Matching NLO with parton shower in Monte Carlo scheme

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Abstract

A new method of including NLO QCD corrections to the hard process in the LO Monte Carlo (MC) shower is discussed. The method is based on a recently proposed MC factorization scheme, which dramatically simplifies the NLO coefficient functions. The NLO corrections are introduced by simple reweighing of the events produced by the LO shower with a single, positive MC weight. A practical implementation of the method is presented for the case of electro-weak boson production in the hadron-hadron collision, and the results are compared with well established approaches to NLO+PS matching.

1. Introduction

Fixed-order perturbative approach is an invaluable tool in our QCD toolbox. However the complexity of calculations increases very quickly when moving to higher orders and that is why most processes are known only up to the next-to-leading (NLO) order, with just a few computed at the next-to-next-to-leading order (NNLO) [1, 2]. That means that we effectively model our final state with just a few quarks and gluons. Moreover, often only an inclusive cross section is available, with no access to the final state kinematics.

For those cases where the differential distributions are available, the fixed-order approach works reliably only at high transverse momentum and fails as $p_T \to 0$. This is because of large logarithms $\ln p_T$ that compensate the coupling at low p_T and yield $\alpha_s \ln p_T \sim 1$. Hence, in the region of small p_T, each order contributes comparably and therefore they should all be summed. One way to achieve this is provided by the parton shower (PS) approach, which sums the dominant, leading-logarithmic contributions $(\alpha_s \ln p_T)^n$ to all orders. The events simulated with parton showers are fully exclusive and contain abundance of partons. The distributions are however computed in the collinear (small-p_T) approximation hence they differ from the exact results at high p_T.

The complementary advantages of the NLO calculations and the parton shower can be used simultaneously in a combined framework that goes under the name of NLO+PS matching. Two approaches to that problem are particularly well established, namely MC@NLO [3] and POWHEG [4].

In this contribution, we shall discuss a new method of including NLO correction to the hard process in a LO shower, which is quite different from the two approaches mentioned above. The method was proposed in [5], where it was applied to an analytic, forward-evolution shower, and it was shown to reproduce the corresponding NLO result exactly.

In our discussion, we shall concentrate on the Drell-Yan process, specifically production of the Z boson decaying into e^+e^- pair in the proton-proton collision. At leading order, $q\bar{q} \to Z$ is the only partonic process that contributes. At NLO, we have the real correction $q\bar{q} \to Zg$, a virtual correction, and an entirely new, tree-level contribution $qg \to Zq$. Here, we shall focus only on the $q\bar{q}$ channel. The corresponding real correction is depicted in Fig. 1, where we also introduce the notation for the 4-momenta and some of the kinematic variables, namely the invariant mass of the incoming partons, s, and that of the produced Z boson, $\hat{s}$. The ratio of the
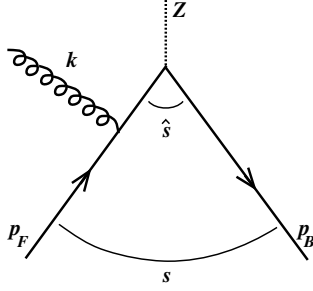
Figure 1: Kinematics of the real correction to Drell-Yan process in $q\bar{q}$ channel.

two $z = \hat{s}/s \to 1$ for the case of soft gluon emissions. It proves useful to work in terms of the light-cone variables

$$\alpha = \frac{2k \cdot p_B}{\sqrt{s}} = \frac{2k^+}{\sqrt{s}}, \tag{1}$$

$$\beta = \frac{2k \cdot p_F}{\sqrt{s}} = \frac{2k^-}{\sqrt{s}}, \tag{2}$$

which are related to z and k_T as follows

$$z = 1 - \alpha - \beta, \tag{3}$$

$$k_T^2 = \alpha\beta s. \tag{4}$$

2. The KrkNLO method

Calculating predictions for hadron colliders proceeds via convolution of the perturbative partonic cross sections (obtained from diagrams such as the one shown in Fig. 1) with the non-perturbative parton distribution functions (PDFs). This is always done within a certain factorization scheme, which also defines the PDFs. The most commonly used factorization scheme is $\overline{\text{MS}}$ [6], hence the parton distribution functions are $\overline{\text{MS}}$ PDFs. The $\overline{\text{MS}}$ scheme is very convenient in the context of fixed-order calculations, however, it leads to a class of purely collinear terms in the partonic cross section. Those terms need to be reproduced by the MC shower generators if one aims at achieving NLO accuracy. This is problematic since Monte Carlo produces particles in 3-space dimensions and forcing it to generate particles in strictly collinear phase space is not straightforward. Unfortunately, this is mandatory if one is to use those generators for NLO+PS matching.

The KrkNLO method, proposed in [5], circumvents this problem by departing from the $\overline{\text{MS}}$ scheme into a new factorization scheme, called the Monte Carlo (MC)

scheme, in which all the problematic collinear contributions are essentially moved to the parton distribution functions that now become PDFs in the MC scheme.

The KrkNLO method proceeds in the following steps:

1. Take a parton shower that covers the (α, β) phase space completely and produces emissions according to an approximate matrix element $K \simeq R$.
2. Upgrade the hardest real emission from PS to the exact matrix element by reweighting with R/K.
3. Upon integration over transverse degrees of freedom, this upgraded PS will contribute an extra term $C_{2q}(z) = \int (R - K)$. Redefine PDFs by subtracting $C_{2q}(z)$ together with all the z-dependent terms from the $\overline{\text{MS}}$ coefficient function. By this procedure, parton distribution functions are transformed to the new MC factorization scheme.
4. Virtual+soft correction, Δ_{S+V}, becomes just a constant. Multiply the whole result by $1 + \Delta_{S+V}$ to achieve complete NLO accuracy.

The method can be used with any shower satisfying the criterion 1. In our implementation, we relied on the Catani-Seymour (CS) shower [7, 8] from the Sherpa MC generator [9]. It evolves with the variable $q^2 = (\alpha + \beta)\beta s$ and the $C_{2q}(z)$ function takes the form

$$C_{2q}^{\text{CS}}(z) = \frac{\alpha_s}{2\pi} C_F \left[-2(1 - z) \right]. \tag{5}$$

In order to upgrade the hardest gluon emission of the CS shower to NLO accuracy, one needs to generates the results with MC PDFs (see next section) and reweight each event with the real weight:

$$W_R^{q\bar{q}}(\alpha, \beta) = 1 - \frac{2\alpha\beta}{1 + (1 - \alpha - \beta)^2}, \tag{6}$$

which is a simple function of kinematic variables, and with $(1 + \Delta_{S+V}^{q\bar{q}})$, where the latter is a constant

$$\Delta_{V+S}^{q\bar{q}} = \frac{\alpha_s}{2\pi} C_F \left[\frac{4}{3}\pi^2 - \frac{5}{2} \right]. \tag{7}$$

3. MC factorization scheme and MC PDFs

As explained earlier, change of the factorization scheme from $\overline{\text{MS}}$ to MC is an essential ingredient of the KrkNLO method. The q and $\bar{q}$ MC PDFs can be obtained from $\overline{\text{MS}}$ PDFs using the following transformation

$$q_{\text{MC}}(x, Q^2) = q_{\overline{\text{MS}}}(x, Q^2) \tag{8}$$
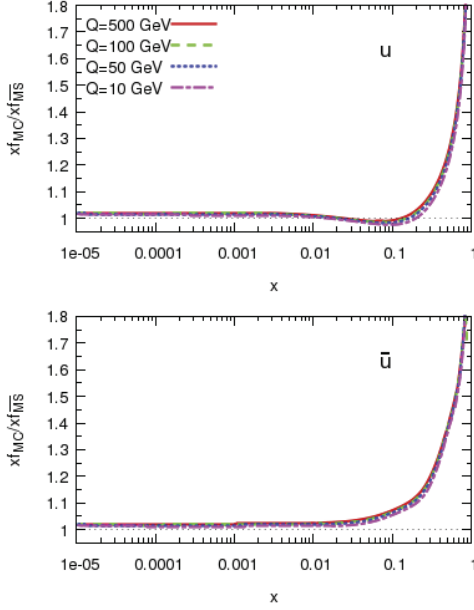$$+ \int_x^1 \frac{dz}{z} q_{\overline{\text{MS}}}\left(\frac{x}{z}, Q^2\right) \Delta C_{2q}(z),$$

Figure 2: Ratios of the u and $\bar{u}$ MC PDFs, obtained with formula (8), to the standard $\overline{\text{MS}}$ parton distributions from the MSTW 2008 LO set [10] for a range of Q^2 values.

where the function

$$\Delta C_{2q}(z) = \frac{\alpha_s}{2\pi} C_F \left[\frac{1+z^2}{1-z} \ln \frac{(1-z)^2}{z} + 1 - z \right]_+ \tag{9}$$

is constructed from $C_{2q}^{\text{CS}}(z)$ of Eq. (5) and all z-dependent terms of the $\overline{\text{MS}}$ coefficient function for the Drell-Yan process. The gluon PDFs is not modified so far in the MC scheme. For processes like DY or DIS, such modifications would change the result only at the order $O\left(\alpha_s^2\right)$, that is NNLO, which is beyond our current accuracy.

Fig. 2 shows the ratios of the MC PDFs to $\overline{\text{MS}}$ PDFs for u and $\bar{u}$ quark for a range of Q^2 values. Here we used the MSTW 2008 LO set of parton distribution functions [10]. We see that the difference is small (up to 2%) for low and moderate x, and it increases for higher values of x. Other quarks show similar picture.

MC PDFs can be used together with the corresponding MC coefficient function, which essentially reduces to Eq. (5), to calculate NLO predictions in the MC scheme.

We studied the fixed-order NLO cross section in both schemes with the MCFM program [11]. For the case of MC scheme, we applied corresponding modifications to the coefficient functions. In all of the MCFM com-

putations discussed in this contribution we set the factorization and the renormalization scales equal to the Z boson mass $m_Z = 91.1876 \,\text{GeV}$, which corresponds to $\alpha_s = 0.1394$. We generated DY events from pp collisions at $\sqrt{S} = 8\,\text{TeV}$ and required presence of the Z boson with invariant mass $50 < m_Z < 150\,\text{GeV}$.

The predictions for physical quantities must be independent of the factorization scheme up to an accuracy claimed in a calculation. In our case of NLO DY process, results in both schemes need to agree exactly up to the order $O(\alpha_s)$. If we look at the total cross section for the production of Z in the $q\bar{q}$ channel only, computed in $\overline{\text{MS}}$ scheme, we can schematically write it as

$$\sigma_{\text{tot}}^{\overline{\text{MS}}} = \sigma_0 \, q \otimes \left(1 + \alpha_s \, C_{2q}^{\overline{\text{MS}}}\right) \otimes \bar{q}, \tag{10}$$

where q and $\bar{q}$ are the standard $\overline{\text{MS}}$ PDFs, $\otimes$ denotes integration over x and 1 corresponds to the Born contribution, whereas the term with the coefficient function $C_q^{\overline{\text{MS}}}$ provides NLO correction.

The same cross section computed in the MC scheme, using the PDFs from Eq. (8), has the following structure

$$\begin{aligned} \sigma_{\text{tot}}^{\text{MC}} &= \sigma_0 \, q^{\text{MC}} \otimes \left(1 + \alpha_s \, C_{2q}^{\text{MC}}\right) \otimes \bar{q}^{\text{MC}} \\ &= \sigma_0 \left(q + \alpha_s \Delta C_{2q}\right) \otimes \left(1 + \alpha_s \, C_{2q}^{\text{MC}}\right)\left(\bar{q} + \alpha_s \Delta C_{2q}\right) \end{aligned} \tag{11}$$

hence it contains some contributions beyond NLO. However, comparing only terms of order $O(\alpha_s)$ between Eqs. (10) and (11) yields the following requirement for the factorization scheme independence of the result

$$\begin{aligned} C_{2q}^{\overline{\text{MS}}} \otimes q \otimes \bar{q} &= \Delta C_{2q} \otimes 1 \otimes \bar{q} + \Delta C_{2q} \otimes q \otimes 1 \\ &\quad + C_{2q}^{\text{MC}} \otimes q \otimes \bar{q}. \end{aligned} \tag{12}$$

We have checked that, indeed, in our particular setup, the l.h.s. of the above equation equals to $(336.36 \pm 0.09)\,\text{pb}$ whereas the terms on the r.h.s. add up to $25.79\,\text{pb} + 25.79\,\text{pb} + 284.77\,\text{pb} = (336.35 \pm 0.09)\,\text{pb}$. Hence, we see that the final result is scheme independent up to $O(\alpha_s)$. This constitutes a nontrivial test of the change of the factorization scheme from $\overline{\text{MS}}$ to MC as the terms on the r.h.s. of Eq. (12) have very different origins, the first two come from the new MC PDFs, whereas the third term comes from the modified coefficient function.

4. Matching NLO with parton shower

As we have demonstrated that the change of factorization scheme works correctly, we are now ready to compute the matched NLO+PS results using the
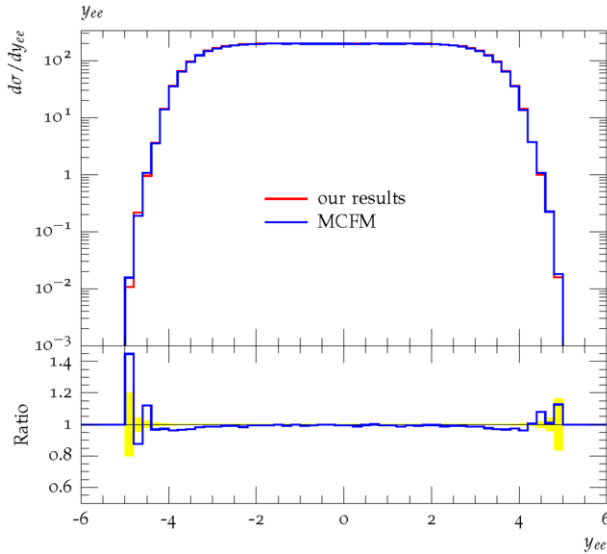
Figure 3: Comparison of the KrkNLO result for distribution of rapidity of the lepton pair with fixed order NLO result from MCFM. Drell-Yan process in $q\bar{q}$ channel at $\sqrt{S} = 8\,\text{TeV}$.
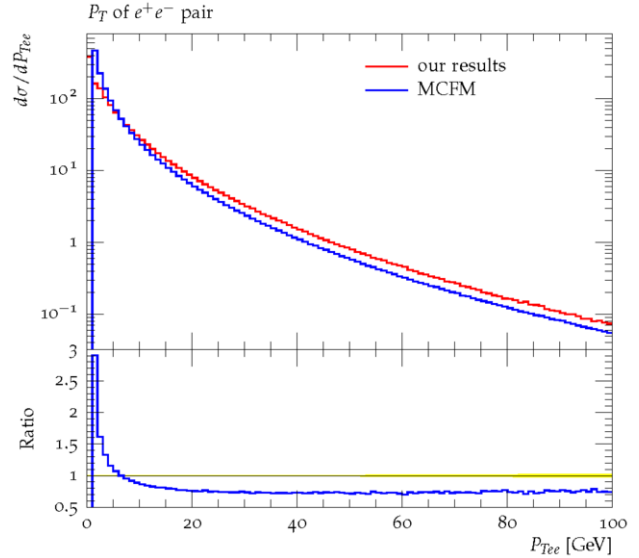

Figure 4: Comparison of the KrkNLO result for distribution of transverse momentum of the lepton pair with fixed order NLO result from MCFM. Drell-Yan process in $q\bar{q}$ channel at $\sqrt{S} = 8\,\text{TeV}$.

	σ_{tot} [pb]
MCFM ($\overline{\text{MS}}$ PDFs)	1344.1 ± 0.1
MCFM (MC PDFs)	1361.6 ± 0.3
PS (MC PDFs)	1044.1 ± 0.9
PS+real (MC PDFs)	1031.1 ± 0.9
KrkNLO [PS+NLO (MC PDFs)]	1355.9 ± 0.8

Table 1: Total (fiducial) cross section for the Z boson production in $q\bar{q}$ channel at $\sqrt{S} = 8\,\text{TeV}$ with $50 < m_Z < 150\,\text{GeV}$.

KrkNLO technique implemented on top of the Catani-Seymour shower.

Each event generated with the Monte Carlo is weighted by $W_R^{q\bar{q}}(\alpha, \beta)\,(1 + \Delta_{V+S}^{q\bar{q}})$, with the real and virtual weights defined in Eqs. (6) and (7) and the α and β variables being functions of the kinematics of the hardest gluon emission.

We start by comparing in Table 1 the total cross sections from the fixed order NLO and from our matched calculation. The first two numbers correspond to the NLO result from MCFM in two different factorization scheme: $\overline{\text{MS}}$ and MC. We see that there is $O(1\%)$ difference between them, which comes from terms beyond NLO, which are present in the MC scheme calculation, c.f. Eq. (11). The following number in the third row is a result of the pure LO shower, only used with MC PDFs. In the next row we see that applying corrections to just the real emissions lowers the cross section slightly w.r.t. LO, which comes from the fact that the tail of exact $p_{T,Z}$ distribution is somewhat lower than what comes out of the CS approximation. Finally, in the last row of Table 1, we give our full matched result for the total cross section of the Z boson production in the $q\bar{q}$ channel. We get the number that is very close to, and actually sits between, the two NLO results from MCFM. The difference between fully corrected PS+NLO is at the level of 0.8% w.r.t. MCFM in $\overline{\text{MS}}$ scheme and 0.4% w.r.t. to MCFM in MC scheme.

Fig. 3 shows the comparison of the lepton pair (hence

Z boson) rapidity distributions between our matched result and that of MCFM. We see excellent reproduction of the NLO distribution by the matched calculation from KrkNLO. In Fig. 4 we compare our matched result for the transverse momentum distribution of the e^+e^- pair with fixed order NLO result from MCFM. At low p_{T,e^+e^-} our curve is below MCFM due to Sudakov suppression built into the parton shower. On the other hand, at high p_{T,e^+e^-} the KrkNLO result is above MCFM. This comes from the fact that in the KrkNLO approach the virtual correction, Eq. (7), is spread over the full p_T range. That lifts up the result at high p_T by $\sim 30\%$ and one can interpret this as including part of the genuine $O(\alpha_s^2)$ order related to mixed real-virtual diagrams.

In Figs. 5 and 6 we compare the KrkNLO results with those from other matching methods, namely POWHEG and MC@NLO. As shown in Fig. 5, the rapidity distribution of the e^+e^- pair comes out essentially identical for all three methods. However, distribution of p_{T,e^+e^-}
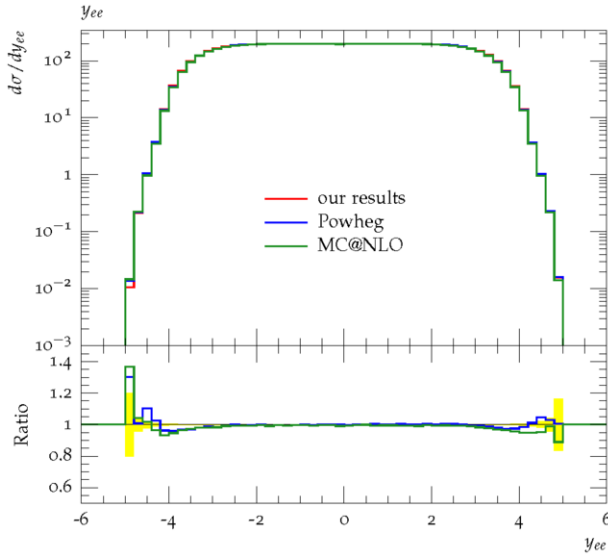
Figure 5: Comparison of the KrkNLO result for distribution of rapidity of the lepton pair with two other matching techniques: POWHEG and MC@NLO. Drell-Yan process in $q\bar{q}$ channel at $\sqrt{S} = 8\,\text{TeV}$.
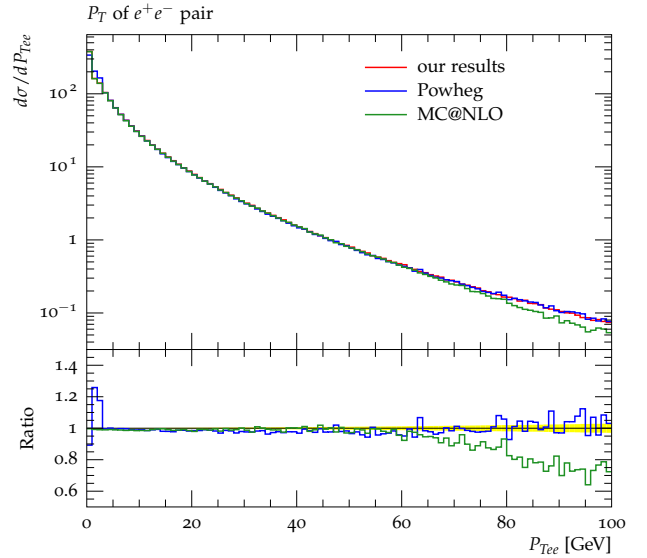

Figure 6: Comparison of the KrkNLO result for distribution of transverse momentum of the lepton pair with two other matching techniques: POWHEG and MC@NLO. Drell-Yan process in $q\bar{q}$ channel at $\sqrt{S} = 8\,\text{TeV}$.

shows some differences. The KrkNLO result is basically equal to that from POWHEG, except for the small difference at very low-p_T, which is an arti-fact of using slightly different evolution variables in the two cases (p_T in POWHEG vs the variable discussed in Sec. 2 in KrkNLO). When compared to MC@NLO, our result is very similar at low and moderate p_T values, whereas at p_T of the order of the Z boson mass and higher, the MC@NLO curve is below KrkNLO and POWHEG. This comes from the fact that MC@NLO is designed such that it recovers fixed-order NLO exactly at higher transverse momenta as the virtual correction is spread only over a range of p_T up to m_Z. In KrkNLO, the virtual correction is spread also over $p_T > m_Z$, which corresponds to inclusion of mixed real-virtual corrections of order $O\left(\alpha_s^2\right)$.

5. Summary

We discussed the KrkNLO method of matching the LO parton shower with fixed-order NLO. The method is based on two elements: change of factorization scheme from $\overline{\text{MS}}$ to the new MC scheme and upgrading the hardest emission to full NLO accuracy by reweighting with a simple, positive weight.

Change of the factorization scheme allows one to eliminate troublesome z-dependent terms from the coefficient function and it effectively amounts to creating new MC PDFs. We have discussed how such PDFs can

be obtained and how they differ from the standard $\overline{\text{MS}}$ parton distributions. We have validated the MC factorization scheme by studying the Drell-Yan process in the $q\bar{q}$ channel at fixed-order NLO and showing that the $\overline{\text{MS}}$ and MC scheme results are identical up to the order $O\left(\alpha_s\right)$.

We have implemented the KrkNLO method on top of the Catani-Seymour shower in Sherpa event generator for the case of production of the electroweak boson (hence, initial-state shower). We have presented comparisons of matched results obtained with our technique to the fixed order NLO results from MCFM. In particular, we have demonstrated that the KrkNLO result recovers fixed-order NLO up to sub-percent, beyond NLO corrections. We have also shown a comparison of the results from our method to the two most commonly used matching technique, namely POWHEG and MC@NLO. All three methods turn out to give essentially identical results for y_Z distributions. The $p_{T,Z}$ spectra are somewhat different between KrkNLO and MC@NLO and we attributed those differences to $O\left(\alpha_s^2\right)$ corrections.

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