EVIDENCE FOR A POSSIBLE π-N RESONANCE IN THE $P_{1/2}$, $T = \frac{1}{2}$ STATE

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Further analysis [1] of the experimental data on the total $\pi^{\pm}p$ cross-sections and the differential elastic $\pi^{-}p$ scattering cross-sections has been carried out for energies between 0.3 and 1.3 GeV. Recognizing that the data are insufficient for a complete and unambiguous analysis in terms of complex phase shifts, restrictive assumptions related to the higher resonances have been made. When the $\pi^{-}p$ amplitudes for the $D^{-}_{3/2}$ and $F^{+}_{5/2}$ states are chosen in agreement with the Breit-Wigner formula having parameters appropriate to the observed peaks in the $T = \frac{1}{2}$ total cross-section, one finds that the best fitting of the angular distributions requires a $T = \frac{1}{2}$, $P_{1/2}$ resonance near 950 MeV plus non-resonant contributions from other states. Within the limitations of the present data it is not possible to eliminate alternate solutions which are somewhat less good on the basis of a least squares analysis. However, our best solutions at 900, 915, 950, 1000 and 1020 MeV are all consistent with a resonant $P_{1/2}$ state.

This result is particularly interesting in view of the fact that the data on ΛK production have for some time indicated a resonance near 950 MeV [2]. Kanazawa [3] concluded that a $P_{1/2}$ or $P_{3/2}$ resonance (pseudoscalar K) at 925-940 MeV of c.m.s. width 100-120 MeV fitted the data available in 1958. He stated that this peak was not related to the πN resonance at 900 MeV which was associated with a higher state of angular momentum (presumably $F_{5/2}$). It now seems that there may be an isobar of $T = \frac{1}{2}$, $J = \frac{1}{2}^{+}$ which has a ΛK decay mode. The most recent data [4] show a pronounced peak in the production cross-section ($\sigma_{\Lambda K} = 1.2$ mb) and also a need for waves higher than S and P to fit the angular distribution and polarization. Analyses with S and P alone show a large $P_{1/2}$ or $P_{3/2}$ amplitude, but unfortunately the data are insufficient for a complete analysis with higher waves included. Fig. 1 shows two possible fits to the production cross-section using a smooth background plus a $P_{1/2}$ resonance of the following characteristics:

TABLE I

	E (MeV)	$\Gamma(E)$ (MeV)	$(2M/\mu)\,\gamma^2$
A	920	120	0.38
B	896	185	0.62

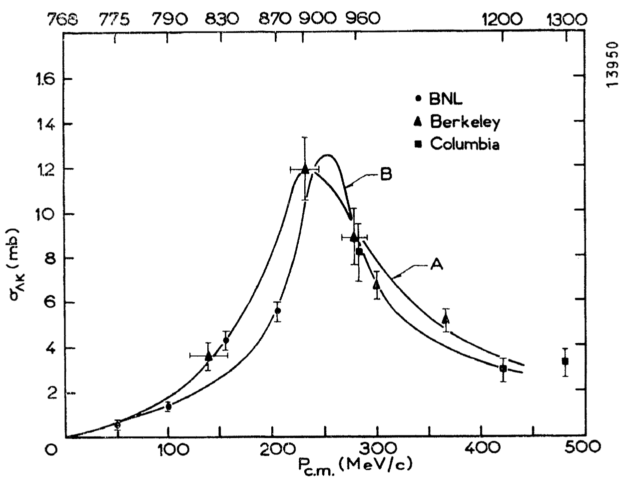

Fig. 1 The data for the ΛK production cross-section given in the reference [4] are displayed for comparison with a smooth background plus a $P_{1/2}$ resonance at 896 MeV (A) or 920 MeV (B)

Reduced width, γ, is precisely defined below. With the Breit-Wigner resonance formula one finds the general result that the angular momentum barrier factor is very important near threshold. In the particular case at hand this means that it is impossible to fit the peak with an $F_{5/2}$ resonance. To summarize the information from ΛK production, one can say there is evidence for a P wave resonance, but it is not possible to determine precise amplitudes. In view of the πp data it is tempting to speculate that there is a $P_{1/2}$ resonance with small $P_{3/2}$ and $F_{5/2}$ amplitudes, the last being significant only because of the 900 MeV $T=\frac{1}{2}$ resonance.

Returning to the πp data analysis, we give a few details of the method [1]. As a first step the total cross-sections were separated into a series of resonances superposed on a monotonically varying background (dependent on the isospin). The peaks were fitted by a relativistic Breit-Wigner formula for the amplitude of a particular angular momentum state, J, between channels α and β:

$$A_J(E,\alpha,\beta)=\frac{\Gamma_J(E,\alpha)\Gamma_J(E,\beta)}{\Gamma_J(E)}\cdot\frac{\frac{1}{2}\Gamma_J(E)}{(E_J-E)-i\frac{1}{2}\Gamma_J(E)} \qquad (1)$$

where E is the pion c.m.s. energy, $\Gamma_J(E,\alpha)$ is a partial width, and $\Gamma_J(E)$ is the total width, assumed to vary like $\Gamma_J(E,\pi)$, the partial width for elastic scattering. $\Gamma_J(E,\pi)$ has the form:

$$\Gamma_J(E,\pi)=\frac{4\mu M\gamma^2 ka}{E_J+E}V_L(ka) \qquad (2)$$

where μ and M are the pion and nucleon masses, respectively, $\hbar K$ is the pion c.m.s. momentum, a is the interaction range chosen as one Fermi, $V_L(Ka)$ is the barrier penetration factor [5] for orbital angular momentum, $L=J\pm 1/2$, and γ is the dimensionless reduced width. The slowly varying energy denominator E_J+E, arising from a relativistic normalization, results in only a small difference from the usual low-energy Breit-Wigner formula. In general, for either resonant or non-resonant states $A_J(E,\alpha,\beta)$ can be written in terms of a complex phase shift $\delta_J(E,\alpha,\beta)$ as follows:

$$A_J(E,\alpha,\beta)=\frac{1}{2i}(e^{2i\delta J(E,\alpha,\beta)}-\delta_{\alpha\beta}) \qquad (3)$$

The parameters E_J and Γ_J in the laboratory system are given below for the three well defined πp peaks.

TABLE II

E(MeV)	$\Gamma(E)$(MeV)	$\Gamma(E,\pi)/\Gamma(E)$	$(2M/\mu)\gamma^2$	J	T
205	165	1.00	1.78	$P_{3/2}$	3/2
605	220	0.67	1.78	$D_{3/2}$	1/2
900	185	0.88	1.78	$F_{5/2}$	1/2

To find the non-resonant amplitudes $A_J=A_J(E,\pi,\pi)$ three types of data have been used; namely, the total cross-section [6], σ, the forward scattering amplitude D as found by Cronin [7], and the expansion coefficients in a cosine power series for the differential cross-section as given by Wood *et al.* [8]:

$$\frac{d\sigma}{d\Omega}=\sum_{n=0} a_n\cos^n\theta \qquad (4)$$

σ, D, and the a_n can be expressed in terms of the A_J or δ_J using well-known formulae [1]. The obvious difficulty in finding the δ_J is the fact that the problem is underdetermined when nothing is known of the polarization. If one accepts the limitation $L\leqq 3$ based on the experimental absence of $n>6$ for $E<1.0$ GeV, there are nine bits of data and seven complex numbers to be determined. The method used to get meaningful solutions was to insist upon a consistent and smooth change in going from one energy to the next, and to restrict the solution by careful choice of the input trial values of the δ_J. The computer programme simply took a set of trial values of δ_J, calculated the nine quantities σ^c, D^c, and the a_n^c, and computed the sum:

$$M=\sum_{n=0}^{6}\left|\frac{a_n^c-a_n}{\Delta a_n}\right|^2+\left|\frac{\sigma^c-\sigma}{\Delta\sigma}\right|^2+\left|\frac{D^c-D}{\Delta D}\right|^2 \qquad (5)$$

where a_n, σ, and D are measured values and Δa_n, $\Delta\sigma$, and ΔD are the uncertainties, directly or indirectly evaluated from experiment. The computer then varied the δ_J cyclically until a minimum of M was obtained. This procedure would be hopeless if data at only one energy were considered; but in the calculation data at all energies were used together as follows:

(1) starting with the results of Pontecorvo [9] at 300 MeV the input δ_J were initially taken as the best-fit results previously obtained for the next lower energy, except as stated below.

(2) for the $D_{3/2}$ and $F_{5/2}$ states the trial δ_J were taken from Eqs. (1) and (3) and Table II.

(3) for the $D_{5/2}$ and $F_{7/2}$ states the trial δ_J were chosen to conform to the measured values of a_5 and a_6 Eq. (4) in view of the previous choice for the $D_{3/2}$ and $F_{5/2}$ states.

(4) finally in succeeding trials other inputs for $L<2$ were chosen to the extent of 93 different calculations some of which involved data considered " less reliable " or " alternate " to the " preferred " sixteen sets.

The results of these calculations are shown in Fig. 2 where the points represent the 29 " best-fit " solutions with the smallest values of M (≈ 1) except for those cases, where consistency with neighbouring energies demanded that the " second best " be accepted. The figure shows that the input restrictions were sufficient to retain the $D_{3/2}$ and $F_{5/2}$ resonances in the solutions. There are other non-resonant solutions, but the ones shown are " best " when the two resonances and consistency are required. One concludes that *the available πp data are consistent with the photoproduction results of Peierls* [10] *and, in addition, suggest a* $P_{1/2}$ *resonance near 950 MeV.*

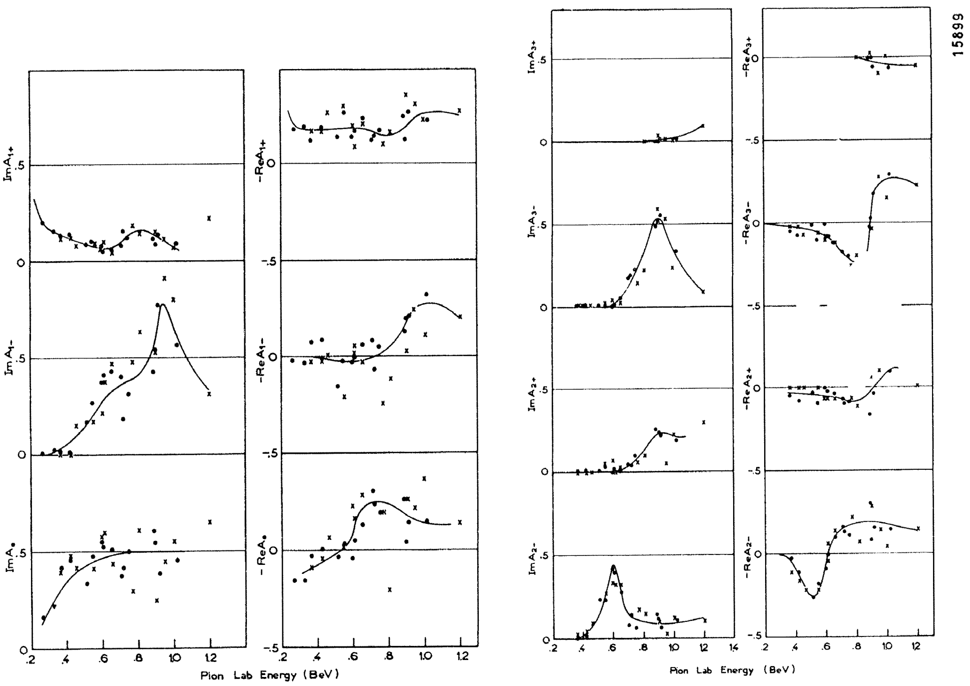

Fig. 2 The πp scattering amplitudes, A_J, are plotted against laboratory pion kinetic energy. The real and imaginary parts of $A_J = A_{L\pm\frac{1}{2}} = A_{L\pm}$ deduced from a " least squares fit " of the experimental data are marked by points (.) for "preferred solutions" and by crosses (×) for " alternate solutions ".

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DISCUSSION

FELD: Fig. 1 indicates that the ΛK production data are consistent with a $P_{1/2}$ resonance interpretation such as Dr. Layson has pointed out.

LAYSON: Yes, here all data are included which were available up to three months ago and you see two possible fits, one has a resonance energy of 900 and the other 920 MeV with widths of the order of 100 MeV. There is certainly some kind of bump there and it could be fitted in several different ways.

HÖHLER: I would like to mention that the sum rule has not been taken into account in Cronin's calculations. If an unsubtracted dispersion relation is used, the result for $D^{(-)}$ is changed appreciably at high energies. There is a slight indication of a new resonance in the isospin one-half amplitude at 900 MeV in the diagram, which has been shown in the talk.

MANDELSTAM: I am just a little bit puzzled as to how, even with the energy dependence, you can get information about more things than you put in the data. First, am I right in saying that even after the assumptions, about $D_{3/2}$ and $F_{5/2}$, you still want to get out more numbers—taking the real and complex part as two numbers—than the number of things you put in?

LAYSON: I think we should explain that. As I said we put in the $D_{3/2}$ and $F_{5/2}$ as resonant phase shifts direct from the Breit-Wigner formula. It turned out that the $F_{7/2}$ was negligible so this is eliminated. Now the other phase shifts were not fit at just one energy. It would certainly be hopeless to try to do an analysis at one energy, so we started with the results at 300 MeV as reported by Pontecorvo, at the Kiev Conference three years ago and we worked up from there. Each time in the calculation we used as input, the results from the next lower energy in continuing a consistent phase shift fit, so in fact we did not get out more than we put in.

MANDELSTAM: I am puzzled that you did not get out more than you put in. You have, say, n pieces of data which vary smoothly with the energy. Now you want to get out $n+m$ results which also vary smoothly with the energy. Surely you can take m of those extra data to be arbitrary, provided you make them vary smoothly with the energy and still get n more curves out. Would you not have an m fold arbitrariness restricted by the fact that this m fold arbitrariness has to vary smoothly with the energy?

LAYSON: No, let us count the number. We will assume there are 7 of the A's, but we are dropping the $F_{7/2}$ because there is no evidence of this, so we have then $6\times 2 = 12$ unknowns, but the $D_{3/2}$ and the $F_{5/2}$ assumption takes out 4. We have the data which are 6 a_n, the forward scattering amplitude, and the total cross-sections, so it corresponds. At one energy we would not trust this method but when we are considering smoothly varying solutions, I think it makes sense.

MANDELSTAM: I think the original analysis which gave those 4 pieces of data comes out of the 8 pieces of data you quote. So of those 8 pieces of data you have, 4 of them originally went into the calculation of these $D_{3/2}$ and $F_{5/2}$ resonance, and you have 4 over which you can play with.

LAYSON: No, I disagree. The assumption of the amplitudes is independent. It came from the shape of the resonance in the total cross-section curve but it did not have anything to do with the angular distribution.

MANDELSTAM: What! How did you get the J assignment without knowing angular distributions?

LAYSON: We took it from photo-production; this is an assumption.

MANDELSTAM: Yes, but photo-production and scattering are surely correlated? You cannot get twice as much information about which phase shifts are predominant by examining photoproduction and scattering.

FELD: I think that this only means that the data are consistent, that is to say that the photo-production and the scattering are consistent. What we looked for was, if you wish, local minima in the solutions, local least square-fit minima in the 12 dimensional array which were statistically completely consistent with the data and which also were in the region which would correspond to $D_{3/2}$ and $F_{5/2}$ resonances. This does not mean that one could not have got, by looking at other regions of this 12 dimensional space, other equally good if not better solutions (although these were perfectly satisfactory), which would also have varied monotonically with energy, and been equally consistent. We did not explore those regions just for the reason that we did not have enough independent data to do this. We did explore, we thought, far enough, in the vicinity of the general region corresponding to these resonances to satisfy ourselves that there were not any spurious solutions which would in fact have different behaviour than the ones we were looking at. This means that in some not very uniquely, nor very precisely defined region of this 12 dimensional space which contains within it amplitudes consistent with the $D_{3/2}$ and $F_{5/2}$ resonances there are solutions which have the smoothly behaving character which was shown in the curves. But these are not unique solutions, as far as we know and we will not be able to tell anything about the uniqueness until we have many more pieces of data than are now available.

MANDELSTAM: So in other words you are saying that the data are consistent with the $P_{1/2}$ resonance at that energy, but do not prove it.(*)

(*) The authors of the paper later explained that the angular distribution in the bump at 900 MeV is not consistent with one resonance only and that, if one tries to fit it with the $F_{5/2}$ wave and one other, the only possibility is the $P_{1/2}$. In that case a $P_{1/2}$ resonance would be very plausible.

LINDENBAUM: I wonder, you have resonances at the same energy in different angular momentum states for both the π interaction and the states that are responsible for the ΛK. Now the question I would raise, is that it would be simpler to understand what is going on if there were one well-defined resonance or if as indicated a couple of well-defined resonances, one would expect that they fed both the π interaction and the strange particle interaction. Yet they are separate and come at the same energy. I wonder what your comment is on this?

FELD: I would say that for whatever reasons, and there have been a number of discussions of this possibility, it would appear to us very likely from these analyses that the second $T = \frac{1}{2}$ resonance is not in a single angular momentum state but rather in at least two. And that, in fact, these complicated resonances (resonance in the sense that the imaginary part of the scattering amplitude goes through a maximum and the real part shows a dispersive form) both feed the inelastic scattering processes and also the strange particle production processes. However in the case of the ΛK production process, the threshold is so close to the resonance, and the momentum dependence on the outgoing K-meson is so strong, that it decreases the effective width (although not the intrinsic width) of the $F_{5/2}$ resonance, as far as feeding the ΛK process is concerned. Therefore the $F_{5/2}$ resonance shows itself only in effects on the angular distribution of ΛK where a small F-wave amplitude can have a large effect, but not in the total cross-section.

FRAZER: With respect to the coincidence of two resonances and also with respect to the large absorption you found in the D-wave around the position of the second resonance, it may be that all this can be understood, at least qualitatively on the basis of the model in which one concentrates one's attention primarily on the inelastic processes, particularly ϱ-meson production as was discussed by Ball and myself.

ON THE APPROXIMATE γ_5 INVARIANCE OF STRONG INTERACTION THEORY

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Recently questions of the manifestation of symmetry properties at high energies have been widely discussed. For example, Gell-Mann considers the symmetry properties following from the interaction invariance under the three-dimensional unimodular group. On the other hand, a number of authors [2, 3] *et al.* have raised the question of the existence of vector bosons with strong interactions of the form:

$$L_{\text{int}} = \sum_{\alpha=1}^{4} I_\alpha(x) B_\alpha(x) \tag{1}$$

where $B_\alpha(x)$ is the vector boson field, $I_\alpha(x)$ is the strong interaction vector current equal to $I_\alpha(x) = \Sigma g_i \overline{\psi}_i \gamma_\alpha \psi_i$. It should be noticed that interaction Lagrangians for weak, electromagnetic and strong processes of the form (1) possess one common property: they are invariant under γ_5 transformations of spinor particles

$$\psi_i \to \gamma_5 \psi_i \quad \overline{\psi}_i \to \overline{\psi}_i \gamma_5 \quad \gamma_5^2 = -1 \tag{2}$$

In the present note we discuss the following hypothesis: for high energies and large momentum transfer $s, t \gg m^2$ the matrix elements of all physical processes are invariant under the γ_5 transformations of spinor particles.

The exact meaning of the γ_5 invariance will be illustrated in the following for different physical processes. We note that in the case of an γ_5 invariant interaction, non-invariant terms appear in the scattering amplitude due to the presence of mass terms of the spinor particles in the free Lagrangian (*).

(*) Here we deliberately make an assumption about the absence of degeneration in the theory, since the latter, as has been shown in [4] can lead to the appearance of γ_5-invariant terms.