Multijet Production in W, Z Events at $p\bar{p}$ Colliders

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Abstract

We calculate cross sections for the production of 0,1,2 and 3 jets in W, Z events at $p\bar{p}$ collider energies. Full leptonic decays of the weak bosons are included. We analyse in particular the relative rate of multijet production, the contribution of the different subprocesses and some overall properties of the events.

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crossing all the relevant subprocesses can readily be obtained. In addition, the decay of the vector boson into leptons is very easy to implement in this scheme. The calculation makes use of a recursive method [8], to obtain compact expressions for the matrix elements. The usual method of calculating the helicity amplitudes by choosing special polarization directions for the gluons in order to reduce the number of contributing Feynman diagrams [9] is not used here. This latter method gives, in the case of the $q\bar{q}ggg$ amplitudes, insufficient simplification to obtain compact expressions. Therefore a more powerful method, based on the recursion relation, is used. This relation allows a rearrangement of the matrix element in such a way that the gluon polarization vector ϵ_μ^λ only appears in the following combination with its corresponding momentum K_μ:

$$F_{\mu\nu}^\lambda = K_\mu \epsilon_\nu^\lambda - K_\nu \epsilon_\mu^\lambda. \tag{2.1}$$

This "abelian" field-strength F is gauge invariant. The expression for the amplitude is written entirely in terms of these field-strengths and no explicit helicity vectors are present. The resulting expressions are very compact and systematic. If one specifies the helicity of the gluons, the matrix element directly reduces, without much calculational effort, to a short expression involving spinorial inner-products, which can be evaluated numerically. All the expressions are extensively tested for the correct collinear and soft behaviour [10]. The abelian limit of $q\bar{q}q'\bar{q}'g$ is also compared with known QED amplitudes [12] by replacing the vector boson by an on-shell photon. The matrix elements with five partons have also been checked with those of [5] and [11], the latter concerning the decay of γ^* into five partons.

The matrix elements thus obtained are incorporated in a computer program which calculates the cross section for a $W^\pm$ or Z and up to 3 jets. The W decays into $e\nu$ and the Z into e^+e^- or $\nu\bar{\nu}$. In both cases we take four quark flavours in the initial state and add to that set the bottom quark for the final state. As discussed in the next section, rapidity, transverse momentum and separation cuts are applied to the final state leptons, quarks and gluons.

There are a number of relevant process statistics, shown in Tables 1-3. In Table 1 the number of subprocesses for each process is presented. In the $W^\pm$ case we have also counted the number of subprocesses when the KM matrix is set to unity. This approximation has been used in the cross section calculations of [7]. We have checked that using this approximation gives a $O(2\%)$ overestimate of the 0 jet cross section, and that this overestimate becomes smaller for higher numbers of jets as initial state gluons become more important.

The time needed to calculate the various matrix elements at the parton level is presented in Table 2. There is no difference in time when V is a Z or a W, except in the four quark case. In this case we make explicit use of the fact that the W has only a left-handed coupling to fermions. We also give the number of times that we have to call a particular matrix element for each accepted event. This represents the number of different kinematic situations which we have to calculate before we are able to compute all the subprocesses. For example, to be able to calculate all 205 subprocesses for the $Z + 3$ jets cross section, we have to call the $q\bar{q}q'\bar{q}'g$

1 Introduction

One of the most important Standard Model physics processes at high energy hadron-hadron colliders is the production of a W or Z with accompanying hadronic (quark or gluon) jets. The importance of such processes has already been established at the CERN $p\bar{p}$ collider [1]. Essentially *any* "new physics" process (heavy quarks, SUSY, ...) can be mimicked by $W, Z +$ jets production, and it is therefore vitally important to be able to estimate these background processes accurately. At the same time, quantitative tests of the Standard Model are possible – the UA2 collaboration has recently reported a measurement of the strong coupling α_s from the relative rate of $W + 1$ jet production [2].

In the last few years, some of us have been involved in detailed studies of the production of $W, Z+0,1,2$ jets. This work involved first calculating the appropriate matrix elements [3] – using "spinor techniques" – and then combining these with phase space and parton distribution integrations to calculate physical cross sections [4]. Recently, the calculational techniques have been extended to allow the matrix elements for $W, Z + 5\{q, g\}$ to be evaluated [5,6]. A complete analysis of up to 3 jet production with W or Z is therefore now possible.

In this letter we summarise the important features of $0, 1, 2$ and 3 jet production in W, Z events at the CERN ($\sqrt{s} = 630\ GeV$) and FNAL Tevatron ($\sqrt{s} = 1.8\ TeV$) $p\bar{p}$ colliders. Our analysis is not intended to be exhaustive – we concentrate rather on the overall properties of the cross sections such as relative total rates, subprocess contributions, etc. Detailed phenomenology requires a proper simulation of the events in a given detector situation.*

In section 2 a summary of how the various elements of the calculation are obtained and incorporated in the computer program is presented. Section 3 describes some of the phenomenological results.

Finally, we should mention that a similar and independent analysis of multijet production with W and Z – based on the matrix element calculation of [5] – has recently been reported [7]. We believe that the importance of an accurate calculation of these processes warrants a complementary analysis. In fact we focus, in general, on different properties of the cross sections and present results at *both* CERN and FNAL collider energies. †

2 Calculational details

In this section we briefly discuss the method used to calculate the matrix element and the implementation in a computer program. For the calculation of the matrix elements we use the results of [6]. All the matrix elements for a vector boson V (γ^*, W or Z) and up to five partons are presented there. Through appropriate

*The computer programs used to generate the cross sections reported here are available and can be obtained via electronic mail by contacting the authors.

†A comparison of the W matrix elements used in the present calculation with those of [7] yields complete agreement. We thank Dieter Zeppenfeld for performing the comparisons.

nr. jets	Z	W full KM	W diagonal KM
0	8	8	4
1	24	24	12
2	125	236	94
3	205	412	158

Table 1. Number of subprocesses for $p + \bar{p} \to V + n$ jets, with 4 flavours incoming and 5 flavours outgoing (W is W^+ or W^-).

process	one call	Per Z event	Per W event
$q\bar{q}$	0.0084	2	1
$q\bar{q}g$	0.016	6	3
$q\bar{q}gg$	0.044	7	4
$q\bar{q}q'\bar{q}'$	0.098	7	
	0.044		6
$q\bar{q}ggg$	0.40	7	4
$q\bar{q}q'\bar{q}'g$	1.51	13	
	0.83		10

Table 2. CPU timing for subprocess calls and number of calls per event. Timing in seconds on a VAX 750.

amplitude 13 times and $q\bar{q}ggg$ amplitude 7 times. For the W it makes no difference whether we only want the W^+, or both the W^+ and the W^-. They are calculated at the same time. Because of the left-handed coupling of the W to fermions the process $\bar{u}d \to W^-$ follows from $\bar{d}u \to W^+$ by means of complex conjugation. For this reason fewer matrix elements have to be evaluated for the W processes than for the Z processes.

In Table 3 we present the times needed to calculate one accepted event. It is evident from Tables 2 and 3 that almost all the computer time is used to calculate the parton matrix elements. In this respect, the number of subprocesses is not important. Furthermore, the time needed to perform the phase-space integration and to call the parton distributions is essentially negligible in the 2 and 3 jet case.

3 Phenomenology

It is neither practical nor meaningful to attempt here a complete phenomenological analysis of jet production in W and Z events at the CERN and FNAL Tevatron $p\bar{p}$ colliders. Detailed phenomenological tests can only be performed in the context of a particular detector, allowing for parton fragmentation, detector reponse, experimental cuts, etc. In what follows, therefore, we will attempt to present an

nr. jets	Z	W
0	0.039	0.031
1	0.12	0.071
2	1.04	0.49
3	22.8	10.2

Table 3. CPU timing per accepted event. Timing in seconds on a VAX 750.

overall picture of multijet production, concentrating on some general properties of the production processes and of the final states.

In order to get a feeling for the multijet cross sections we define a set of "standard" cuts and parameters, designed to typify the UA and CDF detectors. We impose transverse momentum and rapidity cuts on the final state partons and charged leptons, and a cut on "missing energy" (i.e. neutrino transverse momentum) for W events. In addition, the jets and charged leptons are required to have a minimum separation in phase space. At $\sqrt{s} = 630\ GeV$ the cuts are:

$$\begin{aligned} p_T(j) &> 10\ GeV \\ |\eta(j)| &< 2.5 \\ \Delta R(j_1, j_2) &> 1.0 \\ p_T(l) &> 15\ GeV \\ |\eta(l)| &< 2.5 \\ p_T(\nu) &> 15\ GeV \\ \Delta R(l, j) &> 1.0 \end{aligned} \tag{3.1}$$

and at $\sqrt{s} = 1.8\ TeV$ we choose:

$$\begin{aligned} p_T(j) &> 15\ GeV \\ |\eta(j)| &< 2.5 \\ \Delta R(j_1, j_2) &> 0.7 \\ p_T(l) &> 15\ GeV \\ |\eta(l)| &< 2.5 \\ p_T(\nu) &> 15\ GeV \\ \Delta R(l, j) &> 0.7 \quad . \end{aligned} \tag{3.2}$$

Here ΔR is the usual separation in η, ϕ space. Other relevant parameter values are: $M_W = 81\ GeV$, $M_Z = 92\ GeV$, $\sin^2\theta_W = 0.23$, $m_t > M_W$ and, unless otherwise stated, we use the latest MRSB parton distributions [13] ($\Lambda_{\overline{MS}} = 200\ MeV$) with the QCD scale $Q = M_V$ ($V = W, Z$).

Perhaps the most fundamental quantity to consider is the fraction of events with a given number of jets:

$$f_n(V) = \frac{\sigma(V + n \text{ jets})}{\sum_m \sigma(V + m \text{ jets})} \quad . \tag{3.3}$$

In fact the UA1 collaboration have recently presented measurements of f_n from the sample of W events collected up to 1987 [14]. Fig.1 shows the UA1 data and the theoretical expectations for $n \leq 3$. We have chosen a slightly different set of cuts from eqn.(3.1) to try to mimic the actual experimental measurements. For example, we have used $p_T(j) > 5\ GeV$ to try to model the actual jet E_T^{min} of $7\ GeV$. This should *not* therefore be considered a precise comparison: we are calculating at the parton level taking no account of fragmentation, smearing, resolution etc. We show Fig.1 simply to illustrate that the theory and the data are in broad agreement and to show the effect of the theoretical uncertainty coming from the choice of parton distributions and scale. The dashed lines are calculated using the MRSE distributions [13] ($\Lambda_{\overline{MS}} = 100\ MeV$) with a larger scale $Q = \sqrt{\hat{s}}$. Notice that, as expected, the difference increases with the number of jets and is about 30% for f_3. Although the above comparison is rather crude, a more detailed analysis combining the exact matrix elements for $n_{jet} \leq 2$ with a fragmentation model and detector simulation has shown that there is in fact excellent quantitative agreement between theory and experiment [2].

Fig.2(a) shows the predictions for $f_n(W)$ at the two collider energies using the standard cuts listed above. The enhanced rate of multijet events at the higher energy is due to the fact that the jets can be relatively softer, i.e. the dimensionless ratio $p_T^{min}/\sqrt{s}$ is smaller at the higher energy. This effect, then, is simply an artefact of the cuts chosen. The corresponding predictions for $f_n(Z)$ are shown in Fig.2(b). Comparing Figs.2(a) and (b) we see that the differences between W and Z events are very small, and due largely to the different effects of the separation cuts on multijet events with one or two charged leptons in the final state.

We stress again that all these theoretical predictions are afflicted with the usual uncertainties from the choice of parton distributions and scale. The size of this uncertainty can be judged from the predictions given in Fig.1.

The bulk of W and Z events arise from quark-antiquark annihilation. However, initial state gluons contribute at $O(\alpha_s)$. To see the relative importance of gluon-induced contributions to multijet production, we show in Fig.3 the relative contribution of qq, qg and gg induced processes to W production at the two collider energies. (Note that here qq stands for all possible combinations of quarks and antiquarks.) The results are largely as expected, with the gg processes making a small contribution to the ($n_{jet} \geq 2$) cross-sections The gluon-induced contributions are significantly more important at the higher energy, as smaller x values are sampled. For Z production, the results are very similar except that the gluon contributions are slightly reduced.

It is interesting to note that "switching off" the four-quark processes reduces the 2 and 3 jet cross sections (for W production at $\sqrt{s} = 1.8\ TeV$) by approximately 15% and 25% respectively. It follows that the final state jets are predominantly

nr. jets	$< p_T^W >$	$< M(W + \text{jets}) >$	$< M(\text{jets}) >$
1	21	118	-
2	27	152	44
3	30	194	80

Table 4. Average values of event parameters (in GeV) for $p\bar{p} \to W + n$ jets at $\sqrt{s} = 630\ GeV$ with cuts as defined in eq. (3.1).

gluon jets.

The cross sections calculated above are of course sensitive to the cuts on the final state particles, especially where these serve to regulate collinear and soft singularities. As an illustration of this we show in Fig.4 the cross sections for up to 3 jet production at (a) $\sqrt{s} = 630\ GeV$ and (b) $\sqrt{s} = 1.8\ TeV$, as a function of the jet minimum transverse momentum, p_T^{min}. All other cuts are as defined in eqns.(3.1), (3.2). The breakdown of low order perturbation theory becomes apparent as $p_T^{min} \to 0$. Fig.4 can also be used to test the hypothesis – first suggested in [4] – that the multijet cross sections decrease approximately geometrically, i.e. $(\sigma_n/\sigma_0) \simeq (\sigma_1/\sigma_0)^n$. Evidently this very crude approximation works surprisingly well. For example, at $\sqrt{s} = 1.8\ TeV$ and for p_T^{min} values in the $10 - 15\ GeV$ region, it gives 2 and 3 jet cross sections within 20% of their correct values.

Finally, we consider the properties of the multijet events. Because any distribution we might consider (e.g. transverse momentum or rapidity distributions of the final state leptons and jets) would be strongly affected by fragmentation and smearing, we prefer to list only the average values of some important event parameters. Tables 4 and 5 show the results for W production at the two collider energies. (The results for Z production are very similar and are not displayed.) Note that (a) the average W transverse momentum increases with increasing jet multiplicity and is larger at the higher collider energy, (b) the overall final state mass is significantly larger for multijet events at the higher energy, and (c) the overall invariant mass of the jets can be large. This has important consequences when considering backgrounds to new physics processes [7]. For example, we see from Table 5 that at $\sqrt{s} = 1.8\ TeV$ the average jet-jet mass in a two jet event is close to M_W. The jet pair can therefore easily mimic a second $W \to q\bar{q}'$. Likewise a triplet of jets can mimic the decay of a heavy quark: $Q \to qqq$. The multijet cross sections considered in this letter are, therefore, a vitally important component of any search for new heavy states.

nr. jets	$< p_T^W >$	$< M(W + \text{jets}) >$	$< M(\text{jets}) >$
1	31	140	-
2	44	235	80
3	55	310	157

Table 5. Average values of event parameters (in GeV) for $p\bar{p} \to W + n$ jets at $\sqrt{s} = 1.8\,TeV$ with cuts as defined in eq. (3.2).

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Figure captions

[1] Multplicity of jets f_n as a function of n. Data from the UA1 collaboration [14] on $p\bar{p} \rightarrow W(\rightarrow l\nu) + \text{jets}$ are compared with the theoretical predictions. The upper edge of the bands corresponds to the MRSB parton distributions [13] with $Q = M_W$ and the lower edge to the MRSE distributions with $Q = \sqrt{\hat{s}}$. In accordance with [14] there are no lepton cuts. The jet minimum transverse momentum, rapidity and separation cuts are 5 GeV, 2.5 and 1.0 respectively.

[2] Jet multiplicities at $\sqrt{s} = 630\ GeV$ (dashed lines) and 1.8 TeV (solid lines) for (a) $W \rightarrow e\nu$ and (b) $Z \rightarrow e^+e^-$, with the full set of lepton and jet cuts defined in eqns.(3.1), (3.2).

[3] Relative contributions of the qq, qg and gg subprocesses to the $p\bar{p} \rightarrow W + \text{jets}$ cross sections at $\sqrt{s} = 630\ GeV$ and 1.8 TeV.

[4] Cross sections for $p\bar{p} \rightarrow W(\rightarrow e\nu) + \text{jets}$ and $p\bar{p} \rightarrow Z(\rightarrow e^+e^-) + \text{jets}$ as a function of the minimum jet transverse momentum, p_T^{min}, at (a) $\sqrt{s} = 630\ GeV$ and (b) $\sqrt{s} = 1.8\ TeV$. All other cuts are as specified in eqns.(3.1), (3.2).

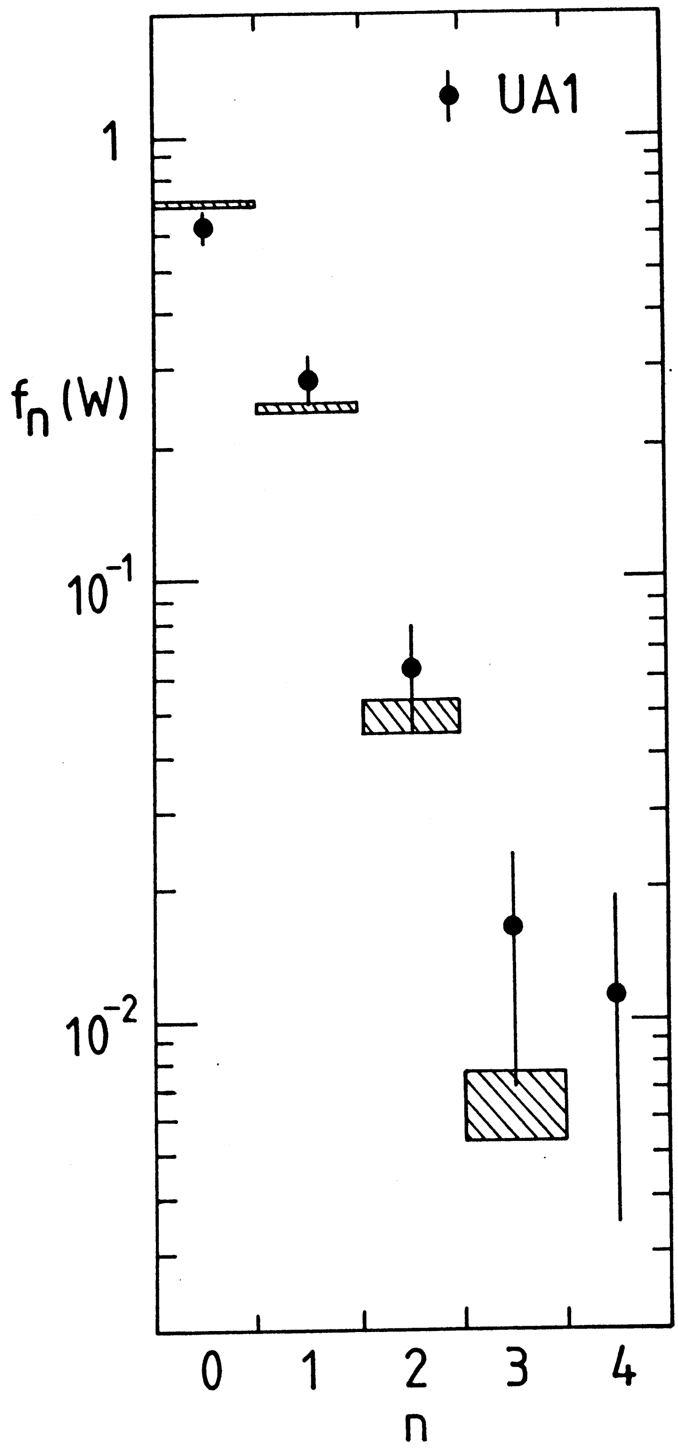

Fig. 1

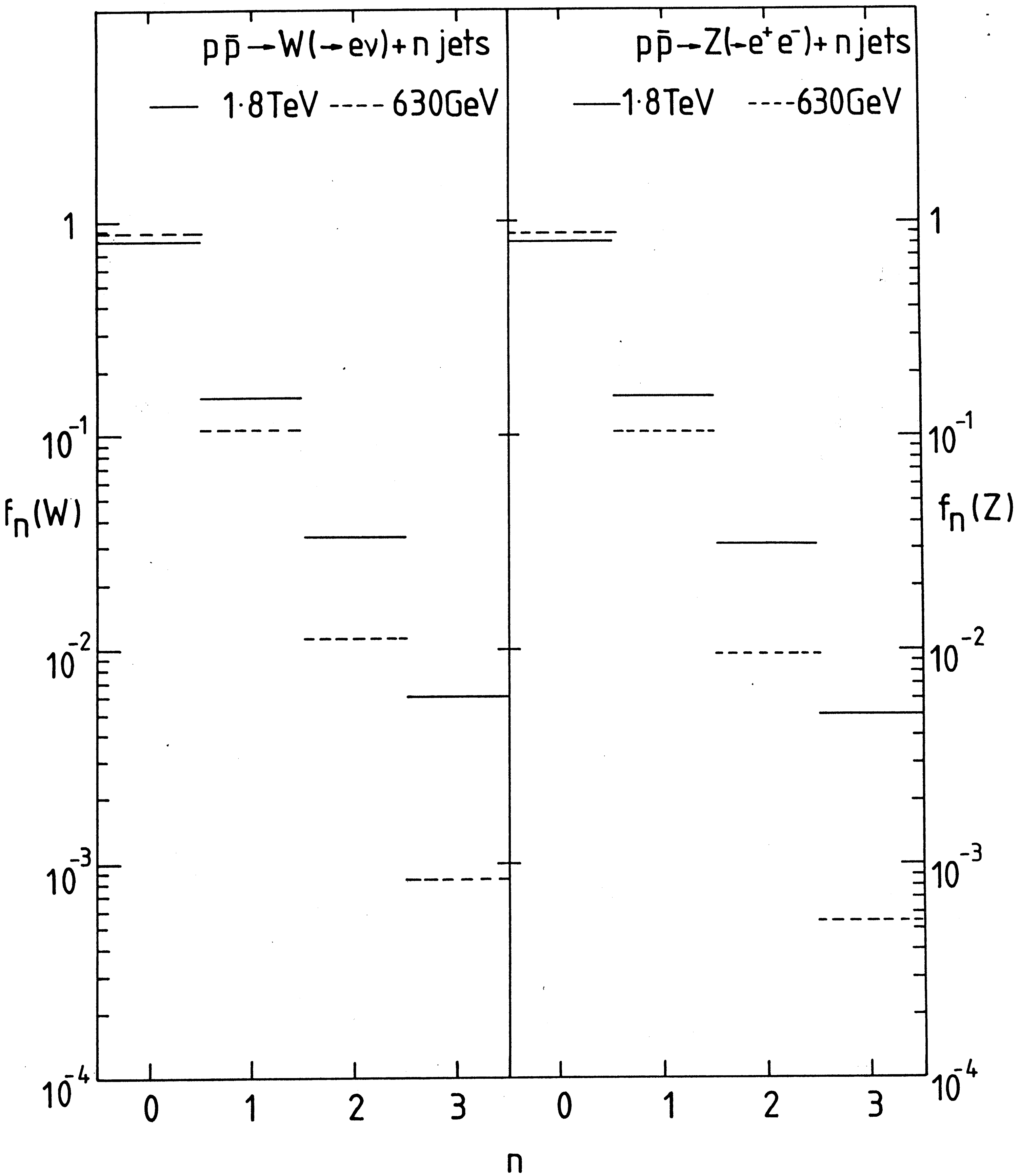
$p\bar{p} \to W(\to e\nu) + n$ jets
— 1·8TeV ---- 630GeV
$p\bar{p} \to Z(\to e^+e^-) + n$ jets
—1·8TeV ----630GeV
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$f_n(W)$
$f_n(Z)$
0
1
2
3
n

Fig. 2