Forward dijet production and improved TMD factorization in dilute-dense hadronic collisions

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We study inclusive dijet production at small x in hadronic collisions. We extend the High Energy Factorization (HEF) framework outside its kinematic window, where the transverse momentum imbalance of the dijets system is comparable to the transverse momenta of the individual jets, to the case of arbitrarily small transverse momentum imbalance. That involves generalizing the Transverse Momentum Dependent (TMD) factorization formula for dijet production to the case of finite N_c and with one of the incoming gluons being off-shell. We discuss the features of this new generalized TMD factorization and relate it to the colour-ordered amplitude formalism.

1 Introduction

Forward dijet production at the LHC offers a unique chance to study QCD dynamics in the regime of small x. By requiring the two jets to be produced in the forward direction one creates an asymmetric situation, in which one of the incoming hadrons is probed at large x, while the other is probed at a very small momentum fraction. Since the number of gluons grows rapidly with decreasing x, the above corresponds to collisions of dilute-dense systems.

This interesting kinematic regime faces various challenges with the most fundamental question concerning the factorization of short- and long-distance dynamics. The standard collinear factorization is not applicable in this case as the dependence on the transverse momentum of the low-x gluon in the target, k_t, cannot be neglected. The latter comes from the fact that at low x, the gluon distribution does not fall quickly with k_t, and a significant part of the contribution to the integrated gluon distribution comes from the region $k_t \gg 0$ [1].

The process of interest is shown schematically in Fig. 1. The energy fractions of the incoming parton from the projectile, x_1, and the gluon from the target, x_2, can be expressed in terms of rapidities and transverse momenta of the produced jets

$$x_1 = \frac{1}{\sqrt{s}}\left(|p_{1t}|e^{y_1} + |p_{2t}|e^{y_2}\right), \qquad x_2 = \frac{1}{\sqrt{s}}\left(|p_{1t}|e^{-y_1} + |p_{2t}|e^{-y_2}\right). \tag{1}$$

In the limit of forward jets production, one obtains

$$y_1, y_2 \gg 0 \qquad \Longrightarrow \qquad x_1 \sim 1 \quad \text{and} \quad x_2 \ll 1\,, \tag{2}$$

hence the target is probed at very low energy fractions and therefore is consists predominantly of gluons with non-vanishing transverse momenta. The target gluon's transverse momentum, k_t, is related to the transverse momenta of the outgoing jets, p_{1t} and p_{2t}, as well as the azimuthal distance, $\Delta\phi$, between the jets

$$|k_t|^2 = |p_{1t} + p_{2t}|^2 = |p_{1t}|^2 + |p_{2t}|^2 + 2|p_{1t}||p_{2t}|\cos\Delta\phi\,. \tag{3}$$

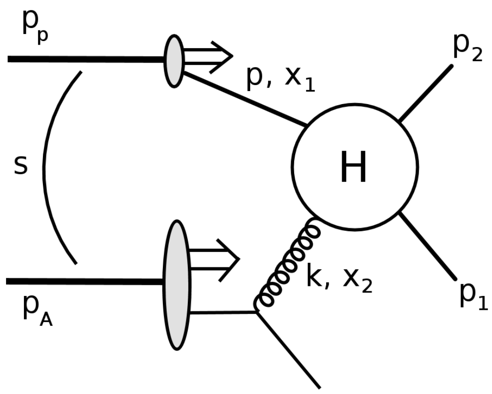

Figure 1 – Inclusive dijet production in dilute-dense collision. The blob H represents hard scattering and the solid lines coming out of H represent partons, which can be either quarks or gluons.

Since the target A is probed at low x_2, the dominant contributions come from the subprocesses in which the incoming parton on the target side is a gluon

$$qg \to qg\,, \qquad gg \to q\bar{q}\,, \qquad gg \to gg\,. \tag{4}$$

2 High energy factorization and generalized TMD factorization

Two approaches to dijet production at small x have been used so far: the high energy factorization (HEF) [2], valid in the limit $Q_s \ll |k_t| \sim |p_{1t}|, |p_{2t}|$, and the transverse momentum dependent (TMD) factorization [3], justified in the range $Q_s \ll |k_t| \ll |p_{1t}|, |p_{2t}|$. Here, Q_s is the so-called saturation scale marking the transition between linear and non-linear regimes of QCD.

The HEF formula for inclusive forward dijet production reads [4]

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{1}{16\pi^3 (x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\, |\overline{\mathcal{M}_{ag^*\to cd}}|^2 \mathcal{F}_{g/A}(x_2,k_t) \frac{1}{1+\delta_{cd}}\,, \tag{5}$$

where $\mathcal{F}_{g/A}(x_2,k_t)$ is the unintegrated gluon distribution in a dense target A, the same that enters the F_2 structure function formula in deep inelastic scattering. The other parton distribution, $f_{a/p}(x_1,\mu^2)$, is a standard collinear PDF of a projectile. The matrix element $|\overline{\mathcal{M}_{ag^*\to cd}}|^2$ is calculated with the incoming gluon kept off-shell using a special set of rules that guarantee gauge invariance of the result [2,5]. The limitation of the HEF formula (5) lies in the fact that it breaks down when the gluon's transverse momentum k_t (or equivalently the transverse momentum imbalance between the two jets) gets much smaller than the average transverse momentum of the jets. Hence, this formula is in principle applicable away from the non-linear regime of the target A and it is not valid in the strict saturation domain.

The TMD formula for our process of interest, so far obtained only in the large-N_c limit [3], takes the form

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_i H^{(i)}_{ag\to cd} \mathcal{F}^{(i)}_{ag}(x_2,k_t) \frac{1}{1+\delta_{cd}}\,. \tag{6}$$

We see that factorization in a strong sense does not hold in this case and it is replaced by generalized factorization with several unintegrated gluon distributions, $\mathcal{F}^{(i)}_{ag}(x_2,k_t)$, accompanied by corresponding hard factors $H^{(i)}_{ag\to cd}$. The hard factors were so far calculated in the limit of vanishing gluon's transverse momentum [3] and the k_t dependence is present in Eq. (6) only via the unintegrated gluon distributions. The generalized TMD formula (6) has the advantage that it can be used to study the saturation domain where $|k_t| \sim Q_s$. It breaks down however when $|k_t|$ becomes comparable to the average jet transverse momentum, which corresponds to $\Delta\phi \ll \pi$, *c.f.* Eq. (3).

$K^{(i)}_{gg^*\to gg}$	$\frac{N_c}{C_F}\,\frac{(\bar{s}^4+\bar{t}^4+\bar{u}^4)(\bar{u}\hat{u}+\bar{t}\hat{t})}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}}$	$-\frac{N_c}{2C_F}\,\frac{(\bar{s}^4+\bar{t}^4+\bar{u}^4)(\bar{u}\hat{u}+\bar{t}\hat{t}-\bar{s}\hat{s})}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}}$
$K^{(i)}_{gg^*\to q\bar{q}}$	$\frac{1}{2N_c}\,\frac{(\bar{t}^2+\bar{u}^2)(\bar{u}\hat{u}+\bar{t}\hat{t})}{\bar{s}\hat{s}\hat{t}\hat{u}}$	$\frac{1}{4N_c^2C_F}\,\frac{(\bar{t}^2+\bar{u}^2)(\bar{u}\hat{u}+\bar{t}\hat{t}-\bar{s}\hat{s})}{\bar{s}\hat{s}\hat{t}\hat{u}}$
$K^{(i)}_{qg^*\to qg}$	$-\frac{\bar{u}\,(\bar{s}^2+\bar{u}^2)}{2\bar{t}\hat{t}\hat{s}}(1+\frac{\bar{s}\hat{s}-\bar{t}\hat{t}}{N_c^2\;\bar{u}\hat{u}})$	$-\frac{C_F}{N_c}\,\frac{\bar{s}(\bar{s}^2+\bar{u}^2)}{\bar{t}\hat{t}\hat{u}}$

Table 1: The hard factors accompanying the gluon TMDs $\Phi^{(i)}_{ag\to cd}$ entering the improved TMD factorization formula (7). The Mandelstam variables are defined according to Eqs. (8) and (9).

3 Improved TMD factorization

We have reconciled the two complementary approaches to forward dijet production, presented in the previous section section, into a unified framework that we call *improved TMD factorization* [6]. This new framework is valid in the limit $Q_s \ll |p_{1t}|, |p_{1t}|$ with an arbitrary value of $|k_t|$. Moreover, it profits from a number of improvements, as compared to HEF and generalized TMD descriptions, which include: (i) full N_c dependence in the hard factors, (ii) restored k_t dependence in the hard factors, and, (iii) reduced number of TMDs and the corresponding hard factors, which leads to a simpler form of the factorization formula.

The improved factorization formula for inclusive dijet production takes the form

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2}\sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\sum_{i=1}^{2} K^{(i)}_{ag^*\to cd}\Phi^{(i)}_{ag\to cd}\,\frac{1}{1+\delta_{cd}}\,, \tag{7}$$

where $K^{(i)}_{ag^*\to cd}$ are the new hard factors and $\Phi^{(i)}_{ag\to cd}$ are the corresponding, new gluon TMDs. The hard factors retain full dependence on gluon's k_t and are summarized in a concise form in Table 1, where, in addition to the standard Mandelstam variables (*c.f.* Fig. 1 for notation)

$$\hat{s} = (p_1+p_2)^2\,, \qquad \hat{t} = (p-p_2)^2\,, \qquad \hat{u} = (p-p_1)^2\,, \tag{8}$$

we introduced their barred versions, defined only with the longitudinal component of the off-shell gluon

$$\bar{s} = (x_2p_A+p)^2\,, \qquad \bar{t} = (x_2p_A-p_1)^2\,, \qquad \bar{u} = (x_2p_A-p_2)^2\,. \tag{9}$$

The results of Table 1 were obtained using two different methods of calculation. The first method followed the original technique developed in the HEF formulation [2] and consisted of using standard Feynman rules in axial gauge with a special choice of the gauge vector, $n = p_A$, and the polarization vector of the off-shell gluon $\epsilon^0_\mu = i\sqrt{2}\,x_2p_{A\mu}/|k_t|$. This procedure is sufficient to render the result gauge invariant for a class of axial gauges in the high energy limit.

The second independent calculation relied on the frameworks of colour ordered amplitudes and helicity methods. Each $2\to 2$ amplitude in one of the three channels (4) is represented as a sum of two, gauge invariant subamplitudes corresponding to different colour flows. The squared amplitude consists of four terms, which however turn out be multiplied by only two unique colour structures. In terms of factorization formula, that means that only two TMDs are need in each channel, which results in the structure of the improved TMD factorization formula presented in Eq. (7).

Hence, the improved factorization has a simpler form than the generalized TMD formula of Eq. (6). The new TMDs, $\Phi^{(i)}_{ag\to cd}$, are related to the old TMDs, $\mathcal{F}^{(i)}_{ag}$, as follows

$$\begin{aligned}
\Phi^{(1)}_{gg\to gg} &= \frac{1}{2N_c^2}(N_c^2\mathcal{F}^{(1)}_{gg} - 2\mathcal{F}^{(3)}_{gg} + \mathcal{F}^{(4)}_{gg} + \mathcal{F}^{(5)}_{gg} + N_c^2\mathcal{F}^{(6)}_{gg})\,,\\
\Phi^{(2)}_{gg\to gg} &= \frac{1}{N_c^2}(N_c^2\mathcal{F}^{(2)}_{gg} - 2\mathcal{F}^{(3)}_{gg} + \mathcal{F}^{(4)}_{gg} + \mathcal{F}^{(5)}_{gg} + N_c^2\mathcal{F}^{(6)}_{gg})\,,
\end{aligned}$$

$$\Phi^{(1)}_{qg\to qg} = \mathcal{F}^{(1)}_{qg}\,, \qquad \Phi^{(2)}_{qg\to qg} = \tfrac{1}{N_c^2-1}\left(-\mathcal{F}^{(1)}_{qg}+N_c^2\mathcal{F}^{(2)}_{qg}\right)\,,$$

$$\Phi^{(1)}_{gg\to q\bar{q}} = \tfrac{1}{N_c^2-1}\left(N_c^2\mathcal{F}^{(1)}_{gg}-\mathcal{F}^{(3)}_{gg}\right)\,, \quad \Phi^{(2)}_{gg\to q\bar{q}} = -N_c^2\mathcal{F}^{(2)}_{gg}+\mathcal{F}^{(3)}_{gg}\,.$$

Both of the above calculations, the one using traditional approach and the one employing colour ordered amplitude formalism, lead to identical results for the hard factors and the new TMDs. Our formulae reduce to the generalized TMD results if the hard factors are taken in the limit of $k_t \to 0$ and only the leading N_c part is kept. Also, the HEF factorization and the collinear factorization are recovered in the respective limits.

4 Summary

We discussed the problem of theoretical description of the forward-forward dijet production in dilute-dense hadronic collisions. Our main result consists of a new, improved TMD factorization formula (7), which unifies two complementary theoretical frameworks commonly used in the literature: the high energy factorization [2] and the generalized TMD factorization [3].

The improved TMD framework is valid for the jet transverse momenta $|p_{1t}|, |p_{2t}| \gg Q_s$ and an arbitrary value of the incoming gluon transverse momentum, k_t. Our result incorporates full N_c dependence and complete k_t dependence in the hard factors and it is written in terms of a minimal set of the gluon TMDs. Hence, it constitutes a robust framework for studies of saturation domain with hard objects.

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