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TREATMENT OF RADIATION FOR MULTIPARTICLE TRACKING IN ELECTRON STORAGE RINGS

KOHJI HIRATA* and FRANCESCO RUGGIERO
CERN, CH-1211 Geneva 23, Switzerland

Abstract A simple prescription is proposed to treat radiation effects in large electron storage rings. When a linear element is inserted at the interaction point, this prescription reproduces correct responses.

INTRODUCTION

The aim of this paper is to present a simple, but fairly accurate method to reproduce the effect of synchrotron radiation in large electron storage rings, which can be used in multiparticle tracking [see Eqs. (2) and (10)].

The conditions that such an approximate treatment of radiation must satisfy are:

1) Without any insertion, the resulting beam distribution should reduce to a Gaussian with natural (i.e. unperturbed) standard deviations.

2) When a linear element is inserted, a fairly correct beam distribution should be reproduced, as Siemann and Krishnagopal[1] suggested recently.

The second condition is more stringent than the first and requires an accurate treatment of the radiation effects during one turn.

Recently, two independent works appeared, concerning the transient beam behaviour in storage rings; one is a theoretical work[2] and the other is a computer program SAD[3]. We will give a simplified version of them: a preliminary version was published in Ref. 4.

SPECIAL PRINCIPLE OF CAUSALITY

The motion of a particle along the arc can be described by a set of stochastic equations containing noise terms, representing the effect of the radiation diffusion. To treat them, it is simpler to use the envelope matrix R. Let us consider the

*On leave of absence from KEK, National Laboratory for High Energy Physics, Tsukuba, Ibaraki 305, Japan.

horizontal betatron oscillation and let us assume no coupling with the vertical motion. Then R is defined as follows:

$$R = \begin{pmatrix} < x_\beta^2 > & < x_\beta x'_\beta > \\ < x_\beta x'_\beta > & < x'^2_\beta > \end{pmatrix}. \tag{1}$$

Here, x_b and x'_b are betatron variables, related to the physical variables x and x' by $x = x_\beta + D\delta$, $x' = x'_\beta + D'\delta$, where D and D' are the dispersion function and its slope. The change of R, which is deterministic, from the entrance (in) to the exit (out) of the arc is expressed by the linear equation $R_{out} = \mathcal{M}_{arc} R_{in} \mathcal{M}^t_{arc} + \bar{B}_{arc}$, where the matrices $\mathcal{M}_{arc}$ and $\bar{B}_{arc}$ represent the betatron oscillation with damping and the integrated effect of diffusion, respectively. The equivalent single-particle mapping can be written

$$\begin{pmatrix} x_\beta \\ x'_\beta \end{pmatrix}_{out} = \mathcal{M}_{arc} \begin{pmatrix} x_\beta \\ x'_\beta \end{pmatrix}_{in} + \Gamma \begin{pmatrix} \hat{r}_1 \\ \hat{r}_2 \end{pmatrix}, \tag{2}$$

where Γ is a matrix such that $\Gamma\Gamma^t = \bar{B}_{arc}$, and $\hat{r}_1$ and $\hat{r}_2$ are two independent Gaussian noises with unit standard deviation and zero average.

Let us consider a transfer line of length L which contains bending magnets. We assume that the dispersion D is already known. Then, as shown in Refs. 5 and 6, we have

$$\bar{B}_{arc} = \int_0^L ds \mathcal{M}(L,s) B(s) \mathcal{M}^t(L,s), \tag{3}$$

where the diffusion matrix $B(s)$ is

$$B(s) = C_2 \begin{pmatrix} D(s)^2 & D(s)D'(s_1) \\ D(s)D'(s) & D'(s)^2 \end{pmatrix}, \tag{4}$$

with C_2 denoting the r.m.s. energy loss per unit length. It is clear that:

Lemma 1 (Special Principle of Causality) The matrices $\mathcal{M}_{arc}$ and the $\bar{B}_{arc}$ are determined only by the properties of the arc and can not be influenced from outside the arc, provided D and D' are not perturbed.

This is an exact statement. It implies that whatever linear or nonlinear kick is applied outside the arc, there is no need to recalculate $\mathcal{M}_{arc}$ and $\bar{B}_{arc}$, provided this does not change the dispersion in the arc, (which is automatically satisfied if the kick is applied where both D and D' vanish). Since we considered betatron motion alone, the lemma holds only when the dispersion in the arc is not perturbed. When considering a 6×6 envelope matrix with respect to physical variables, this restriction is no longer necessary (General Principle of Locality)[7].

SPONTANEOUS MATCHING OF THE BETATRON DIFFUSION MATRIX

The most accurate prescription would be to use Eq. (2) with the help of SAD[3]: the latter gives $\mathcal{M}$ and $\bar{B}$ correctly. We will, instead, give a simplified prescription which is valid in many realistic cases.

Regular Arc

We first consider a long, periodic, regular arc (RA), composed of a large number N of identical cells. Instead of the betatron variables x_β and x'_β, it is convenient to work in normalized variables X_1 and X_2, defined as follows:

$$\begin{pmatrix} x_\beta \\ x'_\beta \end{pmatrix} = T(s) \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}, \quad T(s) = \begin{pmatrix} \sqrt{\beta(s)} & 0 \\ -\alpha(s)/\sqrt{\beta(s)} & 1/\sqrt{\beta(s)} \end{pmatrix}. \tag{5}$$

The matrix T diagonalizes the symplectic part M of $\mathcal{M}$: $M(s_2, s_1) = T(s_2)U(\phi(s_2, s_1))T^{-1}(s_1)$, where U is a pure rotation matrix and ϕ is the betatron phase advance,

$$U(\phi) = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}, \quad \phi(s_2, s_1) = \int_{s_1}^{s_2} \frac{ds}{\beta(s)}. \tag{6}$$

Note that the Twiss parameters α and β depend only on the RA and are periodic in s with period l (cell length); $T(s + l) = T(s)$.

The envelope can be expressed by $R(s) = T(s)\Sigma(s)T^t(s)$, where Σ is the envelope matrix in normalized variables. The integrated diffusion matrix for one cell is now $\bar{B}_{cell} = T_{cell}\bar{\Theta}_{cell}T^t_{cell}$, where T_{cell} is the diagonalizing matrix at the entrance or exit of a cell. It is convenient to write $\bar{\Theta}_{11} = a+b$, $\bar{\Theta}_{12} = \bar{\Theta}_{21} = c$ and $\bar{\Theta}_{22} = a-b$. Then a is just the trace part of $\bar{\Theta}$. Since $\bar{\Theta}$ is positive definite, we have $|b^2+c^2| \leq a^2$.

Since the regular arc consists of N identical cells, Eq. (3) for the RA becomes

$$\bar{\Theta}_{RA} \equiv \sum_{n=0}^{N-1} U^n_{cell}\bar{\Theta}_{cell}U^{tn}_{cell},$$

where $U_{cell} = U(\phi_{cell})$ and we ignored damping in calculating $\bar{\Theta}$. It is now easy to show that $a_{RA} = Na_{cell} \equiv N\kappa$ and $b^2_{RA} + c^2_{RA} = (\sin N\phi_{cell}/\sin\phi_{cell})^2(b^2_{cell} + c^2_{cell})$. Therefore, provided the factor $(\sin\phi_{cell})^{-1}$ is of order unity, we obtain

$$\bar{\Theta}_{RA} = N\kappa I + O(\kappa)[\text{traceless part}] \simeq N\kappa I, \tag{7}$$

where I denotes the 2×2 identity matrix. This result is valid in many realistic cases and, in the original betatron variables, it implies that $\bar{B}_{RA}$ is matched to the Twiss ellipse, $T_{cell}T^t_{cell}$.

Matched Insertions

The transfer matrix for the (whole) arc is expressed by $M_{arc} = M_1 M_{RA} M_2$, where M_1 and M_2 are insertions at the exit and entrance of the RA, respectively, such as dispersion suppressor and final focusing lattice. When the insertions are matched to the RA, in the diagonalizing basis used before, M_1 and M_2 are represented simply by rotations U_1 and U_2 and the diagonalizing transformation remains the same. Thus, Eq. (7) holds also for the whole arc.

The dispersion suppressor can be another source of radiation. Its contribution, however, does not increase with N, so that when N is large, it can be ignored. The 'spontaneous matching', thus, applies also to the whole arc.

Natural Emittance

Let us close the arc by identifying the entrance and the exit and let us call this point the IP. The change of the envelope at the IP for each turn is $\Sigma' = \lambda^2 U \Sigma U^t + \bar{\Theta}$, where we put $\mathcal{M} \to \lambda U$, with λ being the damping rate. After many turns, the beam will reach an equilibrium envelope Σ_∞, defined by $\Sigma_\infty = \lambda^2 U \Sigma_\infty U^t + \bar{\Theta}$.

When we apply our simplified mapping, with $\bar{\Theta} = N\kappa I$, we obtain $\Sigma_\infty = N\kappa I/(1-\lambda^2)$. Since the emittance ϵ is defined by $\epsilon \equiv \sqrt{\det R} = \sqrt{\det \Sigma}$, we have $N\kappa = (1-\lambda^2)\epsilon$, so that we can express $\bar{\Theta}$ by two parameters.

DYNAMIC BETA AND DYNAMIC EMITTANCE

Mismatched Insertion

Let us add a linear element at the IP so that the whole insertions are no longer matched to the RA. This new insertion can not be diagonalized by the old T. We still use the old T, which diagonalized M_{RA}. The new insertion can be represented by a symplectic matrix K so that the envelope is changed at the IP as $\Sigma_+ = K\Sigma_- K^t$. The equilibrium value of Σ_- satisfies the equation $\Sigma'_\infty = \lambda^2 U K \Sigma'_\infty K^t U^t + (1-\lambda^2)\epsilon I$. Here we have assumed that the IP is dispersion free, i.e. that the dispersion function and its derivative are both zero: therefore the insertion does not affect $\bar{\Theta}_{RA}$.

We define

$$\tilde{U} \equiv UK = \begin{pmatrix} \cos\bar{\mu} + A\sin\bar{\mu} & B\sin\bar{\mu} \\ -C\sin\bar{\mu} & \cos\bar{\mu} - A\sin\bar{\mu} \end{pmatrix} \tag{8}$$

where $\cos\bar{\mu} = \mathrm{tr}\tilde{U}/2$. From symplecticity, $C = (1+A^2)/B$. We restrict K within a range where $\bar{\mu}$ remains real so that B is always positive.

Provided $\bar{\mu}$ is not close to a half integer, the new equilibrium is given by[4],

$$\Sigma'_\infty = \bar{\epsilon} \begin{pmatrix} B & -A \\ -A & C \end{pmatrix} + O(\delta), \tag{9}$$

where $\bar{\epsilon} = \sqrt{\det \Sigma'_\infty} = \epsilon(B+C)/2$ is the new emittance and $\delta = 1-\lambda^2$. Since $B+C$ is always larger than two, we obtain:

Lemma 2 (Emittance Growth due to Mismatch) When the insertion is mismatched, the equilibrium envelope has a larger emittance than in the case of a matched insertion.

The case of a thin quadrupole insertion is of particular interest in connection with the beam-beam interaction[8]. When the beam-beam force is approximated by a linear force, K is given by a kick matrix with $K_{21} = -4\pi\xi$, with ξ being the beam-beam parameter. This case was discussed in Ref. 4. Now, B gives the well known dynamic beta effect[9], $\bar{\beta} = B\beta$, whereas the change of the equilibrium emittance is called the dynamic emittance effect: $\bar{\epsilon} = \epsilon(B+C)/2$. The new equilibrium beam size is given by $< x_\beta^2 >' = \bar{\beta}\bar{\epsilon}$, apart from $O(\delta)$ and $O(\kappa)$ terms.

If we rematch the whole insertion, we can cancel all the linear effect of the beam-beam interaction. This could help the luminosity limit. However, we may not expect too much, because the real difficulty of the beam-beam interaction comes from its nonlinear nature.

DISCUSSION

Our prescription for the treatment of synchrotron radiation in multiparticle tracking is thus

$$\begin{pmatrix} X \\ P \end{pmatrix}_{out} = \lambda U(\mu) \begin{pmatrix} X \\ P \end{pmatrix}_{in} + \sqrt{\epsilon(1-\lambda^2)} \begin{pmatrix} \hat{r}_1 \\ \hat{r}_2 \end{pmatrix}. \tag{10}$$

It was already proposed in Ref. 4, using the smooth approximation. Here we have extended its validity to strong focusing machines. In Ref. 4, this was numerically shown to be accurate enough and the same holds true in case of skew quadrupole insertion[7].

We showed 1) the spontaneous matching of the diffusion matrix and 2) the dynamical emittance effect due to mismatched insertion. We restricted ourselves to the horizontal betatron oscillation. A more extended and detailed paper will be published elsewhere[7].

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