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FIELD THEORY OF NULL STRINGS AND p–BRANES

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ABSTRACT

In analogy with the Feynman–Landau propagator for the massless particle, we find expressions for the field theory propagators of null strings and p–branes, as well as the corresponding supersymmetric versions. These null propagators are presumably useful building blocks for the propagators of the "tensionfull" theories, again in analogy with the massive particle.

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The string appears as a fundamental object whose properties allow us to envisage the unification of all known interactions [1]. The enigma of $N = 1$, $D = 11$ supergravity in the stringy context has motivated the formulation of supermembranes [2] and the classification of all possible Nambu–type theories of extended objects [3]. Now, from the quantum viewpoint, all particle theories are quantum field theories and a long–standing problem is how to formulate such a theory for the string [4], the membrane [5], or, more generally, for the p–brane ($p = 1$ for the string). In this letter we wish to address the formulation of p–brane field theory, along the lines formulated in Ref.[6]. The first and simplest construction to investigate is the p–brane propagator, whose knowledge is a crucial step in the understanding and formulation of a possible p–brane field theory. Of course the inherent non–linearities of these objects make the situation pretty much hopeless, but a significant simplification takes place if we consider the tensionless limit of a Nambu–Dirac p–brane [7], a straightforward generalization of Schild's null string [8, 9].

Just as massless particles are simpler to analyze (though perhaps slightly more paradoxical and less intuitive) than massive ones, and null or tensionless strings simplify usual strings while preserving most of the crucial peculiarities of the theory [9], in this letter we propose to study the null or tensionless limit of p–branes in order to construct the (field theory) propagator. We produce explicit expressions for the propagators of null strings and their supersymmetric extensions [9, 10]. Furthermore, since the Lagrangian for a (world–volume supersymmetric) p–brane with zero tension contains terms at most quadratic in the matter fields, our arguments can be trivially extended to the $p > 1$ cases and their supersymmetric extensions.†

Our approach is based on the simple observation that all the points of a null extended object move at the speed of light, subject to the customary transversality constraints. In general, a null p–brane is a dynamical system defined by the $p + 1$ first–class constraints

$$\mathcal{H}_n = \partial_0 X \cdot \partial_n X = 0 \qquad , \qquad n = 0, 1, \ldots, p \tag{1}$$

where ∂_n is the derivative with respect to the n–th internal co-ordinate. The difference between the null and the usual (massive or tensionful) case is that in the latter case, the first constraint is much more involved:

$$\mathcal{H}_0^{\text{usual}} = (\partial_0 X)^2 + T^2 \det(\partial_i X \cdot \partial_j X) \qquad , \qquad i, j = 1, \ldots, n \tag{2}$$

When we consider null extended objects, we are effectively turning off the self-interactions, which are of order $2p$. This is an enormous simplification for $p > 1$, obviously.

† We avoid the "no–go theorem" of Ref. [11] because the starting Lagrangian density for the null p–brane is g and not $\sqrt{-g}$.

A null string may be approximated by a set of M massless non–interacting particles, and the exact null string is obtained in the $M \to \infty$ limit. Similarly, a null 2–brane can be thought of to be made of $M \to \infty$ null strings, and so on. That the p–branes are null is crucial in this recurrence, because otherwise the tension terms would induce non–linear interactions which would spoil the recurrence. Presumably, just as in the particle case, the tension or mass can be considered as a perturbation, and we expect that this perturbation can be (as in the particle case) summed to all orders and recast as a tensionfull (or massive) propagator.

Let us start by considering a relativistic massless particle in a D–dimensional Minkowski space–time whose action is

$$S = \int_{\tau_1}^{\tau_2} d\tau \, \frac{1}{2N} \dot{X}^2 \tag{3}$$

with N the einbein. The reparametrization symmetry of (3) is, in its infinitesimal form,

$$\begin{aligned} \delta X^\mu &= \epsilon \dot{X} \\ \delta N &= (\epsilon N)^{\cdot} \end{aligned} \tag{4}$$

In order to quantize this system, we fix Teitelboim's proper–time gauge [12] and follow the Faddeev–Popov procedure. The functional integral is

$$Z = \int \mathcal{D}N \mathcal{D}X \det(N)^{-1} \delta[\dot{N}] \det \left(\frac{\delta \dot{N}}{\delta \epsilon} \right) e^{-S} \tag{5}$$

where the factor $\det(N)^{-1}$ is necessary to ensure the gauge invariance of the measure. In order to calculate the Faddeev–Popov determinant, we use the second of Eq. (4); the result is $\det(N\partial_\tau^2)$. Using ζ–function regularization, it is easy to manipulate this determinant and show that

$$\det(N\partial_\tau^2) = \det N \ \det \partial_\tau^2 = \Delta\tau \ \det N \tag{6}$$

where $\Delta\tau = \tau_1 - \tau_2$ and the equalities hold after ζ function regularization. The factor $\delta[\dot{N}]$ indicates that only the zero mode $N(0)$ of N contributes, and Eq. (5) can be written as

$$Z = \int d\nu \int \mathcal{D}X e^{-S} \tag{7}$$

with $\nu = N(0)\Delta\tau$.

The integration limits of ν are 0 and ∞, as follows from standard discussions [13]. The integration in X can be done with the help of the change of variables

$$X^\mu(\tau) = X^\mu(\tau_1) + \frac{\Delta X^\mu}{\Delta\tau}(\tau - \tau_1) + Y^\mu(\tau) \tag{8}$$

where $Y^\mu(\tau)$ is a quantum fluctuation which, by consistency, satisfies

$$Y^\mu(\tau_1) = Y^\mu(\tau_2) = 0 \tag{9}$$

With this in mind, the generating functional becomes

$$Z = \int_0^\infty d\nu\ \nu^{-D/2} \exp\left[-\frac{(\Delta X)^2}{2\nu}\right] = G(X_2, X_1) \tag{10}$$

which is Landau's propagator for the relativistic particle [13].

This simple result can be easily extended to the case of null strings. Indeed, consider the null string as made of M free massless relativistic particles, each a distance δ apart from its neighbours, moving perpendicularly to the string (in order to satisfy the constraint $\dot{X} \cdot X' = 0$). The world lines of these constituent point particles never intersect. The action for this discretized null string is

$$S = \sum_{j=1}^{M} S_j = \sum_{j=1}^{M} \int_{\tau_1}^{\tau_2} d\tau\ \frac{1}{2N_j} \dot{X}_j^2 \tag{11}$$

where j labels the points of the string. In the continuum limit ($M \to \infty$, $\delta \to 0$) it becomes σ, the space-like co-ordinate of the string.

Using (10), the propagator for the M-particle system is

$$G(X_2[\sigma], X_1[\sigma]) = \int d\nu_1 \cdots d\nu_M\ \nu_1^{-D/2} \cdots \nu_M^{-D/2} \exp\left[-\sum_{j=1}^{M} \frac{(\Delta X)_j^2}{2\nu_j}\right] \tag{12}$$

where $X_1[\sigma]$ and $X_2[\sigma]$ are the initial and final null string configurations, respectively. In the continuum limit, the propagator for the null bosonic string is thus

$$G(X_2[\sigma], X_1[\sigma]) = \int \mathcal{D}\nu(\sigma)\ \nu(\sigma)^{-D/2}\ \exp\left[-\int d\sigma \frac{(\Delta X)^2}{2\nu(\sigma)}\right] \tag{13}$$

It is not hard to check that the propagator (13) satisfies indeed $\bar{\Box} G(X_2, X_1) = \delta(X_2 - X_1)$, with $\bar{\Box}$ the two-dimensional D'Alembertian.

The generalization to higher-dimensional extended objects is straightforward. For instance, take the concrete example $p = 2$ and parametrize the membrane world-volume by the three real co-ordinates $(\sigma_0, \sigma_1, \sigma_2)$. Then discretize σ_2, effectively splitting the membrane into M null strings a distance δ apart. This can always be done because of the first class constraints (1) above. The discretized membrane is thus made out of M non-intersecting null strings, whose world-sheets do not intersect. These null strings move perpendicularly to the membrane, in complete analogy to the lower-dimensional situation discussed above. The discretized null membrane propagator is

$$G(X_2[\vec{\sigma}], X_1[\vec{\sigma}]) = \int \prod_{j=1}^{M} d\nu_j(\sigma_1)\ \nu_j^{-D/2}(\sigma_1)\ \exp\left[-\sum_{j=1}^{M} \int d\sigma_1 \frac{(\Delta X)_j^2}{2\nu_j(\sigma_1)}\right] \tag{14}$$

The general case of a p-brane follows immmediately: it suffices to discretize it into M different $(p-1)$-branes and follow the same steps above. The propagator in the continuum limit is

$$G(X_2[\vec{\sigma}], X_1[\vec{\sigma}]) = \int \mathcal{D}\nu(\vec{\sigma})\ \nu(\vec{\sigma})^{-D/2}\ \exp\left[-\int d^p\sigma \frac{(\Delta X)^2}{2\nu(\vec{\sigma})}\right] \tag{15}$$

We may also extend this result to the locally supersymmetric case. Calculating in the same proper-time gauge, the result for the supersymmetric particle propagator is simply

$$G(X_2, X_1) = \int_0^{\infty} d\nu\ \nu^{-1-D/2}(\gamma^\mu \Delta X_\mu)\ \exp\left[-\frac{(\Delta X)^2}{2\nu}\right] \tag{16}$$

where the γ^μ are D-dimensional Dirac gamma matrices, realizing the zero-modes of the fermionic constraint.

We now consider the null superstring [9] as the continuum limit of an assembly of M massless spinning particles; the action for this system can be written as

$$S = \sum_{j=1}^{M} S_j = \sum_{j=1}^{M} \int d\tau \left(\frac{1}{2N_j}\dot{X}_j^2 - \frac{i}{2}\dot{\theta}_j^\mu \theta_{\mu j} + \frac{\lambda_j}{2N_j}\theta_j^\mu \dot{X}_{\mu j}\right) \tag{17}$$

where the θ's are real fermionic variables and the degrees of freedom embodied in the λ's are those of the gravitino (just like the Lagrange multiplier N embodies the vielbein). The propagator for a null supersymmetric p-brane, in the continuum limit, is

$$G(X_2[\vec{\sigma}], X_1[\vec{\sigma}]) = \int \mathcal{D}\nu(\vec{\sigma})\ \nu(\vec{\sigma})^{-1-D/2}\ \gamma^\mu \Delta X_\mu(\vec{\sigma})\ \exp\left[-\int \frac{(\Delta X)^2}{2\nu(\vec{\sigma})} d^p\sigma\right] \tag{18}$$

All of the above expressions satisfy the equation $\Box G = \delta(X_2 - X_1)$, as can be easily checked explicitly. The propagator for the null spinning string, Eq. (18) with $p = 1$, agrees with the limit $\alpha' \to \infty$ of the usual spinning string propagator calculated in Ref. [14]. This lends further evidence that the null string embodies indeed the $\alpha' \to \infty$ limit of usual strings. The propagator found above is a necesary and crucial ingredient in the search for a p-brane field theory. The introduction of a dimensionfull constant as a perturbation is a separate question, requiring further study.

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