On the order of the deconfining phase transition in SU(3) LGT *

The APE Collaboration

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We measure the correlation length of Polyakov loop in the region of the deconfinement transition for a pure gauge $SU(3)$ theory on lattices of different sizes. The correlation length is found to be of the order of lattice size, thus excluding a strong first order transition.

In this work we address the question of the order of the deconfinement transition in the $SU(3)$ pure gauge lattice theory. The reasons that led us to address again a question which seemed definitively solved (in favour of a strong first order transition[1]) have been described by M. Fukujita[2] in his review talk. Let me just recall that our aim has been putting the study of the transition on a quantitative ground, comparing the critical behavior of the system on different lattices sizes with the predictions of finite size scaling analysis[3].

This approach, thougth conceptually simple, is made difficult by critical slowing down which makes measurements in the transition region not trivial at all, unless both very high statistics and improved measure techniques are used.

Let us consider the behaviour of the correlation length ξ *vs.* the lattice size at the transition point β_c : in the 2^{nd} order case, the correlation length at the critical point ξ_c is divergent in the bulk, and finite size scale analysis predicts a linear dependence of ξ_c on the size in finite lattices. On the other hand, ξ_c is finite for 1^{st} order transitions. So, a firm conclusion in favour of a first order transition can be reached only if ξ_c stops increasing while increasing the lattice size. Analysis of our results on lattices up to 16 in size does not show any deviation from a linear behaviour of ξ_c *vs.* size [4].

We studied the decay of the Polyakov loop from a cold source in three different asymmetric lattices: $8^2 \times 32 \times 4$, $12^2 \times 48 \times 4$, $16^2 \times 64 \times 4$. The source method[5] is quite effective in measuring correlations length in large lattices, since the wall generates a strong signal up to distances which are unreachable when measuring loop loop correlations. The source has been set up at $z = 1$ fixing all the links in x, y, t directions to identity.

Working at finite temperature, the source has another useful property, namely it reduces tunneling effects. Among the three different vacua corresponding to $\arg(P) = \frac{2\pi}{3}n$, $n = 0, 1, 2$ the system will choose, in the broken phase, the one corresponding to $n = 0$, the same of the wall.

Besides the source method , the key points which allowed us to obtain an accurate estimation of the correlation length in the critical region have been the careful choice of operators and, of course, very high statistics; we spent about 4500 hours of a 256 Mflops APE-tto (see the contribution of E. Remiddi to these proceeedings for a description of Ape-supercomputers[6]), updating links with an overrelaxed[7] algorithm (30μ sec. for one link updating) and measuring each 10 sweeps on "true" links as well as on "smeared" ones.

We estimate the β of the transition as 5.692 ± 0.002 . In the $8^2 \times 32 \times 4$ lattice we scan the β region from 5.620 to 5.730 with a 0.010 step far from criticality, and a 0.005 step near the critical point, performing up to 170.000 sweeps for each β. In the $12^2 \times 48 \times 4$ lattice up to 240.000 sweeps for each β have been performed in five different β values ranging from 5.675 to 5.695. In the $16^2 \times 64 \times 4$ we run just at one β value, performing 150.000 sweeps.

Every ten sweeps a smearing and measuring step takes place; we start measuring the z-dependence of the zero momentum Polyakov line on the configuration of true links :

$$P^{(1)}(z) = \mathrm{Tr} \sum_{x,y} L(x, y, z) \qquad (1)$$

where

$$L(x, y, z) = \frac{1}{3} \mathrm{Tr} \prod_{t=1,L_t} U_\tau(x, y, z, t) \qquad (2)$$

and U_τ is the link variable in the t direction. Then we iteratively transform each link according to the smearing prescription:

$$U_\mu^{(s+1)} = \Pi \left(U_\mu^{(s)} + \epsilon \sum_{\eta=1,-1} \sum_{\nu \neq \mu} S_{\mu\nu}^{\eta} \right) \qquad (3)$$

where $S_{\mu\nu}$ is the staple (incomplete plaquette) for the link μ in the ν direction (along x, y or t), with orientation η, and $\Pi(A)$ stands for a projection of A over a $SU(3)$ element. On the configuration of smeared links we measure

$$P^{(s)}(z) = \frac{1}{3} \sum_{x,y} \mathrm{Tr} \prod_{t=1,L_t} U_\tau^{(s)}(x, y, z, t) \qquad (4)$$

General features and utility of the smearing procedure in the zero tempeature case have been described by E. Marinari[8] in his talk. Let us just check the correctness of the extension of the procedure to the finite temperature case, i.e. the behaviour under global Z_3 transformation. If we make a global Z_3 tranformation on the true links , then

$$U_\tau^{(s)}(\vec{n},t) \longrightarrow zU_\tau^{(s)}(\vec{n},t)\,\forall s,\vec{n} \qquad (5)$$

and the smeared Polyakov loops verify

$$P^{(s)}(\vec{n}) \longrightarrow zP^{(s)}(\vec{n})\,\forall s,\vec{n} \qquad (6)$$

that is, they remain good order parameters for each value s of the smearing step.

From the set of the $P^{(s)}$, $s = 1,11$, we obtain the correlations with the wall $C^{(s)}(z)$:

$$C^{(s)}(z) = \frac{1}{L_x L_y}\Re\,\mathrm{Tr}\sum_{x,y} P^{(s)}(z) \qquad (7)$$

Their expected asymptotic behaviours in z are:

$$C^{(s)}(z) \underset{z\to\infty}{\rightarrow} Ae^{-z/\xi^s} \qquad (8)$$

in the unbroken phase and

$$C^{(s)}(z) \underset{z\to\infty}{\rightarrow} Ae^{-z/\xi^s} + B^{(s)} \qquad (9)$$

in the broken one.

$B^{(s)}$ (the background) is the average value of the $P^{(s)}$ without wall (system broken to the $n = 0$ vacuum) and serves as an order parameter.

From the general properties of smeared operators we know that the z asymptotic behaviour (first reached by operators corresponding to larger smearing number) gives a smearing independent value for ξ, since the correlation length ξ_s is a property of the transfer matrix and not of the operator. From the behaviour under Z_3 transformation we can conclude that $B^{(s)}$ is a true order parameter of the transition for each s. So, the smearing procedure allows us to check our results as well as to increase accuracy: all our results, which could have been obtained also working with true links only, are reproduced by measurements on smeared variables, and the overall error can be significantly reduced.

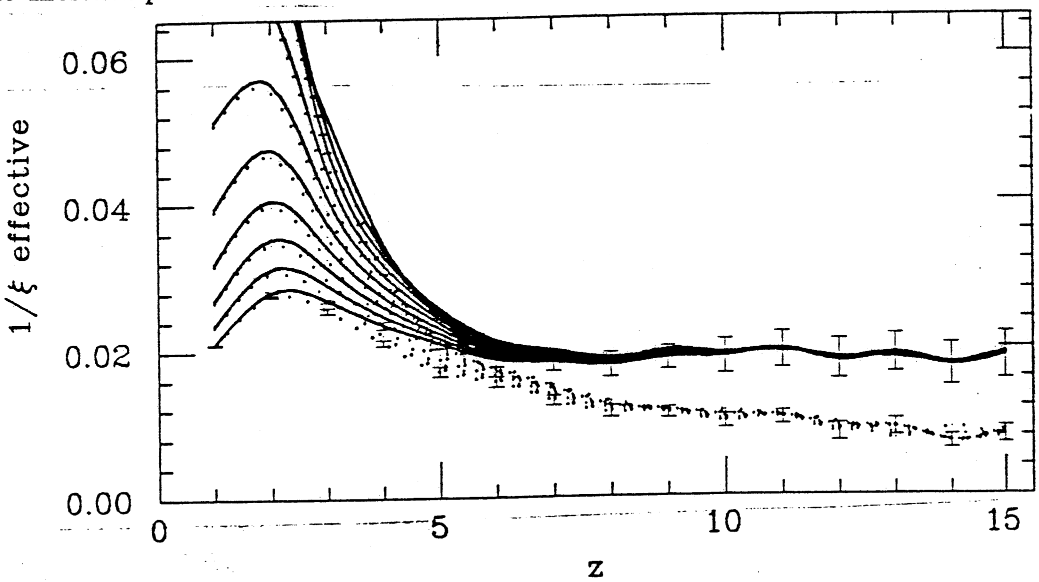

Fig.1 Values of the inverse of the correlation length obtained with an effective ξ procedure , at different values of the smearing number. The continous (dotted) lines are for β= 5.690 (5.695).

We analyze our data in two different ways : global fits and effective ξ analysis. Global fit procedures allow a determination of ξ and background taking into account *almost* all the points in zeta simultaneously (because of the coupling of the "excited" states with the wall, some points near the wall must be discarded). On the other hand, in a global fitting procedure, a very large correlation length can mimick a strong background, and, more dangerously perhaps, vice-versa. An effective ξ analysis offers a criterium free from these possible ambiguities : for each smearing and for each z we compute

$$1/\xi^{(s)}_{eff}(z) = ln\left(C^{(s)}(z)/C^{(s)}(z+1)\right) \qquad (10)$$

After $z \simeq 6$ all the $1/\xi^{(s)}_{eff}(z)$ coincide within the errors, meaning that an asymptotic behavior in z has been reached. This fact holds for all β's. For $\beta < \beta_c$ $1/\xi^{(s)}_{eff}(z)$ is asymptotically a constant (the inverse of ξ), corresponding to an exponential behavior for the correlation functions, and thus a true zero background. For $\beta > \beta_c$ $1/\xi^{(s)}_{eff}(z)$ is no longer a good estimator of $1/\xi^{(s)}_{true}$. This is clearly seen from the decreasing behavior of $1/\xi^{(s)}_{eff}(z)$ with z. The two situations are shown in fig.1, where it is possible to appreciate the effectiveness of the method to discriminate between the broken and the unbroken phase over an interval of 0.005 in β.

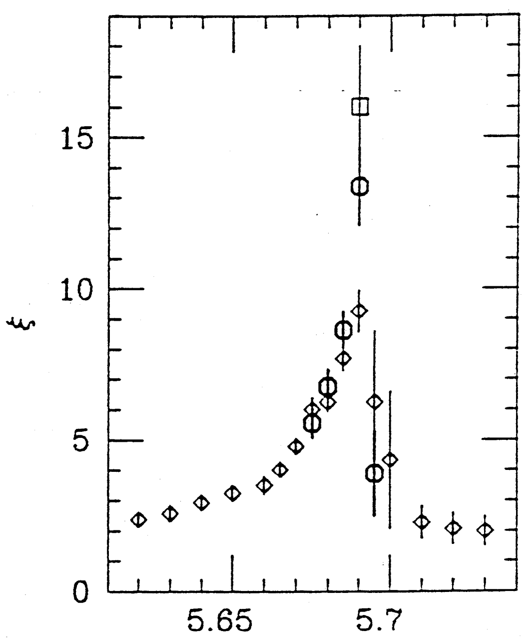

Fig.2 Correlation lengths as a function of beta. Diamonds are data from a lattice of size $8^2 \times 32 \times 4$, circles from $12^2 \times 48 \times 4$, and squares for $16^2 \times 64 \times 4$.

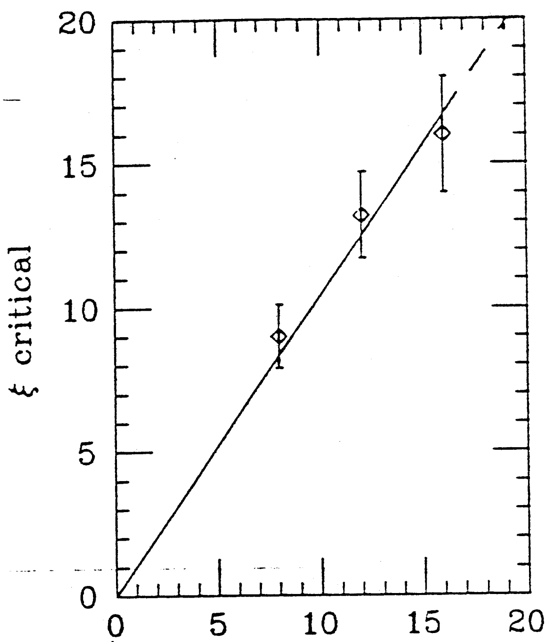

Fig.3 Value of the correlation length at the critical point as a function of the lattice size.

The full set of results for ξ is plotted in fig.2

The dependence of the correlation length at $\beta = 5.690$ (the largest one which we measured) on lattice size (the main aim of our investigation!) is shown in fig. 3.

Let us summarize these results: combining high statistics and technical refinements we obtain a precise measurement of the correlation length near the critical point . "Far" from the critical point the values of the correlation length for the two smaller lattices are the same, and start to be different as soon as ξ of the smaller one exceeds the size of the lattice: the ξ of $8^2 \times 32 \times 4$ lattice saturates, while the one of the $12^2 \times 48 \times 4$ continues to increase.

In all three lattices ξ_c is bounded from lattice size, so it linearly depends on size: the behavior of ξ at critical point fully corresponds to the predictions of finite size scale analysis for second order (or continuous) phase transitions. First order transition with very large correlation length are not excluded by our data: a study of the rounding of the transition *vs.* the lattice size will tell us if the rounding itself is going to disappear in the $L \to \infty$ limit (as expected for a discontinous transition) or if the transition is actually continuous. Work is in progress in this direction[9].

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*Presented by M.P. Lombardo