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THE CONVOLUTION INTEGRAL FOR THE FORWARD-BACKWARD ASYMMETRY IN e^+e^-- ANNIHILATION

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ABSTRACT

The complete convolution integral for the forward-backward asymmetry A_{FB} in e^+e^--annihilation is obtained in order $O(\alpha)$ with soft photon exponentiation. The influence of these QED corrections on A_{FB} in the vicinity of the Z peak is discussed. The results are used to comment a recent ad-hoc ansatz using convolution weights derived for the total cross-section.

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Nearly twenty years ago, the convolution integral for first-order QED initial state radiation in e^+e^--annihilation was derived [1]:

$$\sigma_T^e(s) = \int_o^1 dv\ \sigma_T^o[\ s(1-v)\ ]\ \rho_e(v), \tag{1}$$

where σ_T^o is the Born cross-section, $v \equiv 1-s'/s$, $s = 4E_b^2$, s' the effective mass of the produced particles and

$$\rho_e(v) = [\ \delta(v)\ S_e + \theta(v-\epsilon)\ H_e(v)\ ], \tag{2}$$

$$S_e = Q_e^2\ \frac{\alpha}{\pi}\ [\ (L_e-1)\ (2\ln\epsilon + 3/2) + \pi^2/3 - 1/2\ ], \tag{3}$$

$$H_e(v) = Q_e^2\ \frac{\alpha}{\pi}\ \left[\ \frac{1+(1-v)^2}{v}\ (\ L_e-1\ )\ \right], \tag{4}$$

$$L_e = \ln s/\ m_e^2\ , \qquad Q_e = -\ 1, \tag{5}$$

and v is the energy of the emitted photon in units of the beam energy E_b. Since then, the treatment of QED corrections to annihilation processes has been considerably improved. One of the best studied reactions is muon production,

$$e^+\ e^-\ \longrightarrow\ \mu^+\ \mu^-\ (\gamma), \tag{6}$$

which at the storage ring LEP will be measured in the vicinity of the Z-boson resonance with an accuracy of the order of 0.1%. In addition to (3,4), further QED radiative corrections have been calculated for this energy region: leading logarithmic corrections to initial state radiation [2]; first order corrections also for initial-final interference and final state radiation [3]; and analytic higher order corrections of the initial state radiation for the total cross section [4,5].

As interesting as σ_T is the C-odd forward-backward asymmetry A_{FB},

$$A_{FB} = \sigma_T^{-1} \left[\int_o^1 dc \frac{d\sigma}{dc} - \int_{-1}^{o} dc \frac{d\sigma}{dc} \right], \tag{7}$$

where $c = \cos\theta$ is the scattering angle of μ^+ with respect to e^+. Besides numerical results obtained with Monte Carlo programs, there exists a complete analytic calculation of the $O(\alpha)$ QED corrections to A_{FB} [6]. Recently, leading logarithmic corrections to initial state radiation have been reported [7]. Both results are obtained without cuts on the photon phase space. In this letter, we present the convolution integral for bremsstrahlung corrections to A_{FB}. Of course, one cannot expect that it is characterized by the weight functions $\rho_a(v)$, $a = e,i,f$, which have been derived for the C-even total cross-section (a=e: initial state radiation, a=f: final state radiation, a=i: their interference). Leaving out all technical details [8], we present here the result of an integration of the corresponding squared matrix element over the photon momentum phase space. Hereby we will use a sufficiently general notation which allows a common treatment of the complete $O(\alpha)$ correction:

$$A_{FB}(s) = \frac{\sigma_{F-B}(s)}{\sigma_T(s)} , \tag{8}$$

$$\sigma_{F-B}(s) = \sum_{\substack{a=e,i,f \\ k,l=1,2}} \mathrm{Re} \int_o^1 dv \, \sigma_{F-B}^{a,o}(s,s';B_k,B_l) \; r_a(v;B_k,B_l) \; . \tag{9}$$

Here, the B_k are two interfering vector bosons (B_1=Z, B_2=γ) and:

$$\sigma^{f(e),o}_{F-B}(s,s';B_k,B_l) = \sigma^{o}_{F-B}(s^{(\prime)},s^{(\prime)};B_k,B_l), \tag{10}$$

$$\sigma^{i,o}_{F-B}(s,s';B_k,B_l) = \frac{3}{4}\,\sigma^{o}_{T}(s,s';B_k,B_l), \tag{11}$$

$$\sigma^{o}_{A}(s,s';B_k,B_l) = \frac{4\pi\alpha^2}{3s'}\, C_A(B_k,B_l)\,\frac{1}{2}\left[\kappa_k(s')\kappa^*_l(s)+\kappa_k(s)\kappa^*_l(s')\right],$$

$$A = T, F\text{-}B, \tag{12}$$

$$\kappa_l(s) = \frac{s}{s - m_l^2}, \tag{13}$$

$$m_l^2 = M_l^2 - i\, M_l\Gamma_l(s), \tag{14}$$

$$\Gamma_l(s) = \frac{s}{M_l^2}\,\Gamma_l, \tag{15}$$

where $C_A(B_k,B_l)$ are corresponding coupling constant combinations; see e.g. [6].

The convolution weights for A_{FB} are:

$$r_a(v;B_k,B_l) = \delta(v)\, s_a(B_k,B_l) + \theta(v-\epsilon)\, h_a(v), \quad a = e,i,f. \tag{16}$$

Up to a normalization, the soft plus vertex contributions $s_a(B_i,B_j)$ for a = e,f are the same as introduced in (3). Of course, for s_f one has to use the charge Q_f and mass m_f of the produced fermions :

$$s_a = Q_a^2\,\frac{\alpha}{\pi}\left[(L_a-1)\,(2\ln\epsilon + 3/2) + \pi^2/3 - 1/2\right], a=e,f. \tag{17}$$

For the initial-final interference, the soft plus $\gamma\gamma$, γZ - box parts deviate from the C-even corrections. They also depend on the interfering intermediate particles due to the box terms:

$$s_i(B_k,B_l) = Q_e Q_f \frac{\alpha}{\pi} \left\{ - (1+8\ln 2)\ln\frac{2\epsilon}{\lambda} + 4\ln^2 2 + \ln 2 + \frac{1}{2} + \frac{\pi^2}{3} \right.$$
$$\left. - \quad B(B_k,B_l) \right\} , \tag{18}$$

$$B(Z,Z) = H^T_{1,\text{box}} , \qquad B(\gamma,\gamma) = F^T_{1,\text{box}} , \tag{19}$$

$$B(Z,\gamma) = \frac{1}{2} \left[B^*(\gamma,\gamma) + B(Z,Z) \right] , \tag{20}$$

where the functions $H^T_{1,\text{box}}$, $F^T_{1,\text{box}}$ are from [6]. The infra-red regulator λ has also been defined there. Finally, the C-odd weight functions due to hard bremsstrahlung are:

$$h_e(v) = Q_e^2 \frac{\alpha}{\pi} \frac{[1+(1-v)^2]}{v} \frac{(1-v)}{(1-\frac{v}{2})^2} \left[(L_e-1) - \ln\frac{(1-v)}{(1-\frac{v}{2})^2} \right] , \tag{21}$$

$$h_i(v) = \frac{2}{3\ k} \left[2(1-v)(v^2+2v-2) + (1-v)(5v^2-10v+8)\ln(1-v) \right.$$
$$\left. + (5v^3-18v^2+24v-16)\ln(2-v) \right] , \tag{22}$$

$$h_f(v) = \frac{2}{v} \left[(1-v)(L_f-1) + \ln(1-v) + \frac{1}{2} v^2 L_f \right] . \tag{23}$$

Soft photon emission from the initial state is isotropic and thus does not change the C-parity behaviour of the convoluted Born amplitude (the same is true for the final state soft photon emission but not for corrections due to the interference). Thus, the exponentiation of the soft photon part for initial state radiation can be carried out for A_{FB} in exactly the same manner as for σ_T [2,4-5]. To do so we perform the following replacements:

$$\int_0^1 dv\ \sigma^{F-B,o}(s')\left[\ \delta(v)\ (1+s_e) + \theta(v-\epsilon)\ \frac{\beta_e}{v}\right] \longrightarrow$$

$$\longrightarrow \left(1+\bar{s}_e\right)\int_o^1 dv\ \sigma^{F-B,o}(s')\left\{\ \delta(v)e^{\beta_e \ln\epsilon} + \theta(v-\epsilon)\ \frac{\beta_e}{v}\ e^{\beta_e \ln v}\right\}$$

$$= \left(\ 1+\bar{s}_e\right)\ \int_o^1 dv\ \sigma^{F-B,o}(s')\ \beta_e\ v^{\beta_e - 1}\ , \qquad (24)$$

$$\beta_e = Q_e^2\ \frac{2\alpha}{\pi}\ \ (\ L_e - 1\ )\ , \qquad (25)$$

$$\bar{s}_e = Q_e^2\ \frac{\alpha}{\pi}\left[\ \frac{3}{2}\ (\ L_e - 1) + \frac{\pi^2}{3} - \frac{1}{2}\ \right]. \qquad (26)$$

The soft photon exponentiated correction (24) contains a piece from the hard photon function which is remnant of the cancelled infra-red divergency, so that we must replace in parallel:

$$h_e(v) \longrightarrow \bar{h}_e(v) = h_e(v) - \frac{\beta_e}{v}. \tag{27}$$

Let us now discuss the formulae derived above. An integration of (9) without restriction on the photon energy ($k_o^{max}/E_b = v^{max} = \Delta = 1$) leads to the explicit analytic expressions of [6] for A_{FB} which had been derived there using another Lorentz frame and different variables. An integration with superimposed photon energy cut ($v \leq \Delta < 1$) may also be done explicitly [8]. The numerical influence of the different contributions to A_{FB} at $\sqrt{s} = M_Z$ is shown in table 1 as a function of the photon cut energy k_o^{max}. For tight cuts, the influence of interference and final state radiation rises due to rising imbalance of bremsstrahlung with box and vertex contributions, resp. In principle one should also exponentiate final state soft photon radiation [9], though this would be numerically only a minor

Table 1 Forward-backward asymmetry in per cent at $\sqrt{s} = M_Z$ as a function of the hard photon energy cut k_o^{max} for $M_Z = 92$ GeV $\Gamma_Z = 2.5$ GeV, $\sin^2\theta_W = 0.23$. The contributions are included stepwise:

- ini. Born plus $O(\alpha)$ initial state radiation,
- exp.ini. exponentiated soft photon initial state radiation,
- fin. $O(\alpha)$ final state radiation,
- interf. $O(\alpha)$ interference bremsstrahlung.

A_{FB} / k_o^{max}	ini.	exp.ini.	fin.	interf.
no cut	-0.5221	0.2469	0.2463	0.2783
10 GeV	-0.4280	0.3365	0.2200	0.2749
1 GeV	0.6974	1.1857	0.9544	2.2725

improvement if one compares the expected effect with the anticipated experimental accuracy at the Z peak of about 0.3%. We applied an additional naive exponentiation of final state soft photons and observed an influence of less than 0.15%.

The interest in a convolution representation for the forward-backward asymmetry as derived here has been stimulated recently by the need for sufficiently effective algorithms for the study of the Z peak. Since the C-odd weight functions (21-23) were not known then, it was proposed in [10] to use instead C-even functions, e.g. (2) for initial state radiation (including higher order corrections [2,5]):

$$\bar{\sigma}_e^{F-B}(s) = \int_o^1 dv \, \sigma^{F-B,o}(s') \quad \rho_e(v) \ . \tag{28}$$

Strictly speaking, an ansatz like (28) is wrong. A C-odd quantity like A_{FB} is not intrinsically related to weights which determine the behaviour of the C-even total cross-section. Nevertheless, for the Z peak region it has been shown numerically [10] and analytically [11] that (28) is in excellent agreement with the correct result. This may be explained by soft photon dominance. To do so, we quote here besides $H_e(v)$ in (4) also the two other C-even hard photon weights:

$$H_i \ (v) = Q_e Q_f \frac{\alpha}{\pi} \frac{3}{v} \ (\ 1-v \ ) \ (\ v-2 \ ) \ , \tag{29}$$

$$H_f \ (v) = Q_f^2 \frac{\alpha}{\pi} \frac{1}{v} \ [\ 1+(1-v)^2 \ ] \ [\ (L_f-1) + \ln(1-k) \ ] . \tag{30}$$

In the soft photon region of the photon momentum phase space, C-even and C-odd weights agree ($H_{e(f)}(v) \sim h_{e(f)}(v)$, $v \longrightarrow o$). As has been mentioned already for the soft contributions (17), photon emission is isotropic there. Consequently, it does not influence the C-parity behaviour of the cross-section part to be convoluted. Applying a photon energy cut in (28) makes the agreement with (9) even better. This is what one really observes even if the final state radiation contribution then does not remain small. In contrast to initial and final state radiation, the interference bremsstrahlung corrections to σ_T and A_{FB} differ even in the soft photon limit due to anisotropic emission (compare (22) and (29) and the corresponding soft parts in (18) and in [6]). So, if the interference bremsstrahlung and/or hard bremsstrahlung become numerically important the ad hoc ansatz (28) fails. At the Z peak both pieces are suppressed. This has been shown for the interference in [6,12]. Further, the Breit-Wigner resonance function for the dominating Z-exchange cross-section is:

$$|\kappa_Z(s')|^2 \sim \{ [s^2(1-v)^2 - M_Z^2] + M_Z^2 \Gamma_Z^2 \}^{-1}. \tag{31}$$

For $s - M_z^2$, the resonance behaviour is completely lost for hard photon emission ($v \leq 1$). This soft photon dominance at the Z peak together with the above discussion proves that the ansatz (28) is completely justified there. Away from the Z peak one has either no hard photon suppression (loose energy cut) or no interference suppression (this also happens for tight cuts at the Z peak). Then (28) fails and has to be replaced by the correct result (9,21-24) which has been presented in this letter.

Table 2 contains a comparison of A_{FB} and the ad hoc $\bar{A}_{FB}$, where we additionally had to leave out the interference bremsstrahlung part in $\bar{A}_{FB}$ because there is no reasonable prediction for it. Everywhere in the LEP1 energy range the agreement of the two definitions is quite good if no very tight cuts are applied. Then the influence becomes large. One should also remark that there is no sense in an inclusion of higher order initial state radiation corrections beyond the soft photon case into $\bar{A}_{FB}$ because for hard photons the ad hoc ansatz is definitely wrong.

To summarize, we have derived the correct convolution representation for A_{FB}. The ad hoc ansatz $\bar{A}_{FB}$ gives in the Z peak region reasonable numerical results. Nevertheless, we see no further need to use $\bar{A}_{FB}$ in view of the results presented here and in [7].

Table 2 Comparison of A_{FB} (9,24) and $\tilde{A}_{FB}$ in per cent.
- upper rows: exact A_{FB} ,
- lower rows: $\tilde{A}_{FB}$ with the ad hoc ansatz for initial and final state radiation using the C-even convolution weight functions (interference neglected); parameters as in table 1.

$\sqrt{s}$ k_o^{max}	82GeV	91GeV	92GeV	93GeV	102GeV
BORN	-68.001	-6.652	1.886	10.129	59.646
(no cut)	-53.584	-8.048	0.278	6.795	20.285
	-54.135	-8.131	0.239	6.784	20.117
10GeV	-67.781	-8.023	0.275	6.748	29.507
	-68.234	-8.177	0.128	6.785	29.956
1GeV	-65.613	-5.147	2.272	9.552	63.134
	-68.211	-7.442	0.954	8.778	59.077

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