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**THE ELLIPTICITY INTRODUCED BY INTERFERENTIAL MIRRORS  
ON A LINEARLY POLARIZED LIGHT BEAM ORTHOGONALLY REFLECTED**

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**ABSTRACT**

Modifications of the state of polarization of a light beam trapped in a multipass cavity formed by interferential mirrors were studied. It was found that the mirrors analysed introduced ellipticities ranging from  $3 \times 10^{-5}$  to  $2 \times 10^{-4}$  per reflection.

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## 1. INTRODUCTION

One of the many applications of interferential mirrors is the possibility of building multipass cavities for visible light with a trapping time of many microseconds [1]. In some specific cases there is interest in a multipass cavity (with a number of reflections of more than 400) (see, for example, Ref. 2) where it can be rather essential to know the perturbations introduced by the cavity on the state of polarization of the trapped light.

In this article we wish to present some experimental results on the effects of interferential mirrors on the state of polarization of linearly polarized light trapped in a multipass cavity. In this type of cavity, in general, the transverse dimension of the region where the beam is contained (of the order of a few centimetres) is much larger than the beam spot (less than a millimetre); therefore each reflection can be spatially separated from all the others. We will deal here with the multipass cavity where the beam enters and exits [1] through the same hole drilled in one of the two concave mirrors and separated by a distance  $D$ .

The theory of this type of multipass cavity has been given in Ref. [1]; here we recall that if  $x_0$  and  $y_0$  are the coordinates of the entrance hole and  $\alpha_x$  and  $\alpha_y$  the initial slopes of the incoming beam, then the coordinates  $x_n$  and  $y_n$  of the  $n^{\text{th}}$  reflection on the mirrors' surface (we are in the limit of the Gaussian approximation), for the case of a cavity formed by mirrors of radius of curvature  $R$ , are given by:

$$\begin{aligned} x_n &= x_0 \cos(n\theta) + [D/(2R - D)]^{1/2}(x_0 + R\alpha_x) \sin(n\theta) \\ y_n &= y_0 \cos(n\theta) + [D/(2R - D)]^{1/2}(y_0 + R\alpha_y) \sin(n\theta), \end{aligned} \quad (1.1)$$

where  $\cos \theta = 1 - D/R$ . The  $z$  axis of the system of coordinates that we are using coincides with the optical axis  $L$ .

From the theory of interferential mirrors [3], it can be shown that for real mirrors, after the  $i^{\text{th}}$  reflection, the two components of the light linearly polarized parallel and orthogonal to the incidence plane are out of phase by a quantity

$$\Delta_i = \Delta_i^P - \Delta_i^O \approx C\Phi_i^2, \quad (1.2)$$

where  $\Phi_i$  is the angle of incidence for the  $i^{\text{th}}$  reflection and  $C$  is a constant depending only on the mirrors' characteristics. Therefore the ellipticity  $\Psi_i$  gained by the light beam in the  $i^{\text{th}}$  reflection is given by

$$\Psi_i = (1/2)\Delta_i \sin(2\delta_i) = (1/2)C\Phi_i^2 \sin(2\delta_i), \quad (1.3)$$

where  $\delta_i$  is the angle between the light polarization vector and the incidence plane in the  $i^{\text{th}}$  reflection.

Using expressions (1.1) and (1.3) it can be shown that the total ellipticity  $\Psi_T$  ('off-0°' total ellipticity) gained by the trapped light during the  $N$  reflections is proportional to  $N$ , so that for this type of contribution we will later take

$$\Psi_T = \text{constant} \times N. \quad (1.4)$$

It has been clearly shown in Ref. [4] that light reflected on interferential mirrors may also gain, in each reflection, an ellipticity  $\epsilon$  independent of the incident angle of the light beam; i.e. such an effect is present also for  $\Phi_i = 0$ .

The interpretation of this phenomenon, given in Ref. [4], is that  $\epsilon$  is related to stresses present within the layers composing the interferential mirrors themselves: thus the mirrors, in each point, act as a birefringent plate with the axis parallel to the mirrors' surface.

The experimental results we present here are mainly focused on finding  $\epsilon$  and have been obtained with a small cavity (D about 45 cm) formed by interferential mirrors<sup>\*)</sup>; the results refer to six mirrors, three of which have a hole; the geometrical characteristics of these mirrors are listed in Table 1.

The mirrors used in these tests are commercially-available mirrors for which neither the exact chemical composition of the layers nor the fabrication processes are given. We know, however, that the layers are deposited on an optical glass BK7 (about 1 cm thick) and that the surface on which they are deposited is worked at  $\lambda/20$ . The reflectivity of these mirrors (measured directly by us) is such that after  $250 \pm 20$  reflections the light intensity is reduced to  $1/e$ : this result is valid at  $\lambda = 514.5$  nm.

These results are in agreement with the firm's specifications.

## 2. EXPERIMENTAL LAYOUT AND METHOD

A simplified sketch of the experimental layout is given in Fig. 1. The light source is a continuous wave (CW) argon-ion laser tuned at the wavelength  $\lambda = 514.5$  nm. The light, linearly polarized by the prism P<sup>\*\*)</sup> enters the optical cavity through the pierced mirror M<sub>F</sub>; after several reflections the light exits the cavity via the same hole. The polarization state of the light is then analysed by the prism A<sup>\*\*)</sup>. With the light reflected only once in the cavity, the extinction factor achieved in air with the prisms we used is

$$\sigma^2 = I_{tr}/I_0 = 2 \times 10^{-7}, \quad (2.1)$$

where  $I_{tr}$  is the light power transmitted by A and  $I_0$  the light power before A.

The transmitted light is detected by the photodiode D<sub>2</sub>; the current signal of the diode is amplified and the tension value  $V_{tr}$  on the feedback resistor  $R_f$  of the amplifier is recorded. The light rays deflected by the prisms A and P are used for power-monitoring purposes in order to normalize data from different measurements.

Both spherical mirrors of the optical cavity are mounted on adjustable mechanical supports. The number of reflections and the distance D between the mirrors chosen in the different measurements are listed in Table 2. The points on the mirrors where the light is reflected are (by our choice) always on a circle with a diameter of about 4 cm within the central part of the mirror. The pierced mirror M<sub>F</sub> must be kept fixed all the time, while the analysed mirror M<sub>R</sub> can be rotated by an amount  $\gamma$  around an axis u chosen coincident with optical axis L of the cavity.

The measurements consist in recording the transmitted light  $I_{tr}$  for different values of  $\gamma$  with an angular scanning of  $5^\circ$ . In more detail, the procedure for the measurements is the following: once the mirrors are initially positioned to some initial value for  $\gamma$ , the extinction factor is measured (as a check) with only one reflection on M<sub>R</sub> and with the prisms P and A crossed; the value usually found was around  $2 \times 10^{-7}$ . The complete optical multipass path is then built by positioning the mirrors' mechanical supports; after this operation, before starting the measurements, the prism A is rotated to minimize again the transmitted light intensity  $I_{tr}$ .

\*) Manufactured by MTO, Palaiseau, France.

\*\*\*) Manufactured by Karl and Lambrecht, Chicago, Ill., USA.

This last operation is necessary since the cavity itself introduces a rotation  $\varrho$  in the polarization plane of the light [4]; this is a purely geometrical effect reflecting the rotational direction of the optical path. Using the expression for  $\varrho$  given in ref. [4] we have, for our case,  $\varrho \ll 10^{-4}$  rad.

After this last operation the light power transmitted by A is measured and recorded for different angular positions  $\gamma$  of the mirror  $M_R$ , the values of  $\gamma$  ranging from  $0^\circ$  to  $360^\circ$ .

During the measurements we have often checked that by introducing a compensator before A it was possible to obtain for  $I_{tr}$  values compatible with the extinction value previously obtained.

Figure 2 shows, as an example, the output voltage  $V_{tr}$  of the amplifier feedback resistance of diode  $D_2$  as a function of the angle  $\gamma$  for a cavity where  $M_F = M_2$  and  $M_R = M_1$  (see Table 1).

The error assigned to each measurement is the sum of a term due to the fluctuation in the laser light intensity and a systematic error due to a slow drift in the dark current of the diode amplifier.

In our case, from what we said above, the total ellipticity gained by the light in the cavity is the sum of the total 'off- $0^\circ$ ' ellipticity  $\Psi_T$  plus the intrinsic ellipticity  $\epsilon$  introduced by the mirrors.

The light intensity transmitted by the analyser A is given by the following expression

$$I_{tr}(\gamma) = I_0\{\sigma^2 + [N_F \epsilon_F + N_R \epsilon(\gamma)_R + \Psi_T]^2\}, \quad (2.2)$$

where  $N_F$  and  $N_R = N_F + 1$  are the number of reflections on  $M_F$  and  $M_R$ ,  $\epsilon_F$  and  $\epsilon_R$  are the average ellipticities per reflection of the mirrors. In writing expression (2.2) we have assumed that the ellipticity parameters are, for each mirror, a slowly varying function of the position on the mirror's surface—at least in the region covered by the beam spots (about 4 cm around the centre).

### 3. DATA ANALYSIS

In agreement with expression (2.2) the experimental results (of each mirror analysed) have been fitted to the following expression as a function of the angle  $\gamma$ :

$$V_{tr}(\gamma) = A + N^2[B + C \sin(2D + 2\gamma)]^2, \quad (3.1)$$

where  $V_{tr}(\gamma)$  is the voltage value on the feedback resistor  $R_f$  of the photodiode amplifier and which we can write as:

$$V_{tr} = I_{tr}R_f\eta \quad (3.2)$$

with  $\eta$  being the efficiency of the diode (ampere per watt);  $N = N_F = N_R$  is the number of reflections on each mirror and A, B, C, and D are parameters determined by performing the best fit of the experimental data. One can show that:

i)  $A = V_0\sigma^2$

where  $V_0 = I_0R_f\eta$ .

ii)  $B = V_0^{1/2}(a + \epsilon_F) = V_0^{1/2}[a + (1/2)\Delta_F \sin 2\beta_{OF}]$ ,

where  $a$  is proportional to  $\Psi_T/N$  and  $\Delta_F$  is the average phase shift per reflection due to the intrinsic birefringence of the fixed pierced mirror  $M_F$ ;  $\beta_{OF}$  is the angle between the polarization plane of the light and the average birefringence axis of  $M_F$ .

iii)  $C = V_0\Delta_R/2 = V_0^{1/2}\epsilon_{OR}$ ,

where  $\epsilon_{OR}$  is the maximum average ellipticity per reflection gained by the polarization state of the light due to the intrinsic birefringence of the rotated mirror  $M_R$ .

iv)  $D = \beta_{OR}$  is the angle at  $\gamma = 0^\circ$  between the average birefringence axis of the rotated mirror  $M_R$  and the polarization plane of the light.

Among the different parameters of the fit we are mainly interested in the quantity  $C$  since this quantity is directly proportional to  $\epsilon_{OR}$ , i.e. the maximum ellipticity  $\epsilon$  per reflection introduced by the mirror  $M_R$  under analysis: from the fit we have also obtained a value for the extinction factor  $\sigma^2$ .

We have evaluated  $\epsilon_{OR}$  for five mirrors, different in curvature and diameter, varying the cavity length and the number of reflections  $N$ ; we have always kept mirror  $M_2$  of Table 1 as  $M_R$  of the cavity (see Fig.1).

The results on  $\epsilon_{OR}$  and on  $\sigma^2$ , obtained by fitting all the experimental results for the mirrors  $M_1$ ,  $M_3$ ,  $M_4$ ,  $M_5$ , and  $M_6$ , are summarized in Table 2.

#### 4. CONCLUSIONS

From our measurements, we found that the interferential mirrors at our disposal introduce, on the state of polarization of the light, an average ellipticity  $\epsilon$ , per reflection, independently of the angle of incidence on the mirrors' surface, ranging between  $3 \times 10^{-5}$  and  $2 \times 10^{-4}$  (see Table 2)—at least in the central part of the mirrors' surface (about  $4 \text{ cm}^2$ ).

Our results agree quantitatively with those obtained by Bouchiat and Pottier [4] who analysed, with a different method, mirrors from other firms.

From our results we can conclude that possible variation from point to point on the surface of the 'birefringence axis' of the mirrors must be small, as also indicated by the excellent extinction factors always obtained for each cavity made (see Table 2). In fact, two of the analysed 7 cm diameter mirrors were further used in a set-up for a long multipass optical cavity ( $D = 450 \text{ cm}$  and with the number of reflections per mirror  $N = 200$ ) with all the optical components in vacuum. In these conditions, by properly adjusting the relative orientations of the mirrors and the light polarization plane, we were able to obtain in optimal conditions an extinction factor better than  $10^{-7}$ .

Finally, we wish to add that we repeated the measurements almost a year later and, within the experimental errors, we found the same results.

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**Table 1**  
Geometrical characteristics of the analysis mirrors

Mirror	Diameter	Radius of curvature (m)	With hole?
M <sub>1</sub>	7	10	no
M <sub>2</sub> <sup>a)</sup>	7	10	yes
M <sub>3</sub>	7	10	no
M <sub>4</sub> <sup>a)</sup>	7	10	yes
M <sub>5</sub> <sup>a)</sup>	10	14	yes
M <sub>6</sub>	11	14	no

a) Pierced mirrors with hole at a third of the diameter from mirror's edge.

**Table 2**  
Results of the best fit for  $\sigma^2$  and  $\epsilon$

M <sub>F</sub>	M <sub>R</sub>	N <sup>a)</sup>	D <sup>b)</sup>	$\sigma^2$	$\epsilon$
M <sub>2</sub>	M <sub>1</sub>	63	43	$2 \times 10^{-7}$	$6.7 \times 10^{-5} \pm 1.5 \times 10^{-6}$
M <sub>2</sub>	M <sub>3</sub>	57	52	$5 \times 10^{-7}$	$3.0 \times 10^{-5} \pm 2.5 \times 10^{-6}$
M <sub>2</sub>	M <sub>4</sub>	57	52	$2 \times 10^{-7}$	$1.2 \times 10^{-4} \pm 2.5 \times 10^{-6}$
M <sub>2</sub>	M <sub>5</sub>	67	51	$2 \times 10^{-7}$	$2.2 \times 10^{-4} \pm 7.0 \times 10^{-7}$
M <sub>2</sub>	M <sub>6</sub>	47	42	$2.5 \times 10^{-7}$	$1.8 \times 10^{-4} \pm 2.0 \times 10^{-6}$

a) N is the total number of reflections in the cavity.

b) D is the cavity length (in cm).

## Figure captions

**Fig. 1** Simplified sketch (not to scale) of the layout; A: analyser prism.  $D_1$ ,  $D_2$ ,  $D_3$ : photodiodes.  $M_F$ : pierced mirror through which the beam enters and exits.  $M_R$ : analysed mirror. P: polarizer prism. S: view of the mirrors' surface with spots of reflecting beam.  $C_1$ ,  $C_2$ : points on the cavity's optical axis. D: cavity length.

**Fig. 2** Voltage  $V_{tr}$  on the amplifier feedback resistor  $R_f$  of diode  $D_2$  plotted against angle  $\gamma$  for a cavity with  $M_F = M_2$  and  $M_R = M_1$  (see Tables 1 and 2). The solid line represents the best fit [Eq. (3.1)].



