch profile of the type [17]

$$\bar{\rho}(z) \sim \mathrm{sech}^2\left(\frac{z}{2L_b}\right) \tag{9}$$

and normalizing to the same root mean square length, the radiation intensity is increased by typically 10%. In the quantum regime, one gains in making the bunches as compact and short as possible. Indeed, if one assumes that the luminosity increases as E^2 in the calculation of l_c, one finds that

$$\delta \sim L_b^{1/3}$$

as opposed to

$$\delta \sim L_b^{-1}$$

in the classical case.

This bunch profile effect picks up only the leading term, that of order of L_b/l_c in δ. It is not sensitive to variations of the density which would vary appreciably over l_c .

A complete treatment of the effects associated with varying densities is represented in ref. [6]. We separate these contributions to δ of orders 1, 0, and –1, respectively, in the large quantity L_b/l_c. This is done through a Mellin transform analysis of the radiation rate [18], which is then calculated in a new streamlined way, short–cutting Airy functions.

The contribution of order one has been mentioned already. The contribution of order zero contains a $\ln L_e$ contribution, which singles itself out there as the leading term when $L_e/L_b >> 1$.

The contribution of order –1 is an integral involving the first (squared) and second derivatives of the density through the combination

$$3\bar{\rho}(z)\bar{\rho}\,''(z) - 4\bar{\rho}\,'^2(z) \quad . \tag{10}$$

In our approach it appears as a contour integral which is well defined and can be readily calculated with a model density such as relation (9). One finds that the corresponding effect on δ is small.

To conclude, the very high luminosity which one has to achieve with very high energy linear colliders imposes a very strong beamstrahlung, and the deep quantum regime, with its special features, is likely to prevail. In any case, the radiative losses are large, although they could be somewhat reduced with ribbon bunches at the cost of extra technical difficulties. Radiation has a rather hard spectrum. This may be considered a gain, an electron–positron collider at these energies being also an intense photon–photon collider [13]. This brings extra problems with the pairs which these photons will generate in the bunch [19]. For annihilation processes, the resonance enhancement factor, so much associated with electron–positron machines, will be seriously eroded. However, for some reactions of great potential interest at these energies, such as $WW \to$ Higgs, this is not a very serious loss.

There is a long list of questions still to be studied in connection with multi–photon radiation, the larger D regime, the transverse density profile, etc. The connection with channeling and transition radiation, where the method followed readily applies, appears to be interesting. The Feynman graph approach should eventually impose itself there. At present, we are extending our approach to electron–positron pair production in the field of the bunch. This is the process shown in fig. 2. It is deemed worth a detailed study [19].

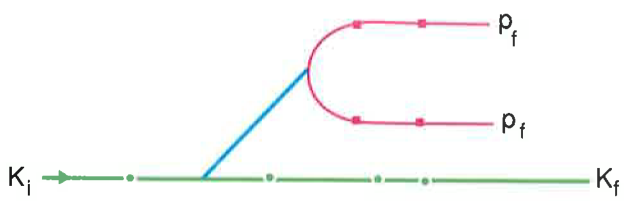

FIGURE 2

Feynman graph for pair production in beamstrahlung.

The calculation of pair production by a photon entering the bunch has now been done [20]. The probability for pair production is of the same order as that for beamstrahlung. It is considerable.

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Work supported in part by the United States Department of Energy under grant DE–FG02–84 ER 40158.

Addresses:

M. Jacob
CERN, CH–1211 Geneva 23, Switzerland

Tai Tsun Wu
CERN, CH–1211 Geneva 23, Switzerland and
Gordon McKay Laboratory, Harvard University, USA

Received and reviewed by R. Klapisch, 1 April 1989.