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BEAMSTRAHLUNG IN THE MULTI-TeV REGIME*)

M. Jacob
CERN - Geneva

ABSTRACT

Electron-positron linear colliders in the multi-TeV range have to operate with large luminosity because of the rapid decrease of many cross-sections with energy. This leads to special features for the radiation emitted as bunches cross each other. When the disruption parameter remains small, the question can be treated analytically. Some recent work is reviewed.

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1. - BEAMSTRAHLUNG IN THE QUANTUM REGIME

Electron-positron collisions offer a particularly clean and powerful way to probe the structure of matter at very small distances. The full energy is available at the constituent level. However, electrons are far more difficult to accelerate than protons. It is well known that LEP will be the largest electron-positron ring ever built. At energies significantly beyond the LEP range, synchroton radiation would become forbiddingly high. Linear colliders have to be considered. At present, studies are actively under way to find the proper design for such machines [1]. We shall here consider linear accelerators which would reach 1 to 5 TeV per beam.

A gain by an order of magnitude from LEP energy has to be associated with a gain in luminosity by typically two orders of magnitude, since the annihilation cross-sections decrease as s^{-1}, s being the centre-of-mass energy squared. One is therefore speaking about luminosities at the level of 10^{33} to 10^{34} $cm^{-2}s^{-1}$ [1].

High luminosities imply high bunch densities. The electrons traversing a positron bunch will therefore undergo an important acceleration, which will imply a rather large bremsstrahlung. Such a bremsstrahlung, at the level of a beam of particles, has been dubbed "beamstrahlung". We shall use the word despite its etymological weakness.

The nature of the beamstrahlung thus expected can be quite different from the one known at lower energies, and soon to be measured at SLC and at LEP. Indeed, for the reason given, there is not only a gain in energy, but also a big gain in acceleration during crossing, which is the price to pay for luminosity.

In the well-known approach to synchroton radiation [2], one introduces a parameter

$$\Upsilon = \frac{\gamma^2}{m\rho_c} \tag{1}$$

where $\gamma = E/m$, E and m are the electron energy and mass respectively, and ρ_c is the radius of curvature. T is the ratio between the classical energy typically radiated, which is defined by the cut-off frequency, and the incident electron energy.

When T << 1, one is in the classical regime and classical relations apply. When T >> 1, the classical approach clearly breaks down. One is in the deep quantum regime and the approach has to be modified accordingly.

It turns out that, while γ increases with energy, ρ_c may not increase as one could expect from the rigidity of the trajectory of a very energetic particle. The luminosity which has to be reached implies a strong bending and therefore prevents ρ_c from increasing much. It follows that very high energy linear colliders can easily lead us into the deep quantum regime ($T \gg 1$).

In the classical approach, which still prevails at present energies, a very important rôle is played by the coherent radiation length L_c. It is defined as the length over which the direction of the trajectory changes by an angle $\theta = \gamma^{-1}$. The radiative length L_e, namely the length over which an electron cannot be distinguished quantum mechanically from an electron dressed with photons, is negligibly small as compared to L_c. It is of course proportional to h, while proportional to γ/m. As the energy increases, however, L_e increases, while L_c does not do so as fast because of the strong bending imposed by the luminosity. As a result, very high energy linear colliders may well lead us to a new regime where $L_e \gg L_c$, which is in essence a quantum regime.

The purpose of this paper is to discuss beamstrahlung in this new regime ($L_e \gg L_c$), which corresponds to $T \gg 1$. In so doing we follow an approach which is particularly well suited to that problem, namely calculating Feynman graphs. While calculating Feynman amplitudes with asymptotic free plane wave states is well known, computation with the particular boundary conditions proper to bunch collisions is somewhat special.

We summarize here calculations following that approach and already presented to a large extent, with full details, in three papers by T.T. Wu and the author [3]. We refer the reader to them for explicit calculations which are merely outlined here. The purpose of this paper is to present a summary, stressing the salient points special to the regime considered.

The amount of radiation lost in beamstrahlung is rather large [3,4]. One may quote mean energy losses at the level of 20%! This imposes severe constraints on the machine. One is pushed to consider short elliptical ribbon bunches in order to minimize radiation once the energy and the luminosity are both in an acceptable range [5]. This imposes severe constraints when discussing the physics accessible with those machines [6].

The most important result refers to the mean fractional energy loss which, in the case of cylindrical uniform bunches, is written as [3]

$$\delta = \frac{\langle E_\gamma \rangle}{E} = K_e \frac{\alpha}{\pi} \ln \frac{L_e}{\ell_c} + K_i \frac{\alpha}{\pi} \frac{L_e}{\ell_c} \qquad (2)$$

where K_e (for "external") and K_i (for "internal") are two numerical factors of order 1.

$\langle E_\gamma \rangle$ is the average energy taken away by the photon and E is the incident electron energy. L_b is the bunch length and ℓ_c, which, as discussed later, plays the rôle of the coherent radiation length in the deep quantum regime, is defined as:

$$\ell_c = \left(L_c^2 L_e\right)^{1/3} \tag{3}$$

Relation (2) is accurate provided that

$$L_e \gg L_c \tag{4}$$

which, as discussed later, is equivalent to T >> 1.

The second term reproduces, in that regime, the more general but less simple expression arrived at by Blankenbecler and Drell [4,5]. The presence of the first term has been confirmed by Bell and Bell [7], who have also computed non-leading terms in edge effects.

As we shall see, introducing ℓ_c eliminates explicit references to the electron mass, so important in the standard approach to radiative phenomena, where the classical radius of the electron α/m sets the scale. This is indeed expected in this new regime where the transverse momentum gained in bunch crossing is very much larger than the electron mass.

The rest of this paper is organized as follows. In Section 2 we show why and how the multi-TeV regime differs so much from the regime met at present energy, namely 50 GeV per beam, and we discuss some consequences, anticipating the results presented in Section 3. In Section 3, we derive the radiation yield using the Feynman graph approach [3]. Finally, in Section 4, we discuss the present outlook. If one tries to summarize in one sentence the original aspect of the work presented in Section 3 [3], it is its use of Feynman graph techniques to solve a macroscopic radiation problem, but more specifically its relying on the fact that the radiative length L_e eventually becomes the largest length. This leads to the simple form (2), with its two different terms and, of course, the introduction of ℓ_c. When it comes to the actual calculation, our present lowest-order calculation can, of course, be looked at as a distorted wave Born approximation.

2. - GENERAL FEATURES OF QUANTUM BEAMSTRAHLUNG

The luminosity of a linear electron-positron collider can be written as

$$L \sim \frac{N^2 f H}{R^2} \tag{5}$$

In this relation, which is written with a "$\sim$" sign, as opposed to an equal sign, thus neglecting geometrical features which vary from design to design, N is the number of particles per bunch, f is the bunch frequency, R is the radius of the bunch and H is a pinch enhancement factor. The latter translates the focusing effect which one bunch plays on the other. It is related to the disruption parameter D. Denoting by F the focal length associated with the focusing of the incoming electrons (positrons) by the positron (electron) bunch, the disruption parameter is defined as

$$D = \frac{L_e}{F} \tag{6}$$

A relatively large value of D (D > 1, say) can result in a relatively large value for H (H >> 1), while a small value of D (0.1, say) gives H $\approx$ 1.

In order to gain in luminosity, one can increase N and f. This, however, costs power, technical constraints notwithstanding! One can increase H. This is the present CLIC philosophy [1]. This brings us into the large D regime, where analytic calculations are not presently possible. This is likely to increase δ beyond the values obtained in the small D regime (2). One can reduce R, which of course imposes high technological achievements. At present the SLC is based on micron-size bunches. The extreme, in the "super" [4], would be to go to 10Å, at small D. The CLIC philosophy is to have D $\sim$ 1 with larger radii [1]. Some parameters of the SLC, CLIC and the "Super" are listed in Table 2.1.

	E(TeV)	γ	L_b	R	N
SLC	0.05	10^5	10^2m	10^{-6}m	5×10^{10}
CLIC	1	2×10^6	4m	6×10^{-8}m	5×10^9
Super	5	10^7	3m	10^{-9}m	3×10^8

Table 2.1

The bunch length L_b is here expressed in the bunch frame. It is equal to $\gamma\sigma_z$, where σ_z (or $\bar{L}_b$) is the bunch length in the machine frame of reference.

From these parameters one can easily derive the other two characteristic lengths, namely the coherent radiation length L_c and the radiative length L_e. They are listed together with L_b in Table 2.2. We here denote with a bar quantities defined in the machine frame. One goes to the bunch frame multiplying them by γ (Table 2.1). The ratios are, of course, independent of the frame.

	$\bar{L}_b$	$\bar{L}_c$	$\bar{L}_e$
SLC	10^{-3} m	10^{-5} m	10^{-7} m
CLIC	5×10^{-4} m	2×10^{-6} m	2×10^{-6} m
Super	10^{-7} m	10^{-10} m	10^{-5} m

Table 2.2

This Table shows a very important change when going from the SLC to the "Super" situation. Whereas for SLC we have

$$\bar{L}_e \ll \bar{L}_c \ll \bar{L}_b \tag{7}$$

for the "Super" we have

$$\bar{L}_c \ll \bar{L}_b \ll \bar{L}_e \tag{8}$$

CLIC is in a "grey" zone with $\bar{L}_c \sim \bar{L}_e \ll \bar{L}_b$.

We see that in all cases $L_c \ll L_b$. This implies that, to a good approximation, one can be tempted to consider the radiation as originating independently from zones L_c deep within the bunch length L_b, hence, eventually, the term proportional to L_b/ℓ_c in (2). However, the fact that L_e becomes the largest length as the energy (and the luminosity!) increase, leads to drastic changes, namely, the introduction of a new coherent radiation length ℓ_c, which becomes much larger than the classical one L_c (3), and to the increasing importance of the first term in (2).

The scale in both terms is indeed set by ℓ_c (3). One readily finds that ($\bar{L}_e = 2\hbar\gamma/m$):

$$\Upsilon = \frac{\gamma^2}{m\,\ell_c} = \frac{L_e}{2L_c} = \left(\frac{\ell_c}{L_c}\right)^3 \tag{9}$$

hence T >> 1 (quantum regime) corresponds to $(\ell_c/L_c)^3 \gg 1$. In the approach of Blankenbecler and Drell [4], there is a parameter C, the quantum regime corresponding to C << 1. One finds that

$$C = \frac{1}{2}\left(\frac{L_c}{\ell_c}\right)^3 \tag{10}$$

The quantum regime corresponds to L_e becoming the largest length. Radiation over L_e before and after bunch crossing can therefore be considered separately. This is what was done in the first paper of Ref. [3], where the first term in (2) was calculated. In the "Super" regime it corresponds to a 3% contribution to δ.

Calculations in Ref. [3] consider a cylindrical bunch with uniform density and sharp boundary. The charge density is:

$$\rho = \frac{Ne}{\pi R^2 L_e} \tag{11}$$

for $|z| \leq L_b/2$ and 0 otherwise. One may wish to depart from such an approximation, considering more realistic bunch profiles.

The longitudinal profile can be parametrized defining a local density $\tilde{\rho}(z)$, with

$$\rho = \frac{Ne}{\pi R^2 L_e}\,\tilde{\rho}(z)$$

$$\frac{1}{L_e}\int \tilde{\rho}(z)\,dz = 1 \tag{12}$$

From the local density $\tilde{\rho}$, one can define a local coherent radiation length $\tilde{\ell}_c(z)$. One finds that [8]

$$\tilde{\ell}_c(z) = \ell_c\left(\tilde{\rho}(z)\right)^{-2/3} \tag{13}$$

where ℓ_c is the nominal coherent length defined for a uniform cylindrical bunch, with the same radius and the same total number of particles. The smaller the local coherent length, the higher the local density.

The radiation rate over that coherent length is the higher, the higher the density is, namely [8]

$$I(z) \sim \left(\tilde{\rho}(z)\right)^{2/3} \tag{14}$$

It therefore makes sense, as a first good approximation, to neglect the variation of $\tilde{\rho}$ over $\tilde{\ell}_c$, while considering different values of $\tilde{\rho}$ and $\tilde{\ell}_c$ over the bunch length $L_b >> \ell_c$.

One then finds that the overall radiative loss δ over L_b [or the term K_i in (2)] is then simply multiplied by a factor [5,8]

$$\frac{\int \tilde{\rho}^{2/3}(z)\,dz}{\int \tilde{\rho}(z)\,dz} \tag{15}$$

This shows that, with any realistic bunch profile, decreasing at both ends from a maximum value at the centre, this corresponds to an increase in radiation losses as compared with a uniform bunch ($\tilde{\rho}$ = 1). In order to minimize radiation, one should therefore work with bunches which are as compact as possible. Also, in order to minimize the field, and hence the radiation loss, one should prefer ribbon bunches to circular bunches [5].

Going beyond the approximation of a constant $\tilde{\rho}$ over $\tilde{\ell}_c$, and taking into account terms of order $\tilde{\rho}'\tilde{\ell}_c$, $(\tilde{\rho}'\tilde{\ell}_c)^2$ and $\tilde{\rho}''\ell_c^2$, is technically involved. One may say that no dramatic effect should occur, since the neglected terms in (15) are then all weighted down by factors of at least $(\ell_c/L_b)^2$, which are small [8]. Work still needs to be done along that line, studying also the effect of transverse distributions.

3. - THE BEAMSTRAHLUNG RADIATION

We now proceed to the calculation of the radiation rate. In the quantum regime, most of the radiated energy is found with photons taking away a rather large fraction of the electron (positron) energy. It is therefore acceptable to calculate the radiative process proper in perturbation theory, keeping the contribution of order α.

We consider the radiation of an electron crossing a positron bunch. The calculated Feynman graph is shown in Fig. 1. The crosses stand for the interaction of the electron with the bunch which, in practice, corresponds to solving the Dirac equation in the field created by the bunch (distorted wave Born approximation). The calculation is performed in the bunch frame of reference, where the incoming electron experiences only a radial electric field. The geometry is sketched in Fig. 2. In practice, $L_b >> R$. In the classical approximation, the electron trajectory with incident momentum k_i is bent inside the bunch, with a resulting final momentum k_f. A photon, if emitted, as here, leaves (almost) tangentially to the trajectory with momentum k_γ. The direction of k_i and k_f defines the x,z plane, the classical bending plane. The photon has only a small momentum component $k_{\gamma y}$ normal to that plane. In practice, the bending momentum

$$\Delta_T \sim \frac{ZN\alpha}{R} \tag{16}$$

is much larger than $k_{\gamma y}$ and the electron mass, which can be neglected altogether in the quantum regime.

Our calculation is done in the small D approximation (with the "Super" parameters $D \simeq 0.1$), and indeed all analytical approaches made so far [3,4] apply to that regime. One may then replace the problem of bunch-bunch collision by the much simpler problem of electron-bunch collision. An electron radiates in the intense field provided by the positrons independently of its fellow electrons. This is no longer the case when D and H are large. One then has to face the full many-body problem of bunch-bunch collisions. It is our guess that the radiation losses can only be larger than what they are calculated to be in the low D approximation.

The radiation amplitude is written as

$$\mathcal{M} = \varepsilon^\mu M_\mu \tag{17}$$

where ε is the polarization vector of the radiated photon. We shall consider separately the component in the classical bending plane $\varepsilon_\parallel$ and, perpendicular to it, $\varepsilon_\perp$. We write

$$M_\mu = -ie \int d^3\vec{x}\, e^{-i\vec{k}_\gamma\cdot\vec{x}}\, \bar{\Psi}_f(x)\, \gamma_\mu\, \Psi_i(x) \tag{18}$$

In the integrand we have the electromagnetic current of the electron, for which we can also use (spinless case) the Klein-Gordon expression, namely

$$j_\mu(x) = i\left(\psi_f^* \partial_\mu \psi_i - (\partial_\mu \psi_f^*)\psi_i\right) \tag{19}$$

One may indeed sometimes wish to study the simpler spinless case before facing the complications due to spin. The calculation proceeds through six successive steps.

i) One calculates ψ_i and ψ_f in the presence of the bunch. This is done by solving the Dirac (Klein-Gordon) equation. One does it in the large E (but rather small D) approximation, writing

$$\psi(x) = A(x)\, e^{i\eta(x)} \tag{20}$$

when the equation defining $\eta(x)$ is defined according to the optical approximation [9]. We write:

$$A(x) = A_0(x) + \frac{A_1(x)}{E}$$
$$\eta(x) = \vec{k}\cdot\vec{x} + \eta_0(x) + \frac{\eta_1(x)}{E} \tag{21}$$

and solve the Dirac (Klein-Gordon) equation with the four different terms thus determined. It turns out that the same function $\eta(x)$ applies to both the Dirac and Klein-Gordon cases. The details are given in the third paper of Ref. [3]. When we appear to start with a high E approximation (21), we actually make a low D approximation. This is discussed in the third paper of Ref. [3].

ii) One studies the conditions for a stationary phase. Once ψ_i and ψ_f are known, the phase ϕ of the integrand in M is known. It is clear that most of the radiation will originate from that region where the phase factor does not vary appreciably. This leads to the definition of a coherent radiation length. We thus find the new length ℓ_c previously mentioned. It is the same in the Dirac and in the Klein-Gordon cases. The details are given in the second paper of Ref. [3]. When $L_e \gg L_c$, ℓ_c can be much larger than the classical coherent radiation length L_c. For the "super" parameters, one has $\ell_c \approx 20\ L_c$.

iii) The stationarity conditions define a stationary zone around a point x_o, y_o, z_o which is determined by (ii). One then computes the radiation matrix elements in the neighbourhood of x_o, y_o, z_o. One computes separately $m_\parallel$ (x,y,z) and $m_\perp$ (x,y,z).

iv) One computes the space integrals, as they appear in (18), to obtain the two radiation amplitudes $\mathcal{M}_\parallel$ and $\mathcal{M}_\perp$. They are the integrals of the matrix elements calculated in (iii) with the phase factors obtained in (i). One uses approximations corresponding to the stationarity conditions obtained in (ii). The form taken by the amplitudes is well known from classical synchroton radiation [10]. One finds an Airy function in the case of $\mathcal{M}_\perp$ and its derivative in the case of $\mathcal{M}_\parallel$. The asymptotic behaviour of these Airy functions imposes a cut-off on $k_{\gamma y}$ which has a simple physics interpretation. It is the transverse momentum collected by the electron over the coherent radiation length ℓ_c. It is in practice much smaller than the typical transverse momentum $\Delta_T \sim 2N\alpha/R$, collected over the full bunch length $L_b >> \ell_c$.

v) Compute the radiation rate per unit area I(X), where X is the fraction of the incident electron energy taken by the radiated photon. The phase space integrals involving squares of the Airy function can be done analytically expressing the Airy function and its derivative in terms of a Bessel function of fractional order. The spectrum shape in the Klein-Gordon case is found to be

$$I(X) \sim \left(\frac{1-X}{X}\right)^{2/3} \tag{22}$$

and hence does not show any strong peaking. In the Dirac case, the non-helicity flip contribution dominates in the quantum regime. It has a spectrum

$$I(X) \sim \frac{1}{2}\left(\frac{1-X}{X}\right)^{2/3} \frac{1+(1-X)^2}{1-X} \tag{23}$$

where the last factor results from the spinology. The details of the calculation are presented in the second and third papers of Ref. [3].

vi) Compute the fractional energy loss

$$\delta = \int_0^1 X\, I(X)\, dX \tag{24}$$

It is obtained as a sum of beta functions, and is written as

$$\delta = \frac{\alpha}{\pi} f\left(L_B, L_c, L_e\right) \tag{25}$$

It is proportional to α (distorted wave Born approximation). It involves a homogeneous function of the three characteristic lengths, which has the simple form given in (2), once ℓ_c is defined (3). This altogether gives the fractional energy loss associated with bunch crossing proper, since the phase conditions obtained in (ii) pertain to conditions deep inside the bunch, a length ℓ_c at least away from the edges. It is numerically equal to 17% (10% in the Klein-Gordon case), with the "super" parameters.

One sees that everything is as if each coherent radiation length was contributing separately. The radiation rate is proportional to L/ℓ_c. However, it is ℓ_c which matters and not the classical coherent radiation length L_c. The integral (18) extends over the whole space and not only the bunch volume. While we can separate out through the phase conditions different slices of the bunch, we can also separate out the contributions originating from the region before the bunch ($-\infty < z < -L_b/2$) and after the bunch ($L_b/2 < z < \infty$). This is radiation before and after bunch crossing. One cannot speak about a clear time sequence. We actually treat a stationary problem and consider rather contributions from different regions to a unique space integral. Nevertheless, we can have a quantum mechanical interpretation of that radiation saying that the electron hits or leaves the bunch off-shell, having emitted, or not yet emitted, a photon. Its off-shell character can be present over a length L_e. As a result, it is natural to find in the rate terms proportional to $(\alpha/\pi)\,\ell n\,(L_e/\ell_c)$ which can be associated with radiation before and after bunch crossing. This is the first term in (2).

The details of the calculation are given in the first paper of Ref. [3], while the third paper puts together the three contributions referred to as radiation before, during and after bunch crossing.

We then proceed to some technical points in order to show how the new coherent radiation length ℓ_c originates.

The stationary phase cannot be satisfied in a real sense. While $\partial\phi/dx = 0$ and $\partial\phi/dy = 0$ define impact co-ordinates, $\partial\phi/dz = 0$ has no real solution. In the Klein-Gordon case, one has the particularly simple relation

$$\frac{\partial \phi}{\partial z} = \frac{1}{8}\,\frac{X}{1-X}\,\frac{1}{k_i}\left\{ |m_{\parallel}(z)|^2 + |m_{\perp}(z)|^2 \right\} \tag{26}$$

where $m_{\parallel}$ and $m_{\perp}$ are the two matrix elements calculated in (iii). They read [3]:

$$m_{\parallel} = -2\,\Delta_{Tx}\left(\tau_i(z) - \tau_i(z_1)\right)$$
$$m_{\perp} = \frac{2}{X}\,k_{\gamma y}\,\frac{k_{fx}}{\Delta_T} \tag{27}$$

The function $\tau_i(z)$ measures the relative amount of bunch charge met by the incoming electron up to the distance z, namely

$$\tau_i(z) = \frac{1}{L_e}\int_{-\infty}^{z} \tilde{e}(z')\,dz' \tag{28}$$

For a uniform bunch, one has simply

$$\tau_i(z) - \tau_i(z_1) = z - z_1 \tag{29}$$

The co-ordinate z_1 is defined so that the second derivative of the phase vanishes:

$$\frac{\partial^2 \phi}{\partial z^2} = 0 \quad \text{at} \quad z = z_1 \tag{30}$$

This is always possible, with a real value [3], and one finds

$$\tau_i(z_1) = \frac{1}{X} \, \frac{-k_{\gamma x}\Delta_{Tx} + k_{\gamma y}^2}{\Delta_T^2} \tag{31}$$

The phase conditions are such that most of the radiation will originate from the neighbourhood of z_1, indeed over a length ℓ_c around z_1, and z_1 is determined by the direction of the final photon and electron. If one neglects $k_{\gamma y}$, this is the point which corresponds to tangential emission of the photon, with the x and y co-ordinates being fixed by the stationary phase conditions.

If one first neglects the variation of $\tilde{\rho}(z)$ inside the bunch, we can obtain the phase of the amplitude through integration [3]:

$$\frac{\partial \phi}{\partial z} = \frac{1}{2X(1-X)k_i} \left\{ \frac{X^2 \Delta_T^2}{L_\ell^2} \tilde{\rho}^2(z_1)(z-z_1)^2 + \frac{k_{\gamma y}^2 k_{\ell x}^2}{\Delta_T^2} \right\} \tag{32}$$

$$\phi(z) = C_3 (z - z_1)^3 + C_1 (z - z_1) + C_0 \tag{33}$$

The coherent radiation length is then defined by $C_3^{-1/3}$, explicitly, for $\tilde{\rho} = 1$:

$$\ell_c = \left(\frac{R^2 L_\ell^2 k_i}{(N\alpha)^2} \right)^{1/3} \tag{34}$$

with

$$\Delta_T \sim \frac{2N\alpha}{R} \tag{35}$$

It is different from the classical coherent radiation length. In terms of the more familiar centre-of-mass variables (referred to by bars), one has:

$$\bar{L}_c = \bar{L}_\ell \frac{Rm}{N\alpha} \qquad \bar{L}_e = 2\frac{\bar{k}_i}{m^2}$$

and hence

$$\bar{L}_c^2 \bar{L}_e = 2\frac{R^2 \bar{L}_\ell^2}{(N\alpha)^2} \bar{k}_i = \bar{\ell}_c^3 \tag{36}$$

One sees that the combination of $\bar{L}_c$ and $\bar{L}_e$ which defines $\bar{\ell}_c$ eliminates the electron mass.

The calculation of the radiation amplitude then proceeds through the space integrals of the matrix elements (27) with the phase factor (33). As already said, it is the same for the Dirac and Klein-Gordon cases, whereas the matrix elements m are not. These integrals are of the type met in the classical theory of synchroton radiation [10]. In the case of $m_\perp$, one obtains an Airy function $A_i(u)$. In the case of $m_\parallel$, one obtains the first derivative of that Airy function, $A_i'(u)$. Up to a numerical factor, the argument is such that

$$u^{1/2} \sim k_{\gamma y} \Delta_T^{-1} \qquad k_{\gamma y} \sim \frac{N\alpha}{R} \frac{\ell_c}{L_\ell} \tilde{\rho}^{1/3} \tag{37}$$

We see that the exponential dependence of the Airy function imposes a cut-off on $k_{\gamma y}$, the photon momentum component perpendicular to the classical bending plane. The typical value of $k_{\gamma y}$ (corresponding to u = 1) is equal to the bending momentum collected by the electron over the coherent radiation length ℓ_c.

We have neglected so far the variation of $\tilde{\rho}$. However, our relations remain valid provided that $\tilde{\rho}$ does not vary appreciably over $\tilde{\ell}_c = \ell_c \tilde{\rho}^{-2/3}$, where $\tilde{\ell}_c$ is the local coherent radiation length. This is in general a good approximation over most of the bunch length, and is the approximation which is used to obtain (15). We shall keep it in the following, the uniform cylindrical bunch case corresponding to $\tilde{\rho} = 1$.

The radiation rates corresponding to a photon polarization normal $I_\perp$ and parallel $I_\parallel$ to the classical bending plane are then readily obtained [3]. In the spinless (Klein-Gordon) case, one finds:

$$I_\perp = \frac{6\alpha}{\pi^3 k_i}\left(\frac{\pi R^2 L_b}{N\alpha}\right)^2 \int_0^\infty v^2 dv\, A_i^2(v^2)$$
$$\int \omega^{2/3} \frac{\Delta_T^2}{L_b}\, dk_{\ell x}\, dk_{\gamma x} \frac{dX}{X}$$
$$I_{\parallel} = \frac{6\alpha}{\pi^3 k_i}\left(\frac{\pi R^2 L_b}{N\alpha}\right)^2 \frac{1}{9}\int_0^\infty dv\, A_i'^2(v^2)$$
$$\int \omega^{2/3} \frac{\Delta_T^2}{L_b}\, dk_{\ell x}\, dk_{\gamma x} \frac{dX}{X} \tag{38}$$

with

$$\omega = \frac{6(1-X)}{X} \frac{k_i}{\Delta_T^2 L_b} \tag{39}$$

where we recall that $\Delta_T \sim 2N\alpha/R$.

The next step is to integrate the integrals over A_i^2 and $A_i'^2$. This is done using the relations [11]:

$$A_i(v^2) = \left(\frac{v^2}{3}\right)^{1/2} K_{1/3}(2v^3)$$
$$A_i'(v^2) = \sqrt{3}\, v^2 K_{2/3}(2v^3) \tag{40}$$

One then readily proceeds with the phase space integrals to obtain the global radiation rate

$$J(X) = K_i \frac{\alpha}{\pi} \frac{L_b}{\ell_c} \pi R^2 \left(\frac{1-X}{X}\right)^{2/3} \tag{41}$$

Dividing by πR^2 to obtain the rate per unit area, I(X), one finds the contribution to δ as given by the last term in (2). This is the contribution resulting from radiation "during" bunch crossing or, more precisely, from the contribution, incoherent to the rest, coming from the integral over the bunch in the region of validity of (32), a length ℓ_c away at least from the edges. Since $\ell_c \ll L_b$, this is a good approximation to the radiation rate. It is proportional to α/π as expected, but multiplied by a rather large factor L_b/ℓ_c (60, say, with the super parameters). The quantity K_i is of order unity. In the quantum regime, one finds $K_i = 1.38$.

We presented the spinless results for their pedagogical simplicity. Indeed, the bulk of the contribution in the Dirac case is simply proportional to the result obtained in the Klein-Gordon case (41). This is the non-helicity flip contribution. One finds in that case

$$J_{NF}(x) = J_{KG}(x)\,\frac{1}{2}\;\frac{1+(1-x)^2}{1-x} \tag{42}$$

where $J_{KG}(x)$ is given by (41).

The helicity flip term is relatively very small with the super parameters. It is proportional to L_b/L_e which is then a small quantity, while L_b/ℓ_c is a large quantity. The complications due to spin are discussed in detail in the third paper of Ref. [3].

The contribution to the amplitude obtained through integration over the bunch does not interfere with those coming from the zones before and after the bunch. As mentioned earlier, they become relevant when $L_e \gg \ell_c$. The overall contribution to δ, as calculated in the first and third papers of Ref. [3], is the second term in (2). The coefficient K_e is of order one. One finds $K_e = 4/3$.

One can then write δ as

$$\delta = \frac{\alpha}{\pi}\left(1.38\,\frac{L_e}{\ell_c} + \frac{4}{3}\ln 4\,\frac{L_e}{\ell_c}\right) + O\left(\frac{\alpha}{\pi}\right) \tag{43}$$

The two terms where α/π is multiplied by a large number have thus been isolated. This is accurate as long as $\ell_c \gg L_c$, or as long as L_e is by far the largest length, this defining the deep quantum regime. Edge effects not treated at that level are relatively small, since they are only of order α/π. We stress again the simplicity of (2) and (43), once the new coherent radiation length ℓ_c is defined.

4. - PRACTICAL CONSIDERATIONS

The radiation intensity and the fractional energy loss can be simply expressed in the quantum regime, provided that one remains in the low D approximation. They are numerically large. The value of δ is of the order of 20% with the super parameter [3,4] and the radiation spectrum is not strongly peaked. Bunch-bunch collisions thus provide a large intensity of very energetic photons. An electron-positron linear collider is also an intense photon-photon collider, with almost comparable energy. Our method, like other analytical approaches [4,5,7], applies to low values of D. This is not the regime selected by CLIC [1]. With large D and large H, the question cannot be approached as a particle-bunch problem. It would be very surprising if the radiation losses were not then higher.

As a first step one could extend our approach [3], which keeps only linear terms in D, to an approximation to order D^2 to see how important corrections are. We saw that in order to minimize radiation losses, bunches should be as compact as possible (15). They should also be as short as possible. Indeed, we can write ℓ_c as (34)

$$\ell_c = \left(\frac{R^2}{(N\alpha)^2} \frac{2s}{m} L_b^2 \right)^{1/3} \tag{44}$$

where s is the centre-of-mass energy squared. The luminosity is proportional to N^2/R^2 and it has to rise as s for any interesting machine. The two first factors should therefore compensate themselves. It follows that

$$\delta \sim \frac{L_b}{\ell_c} \sim L_b^{1/3} \tag{45}$$

One should therefore try to minimize the bunch length. This quantum regime behaviour is very different from that found in the classical regime where

$$\delta_{class} \sim \frac{\alpha}{m^3} \frac{(N\alpha)^2}{R^2} \frac{\gamma^2}{L_b} \tag{46}$$

Whatever the machine parameters, radiation losses have to be high. This is the price to pay for high luminosity.

With a high value of δ and a very broad radiation spectrum, the possibility of tuning on a resonance and getting a very large rate enhancement, so beneficial with lower energy electron-positron colliders, is lost. Resonance peaks will be smeared, or at least greatly eroded, by radiative effects. This is a setback, but not such a serious one. There is a lot of potential physics at these energies where such radiative effects are not very relevant. This is, for instance, the case for reactions of the type $WW \to X$, where the W's are radiated by the incoming electron and positron. This is very important if the Higgs is very massive and there is a strong sector for electroweak interactions. Such processes can, of course, also be studied at pp colliders, but there the background is extremely strong. If such physics is relevant, e^+e^- colliders would offer the only way to study it in any precise way. The machine energy will, of course, have to be high enough.

One should also stress that the photon-photon collider aspect of such machines is a priori very interesting. One could use $\gamma\gamma$ collisions and $e\gamma$ collisions in the bunch to study the Primakoff-type production of hitherto unknown objects (axions for instance). One would use the interaction of the photons with the very high electromagnetic field inside the bunch. One could also use an external photon beam on a fixed target.

Back to the actual study of beamstrahlung: much remains to be done. We worked in the approximation of a local (over ℓ_c) constant bunch density. One should explicitly study terms of order ρ', ρ'^2 and ρ''. They come multiplied by a factor $(\ell_c/L_b)^2$ and should not provide dramatic effects. Work is in progress [8]. One should study the effect of realistic transverse density distributions. This may lead to a rainbow effect in electron (positron) deflection in bunch collision.

One should also study edge effects. They are of order (α/π) without a large factor, as in (2). Nevertheless, one has to understand better how one goes from the external regime with a $\ell n\, k_{\gamma y}$ behaviour to the sharp cut-off $k_{\gamma y}$ behaviour of the internal regime.

We have limited ourselves to single-photon radiation, but with such a high radiation rate, several-photon radiation should be frequent. The distorted wave method has to be extended to such multi-photon processes.

We have considered the radiation of an electron crossing a positron bunch, but have also to study the radiation during an annihilation process. The method must also be extended to such a case with a proper adaptation of the distorted wave approach.

Finally, this method should also apply to the study of high-energy channelling [12] and also to the study of transition radiation. In both cases the Feynman graph approach should lead to a fruitful formulation.

This shows that the question does not end with relation (2), but rather that the study of beamstrahlung is worth developing further.

REFERENCES

1) U. Amaldi, "Introduction to the next generation of linear colliders", CERN EP-87-169. This paper contains the provisional CLIC parameters listed here:
U. Amaldi, Phys. Lett. 61B (1976) 313;
W. Schell, Contribution to the Marseille Workshop, February 1988; the CLIC study group at CERN has produced several CLIC notes which discuss present ideas about such a machine, CLIC Notes, CERN reports, unpublished;
B. Richter, Nucl. Inst. and Methods 136 (1976) 47;
B. Richter, I.E.E.E. Trans. Nucl. Sci. 32 (1985) 3928.

2) J. Schwinger, Proc. Nat. Acad. Sci. 40 (1984) 132;
A.A. Sokolov and I.M. Ternov, "Synchrotron Radiation", Pergamon (1971);
T. Himel and J. Siegrist, A.I.A. Conf. Proc. 130 (1985);
R.J. Noble, SLAC-PUB-3871 (1986);
P. Chen, SLAC-PUB-4931 (1987).

3) M. Jacob and T.T. Wu, Phys. Lett. 197B (1987) 253;
M. Jacob and T.T. Wu, CERN Preprint TH. 4848 (1987), Nucl. Phys. B, to be published;
M. Jacob and T.T. Wu, CERN Preprint TH. 4907 (1987), Nucl. Phys. B, to be published.

4) R. Blankenbecler and S. Drell, Phys. Rev. B36 (1987) 277. This paper contains the provisional "Super" parameters listed here.
M. Bell and J.S. Bell, CERN PS-87-53 (1987).

5) R. Blankenbecler and S. Drell, SLAC-PUB-4883 (1987).

6) M. Jacob, Physics with multi-Tev electron-positron collider, Kazimierz 1988;
U. Amaldi, La Thuile Workshop (1987), CERN-EP 87-95;
J. Ellis and F. Pauss, La Thuile Workshop, CERN-TH.4682/87 (1987);
J. Ellis, ICFA Contribution, Brookhaven, CERN-TH.4888/87 (1987).

7) J.S. Bell and M. Bell, CERN Preprint TH. 4936 (1987). This paper confirms the first term in (2) and discusses an earlier unpublished introduction of ℓ_c by J.S. Bell.

8) M. Jacob and T. T. Wu, in preparation.

9) T.T. Wu, Phys. Rev. 107 (1957) 871;
D.S. Saxon and L.I. Schiff, Nuovo Cimento 6 (1957) 614.

10) J.D. Jackson, "Classical Electrodynamics", John Wiley (1975).

11) Bateman manuscript project: "Higher Transcendental Functions", A. Erdelyi, Vol. 2 (1953).

12) V.V. Beloshitsky and F.F. Komarov, Physics Reports 93 (1982) 119;
J.U. Andersen, E. Bonderup and R.H. Pantell, Ann. Rev. Nucl. Part. Sci. 33 (1983) 453;
X. Artru, Orsay Preprint 87/54 (1987).

FIGURE CAPTIONS

Fig. 1 : Feynman graph for the emission of a photon by an electron in the presence of a bunch.

Fig. 2 : Radiation of a photon by an electron while its trajectory is bent by the presence of the bunch which extends from $-L_b/2$ to $L_b/2$. Fixing the final momenta $\vec{k}_f$ and $\vec{k}_\gamma$ implies radiation from a zone fixed by the coherence conditions around x_0, y_0 and z_0. The transverse component of the photon $k_{\gamma x}$ is primarily due to the bending of the electron trajectory before emission. In practice $L_b \gg R$. For "typical" super parameters $L_b \sim 3$m, $R \sim 10^{-9}$m. The disruption effect shown in the figure is overemphasized. In practice $D \sim 0.1$.

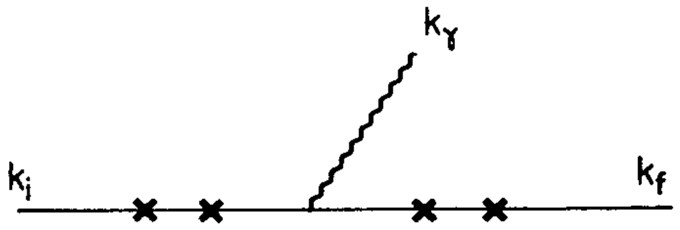

Fig. 1

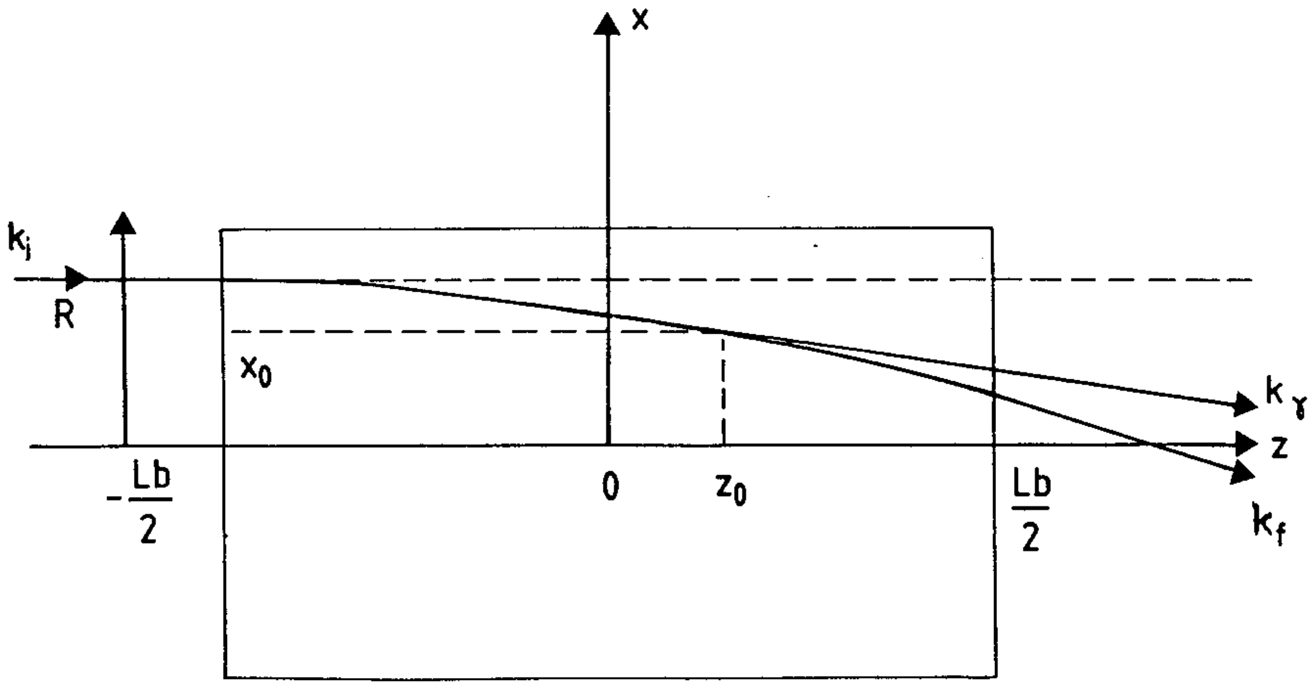

Fig. 2