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TEST OF QCD AND A MEASUREMENT OF Λ FROM SCALING VIOLATIONS IN THE NUCLEON STRUCTURE FUNCTION $F_2(x, Q^2)$ AT HIGH Q^2

BCDMS Collaboration

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A.C.Benvenuti, D.Bollini, G.Bruni, L.Monari, F.L.Navarria
Dipartimento di Fisica dell Universita and INFN, Bologna, Italy

A.Argento[1], J.Cvach[2], W.Lohmann[3], L.Piemontese[4], G.Todorova[5]
CERN, Geneva, Switzerland

A.A.Akhundov, V.I.Genchev, V.G.Krivokhizhin, V.V.Kukhtin, S.P.Kurlovich, S.Nemecek, P.Reimer, V.V.Sanadze, I.A.Savin, A.V.Sidorov, N.B.Skachkov, G.I.Smirnov, G.Sultanov[6], P.Todorov, A.G.Volodko
Joint Institute for Nuclear Research, Dubna

D.Jamnik[7], R.Kopp[8], U.Meyer-Berkhout, A.Staude, K.-M.Teichert, R.Tirler[9], R.Voss, Č.Zupančič
Sektion Physik der Universität, München, Federal Republic of Germany

J.Feltesse, A.Milsztajn, A.Ouraou, J.F.Renardy, P.Rich-Hennion, Y.Sacquin, G.Smadja, P.Verrecchia, M.Virchaux
DPhPE, CEN Saclay, France

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[1]Now at Digital Equipment, Torino, Italy.

[2]On leave from Institute of Physics, CSAV, Prague, Czechoslovakia.

[3]On leave from the Institut für Hochenergiephysik der AdW der DDR, Berlin-Zeuthen, GDR.

[4]Now at INFN, Trieste, Italy.

[5]On leave from Institute of Mathematics, Bulgarian Academy of Sciences, Sofia, Bulgaria.

[6]Now at the Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia, Bulgaria.

[7]On leave from E.Kardelj University and the J.Stafan Institute, Ljubljana, Yugoslavia.

[8]Now at Siemens AG, Münich, BRD.

[9]Now at DPhPE, CEN Saclay, France.

In a previous letter, we have reported on a high statistics measurement of the nucleon structure function $F_2(x,Q^2)$ at large x and Q^2 in deep inelastic scattering of muons on a carbon[1]. Deviations from Bjorken scaling are clearly observed in these data. Here we compare the measured scaling violations to predictions of perturbative Quantum Chromodynamics (QCD).

In this theory, the Q^2 evolution of quark and gluon distributions is described by the Altarelli-Parisi equations[2]. The evolution equation for the structure function $F_2(x,Q^2)$ can be written as

$$\frac{dF_2(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\cdot\int_x^1 [P_{qq}(z)F_2(x/z,Q^2) + 2\sum_{i=1}^{f} e_i^2 P_{Gq}(z)\, x/z\cdot G(x/z,Q^2)]\,dz, \tag{1}$$

where $xG(x,Q^2)$ is the gluon momentum distribution, P_{qq} and P_{Gq} are QCD splitting functions[3], f is the number of effective flavours, and e_i are the quark charges. The strong coupling constant α_s is given in next-to-leading order by

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2)}\cdot\left[1 - \frac{\beta_1 \ln\ln(Q^2/\Lambda^2)}{\beta_0^2\cdot\ln(Q^2/\Lambda^2)}\right], \tag{2}$$

where $\beta_0 = 11 - {}^2/_3 f$, $\beta_1 = 102 - {}^{38}/_3 f$, and Λ is the mass scale parameter of QCD[4]. Estimates of the gluon distribution from neutrino[6] and from muon scattering[7,8] indicate that it is a rapidly decreasing function of x and can be neglected to a good approximation for $x > 0.3$. In this region, equ.(1) is then simplified to the flavour nonsinglet expression

$$\frac{dF_2(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\cdot\int_x^1 P_{qq}^{NS}(z)\,F_2(x/z,Q^2)\,dz. \tag{3}$$

Alternatively, the Q^2 evolution of the structure functions can be expressed through the Q^2 dependence of their moments[5].

Our data[1] are well suited for a precise test of the evolution equations. "Higher twist" contributions to F_2 from quark-quark interactions which are not described by equ.(1) are expected to vary like power series in $1/Q^2$ [9] and are therefore unimportant due to the large Q^2 of the data ($Q^2 > 25$ GeV^2). Furthermore, the data extend up to $x = 0.75$, thus requiring only little extrapolation to calculate the evolution integrals.

Several numerical methods have been developed to fit the predictions of the evolution equations to the experimental data. We have mainly employed two methods which have been developed within our collaboration. They allow to use the singlet and the nonsinglet form of the evolution equations, both in a leading order (LO) perturbation expansion and in a next-to-leading order expansion in the $\overline{MS}$ renormalization scheme. The first of these[10] is based on an expansion of the structure functions into Jacobi polynomials with coefficients given by linear combinations of the QCD moments. In this method, most computations are analytic thus requiring only modest computing time. The second method[11] uses a numerical evaluation of the evolution equations which does not make any assumptions about the analytic behaviour of the Q^2 evolution of $F_2(x,Q^2)$ or of the integration kernels. This is conceptually the most direct approach but is normally considered impractical since it is expensive in computing time. This difficulty was overcome by vectorizing in Q^2 the integrations in equ. (1) and performing the analysis on a fast vector processor. We have also used the programs by Gonzàles-Arroyo, López, and Yndurain[12], by Abbott, Atwood, and Barnett[13], and by Furmanski and Petronzio[14].

The experimental data shown in Fig. 1 of Ref.[1] were used for the fits. Points with $y < 0.2$ were excluded to reduce the sensitivity to spectrometer calibration uncertainties. This cut removes 9 data points at large x and small Q^2 out of a total of 166 points. No correction was applied for Fermi motion.

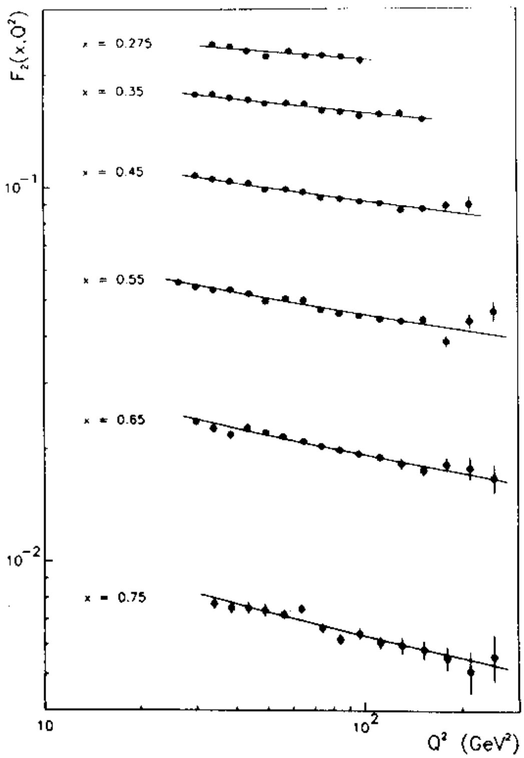

Fig.1. Next-to-leading order QCD fit in the nonsinglet approximation, corresponding to $\Lambda_{\overline{MS}}$ = 230 MeV, superimposed to the experimental F_2 data.

The results of flavour nonsinglet fits are summarized in the Table.

Table. Results of nonsinglet QCD fits to $F_2(x,Q^2)$

Program	Λ_{LO} (MeV)	χ^2/DOF	$\Lambda_{\overline{MS}}$ (MeV)	χ^2/DOF
BCDMS [10]	226±22	174/152	231±20	176/152
BCDMS [11]	215±20	174/150	235±20	178/150
Gonzàles-Arroyo et al. [12]	220±20	178/151	220±20	180/151
Abbott et al. [13]	224±20	190/151	233±20	198/151
Furmanski-Petronzio [14]	225±25	178/152	270±25	181/152

They were obtained using $R = \sigma_L/\sigma_T = R_{QCD}$ and assuming $f = 4$ flavours. With the exception of the method of ref./14/, we remark the excellent agreement between the values of Λ obtained with different programs and the small differences between the leading and next-to--leading order fits. The next-to-leading order fit with our second program/11/ is superimposed to the experimental data in Fig.1. Fits to the data evaluated with R=0 instead of R_{QCD}/1/ increase Λ by 15 MeV and give a slightly larger χ^2.

To evaluate a systematic error on Λ , the individual systematic uncertainties on F_2 were added to the data points and the fits repeated. This was done for each contribution to the systematic errors in turn and the resulting changes in Λ were combined in quadrature. The final systematic error of $\Delta\Lambda$ = 60 MeV is dominated by the 1% uncertainty on the relative normalization between data taken at the three different beam energies.

Our best estimate for the QCD mass scale parameter is the average value of the nonsinglet fits with methods/10-13/

$$\Lambda_{\overline{MS}} = 230 \pm 20 \text{ (stat.)} \pm 60 \text{ (syst.) MeV} .$$

This corresponds to a strong coupling constant at next-to-leading order of

$$\alpha_s = 0.160 \pm 0.003 \text{ (stat.)} \pm 0.010 \text{ (syst.)}$$

at $Q^2 = 100$ GeV^2.

It is important to remark that this numerical value of α_s depends only slightly on the conventional hypothesis of 4 flavours. On the contrary, the value of Λ determined from the fit depends strongly on this choice. The assumption of 5 flavours yields $\Lambda_{\overline{MS}}$= 160 MeV.

Results on Λ from deep inelastic muon scattering experiments on iron targets have been presented earlier by the BFP and EMC collaborations. The BFP group[15] finds Λ_{LO}= 230 ± 40 ± 80 MeV from a leading order singlet fit to their F_2 data. The EMC result[8] from a nonsinglet fit for $x \geqslant 0.35$ is $\Lambda_{\overline{MS}} = 115 \pm 10 ^{+90}_{-45}$ MeV.

A global QCD fit to $F_2(x,Q^2)$ does not, however, constitute a sensitive test of Quantum Chromodynamics. The χ^2 of such a fit describes also its agreement with the x dependence of F_2 which is not predicted by the theory. A more stringent test is obtained by comparing the x dependence of the scaling violations observed in the data to the one expected from the QCD evolution. This is the only specific prediction of perturbative QCD for deep inelastic scattering which can be tested experimentally. In the nonsinglet approximation, this comparison depends on Λ as the only free parameter whereas for singlet fits it is also sensitive to the gluon distribution. The nonsinglet case is shown in Fig. 2a where the logarithmic derivatives $d\ln F_2(x,Q^2)/d\ln Q^2$ are compared to the next-to-leading order prediction for $\Lambda_{\overline{MS}}$ = 230 MeV.

The logarithmic derivatives in Fig. 2 are the slope parameters of staight line fits $\ln F_2 = a \cdot \ln Q^2 + b$ to the data. To calculate the theoretical predictions shown in the same figure, the result of the QCD fit F_2' was assigned at each (x,Q^2) point the statistical error of the corresponding experimental F_2. The logarithmic derivatives $d\ln F_2 / d\ln Q^2$ were then obtained by the same straight line fit as for the experimental data. Within the errors, this linear representation is an excellent approximation of both the experimental and the predicted Q^2 evolution. The measured x dependence of the scaling violations in Fig.2a is in full agreement with the predicted curve within statistical errors (χ^2/DOF = 4.6/5). This shows that our determination of Λ is based on a very good overall description of the data by QCD.

We have also performed flavour singlet fits to our data, imposing a gluon distribution $xG(x,Q_o^2) = {}^1/_2(\eta + 1)\cdot(1 - x)^{\eta}$ at Q_o^2= 25 GeV^2 and fitting $\Lambda_{\overline{MS}}$ for fixed values of η. As an example, the singlet prediction corresponding to the fit for η = 7 is also shown in Fig.2a. Results for $\Lambda_{\overline{MS}}$ from these fits and the corresponding χ^2's from the comparison of measured and predicted scaling violations are shown as a function of η in Fig.3. It is clearly seen that the singlet terms do not improve the quality of the fits and that a soft gluon distribution is favoured by the data. From statistical errors, we find a lower limit of $\eta = 7$ at $Q_o^2 = 25\ GeV^2$ (90% confidence level). This corresponds to a limit of $\eta \approx 6$ at $Q_o^2 = 5\ GeV^2$.

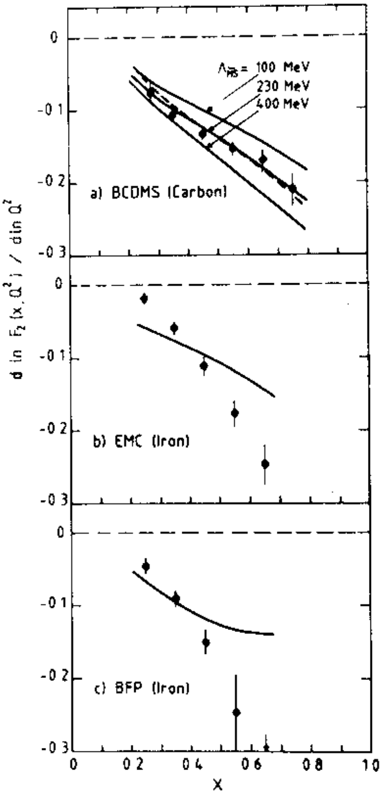

Fig.2. (a) Scaling violations observed in this experiment, expressed as logarithmic derivatives $d\ln F_2(x,Q^2)/d\ln Q^2$. The errors are statistical only. The solid lines are nonsinglet QCD predictions for $\Lambda_{\overline{MS}}$= 230 MeV corresponding to our fit (χ^2/DOF = 4.6/5), and for two other values of Λ. The dashed line corresponds to the singlet fit for a gluon distribution $xG(x,Q_o^2) = 4 \cdot (1-x)^7$ at $Q_o^2 = 25\ GeV^2$, giving $\Lambda_{\overline{MS}}$ = 252 MeV.

(b) As Fig.2a, for the EMC [8] data on iron for $Q^2 > 10\ GeV^2$. The solid line represents a next-to-leading order QCD fit giving $\Lambda_{\overline{MS}}$ = 91 MeV and a χ^2/DOF = 65.9/4.

(c) As Fig. 2b, for the BFP [15] data on iron for $Q^2 > 10\ GeV^2$. The QCD fit here gives $\Lambda_{\overline{MS}}$ = 167 MeV and a χ^2/DOF = 14.2/4. The arrow indicates the upper error bar of the data point at x = 0.65($d\ln F_2/d\ln Q^2$ = -0.687 ± ± 0.410).

We have done the same nonsinglet analysis for the EMC [8] and BFP [15] iron data for $Q^2 > 10\ GeV^2$ (Figs. 2b and 2c). This comparison is incomplete because it does not include the systematic errors of the experimental slope parameters which we are unable to evaluate. However, it is clearly seen that the EMC data are in statistical disagreement with the QCD prediction. The value of Λ determined from these data depends therefore strongly on the x range included in the fit. Increasing further the minimum Q^2 to suppress possible higher twist effects increases the value of Λ but does not improve the agreement with the QCD predictions. For the BFP data, the agreement is better but the statistical errors are large in the region $x > 0.25$ where the nonsinglet approximation applies.

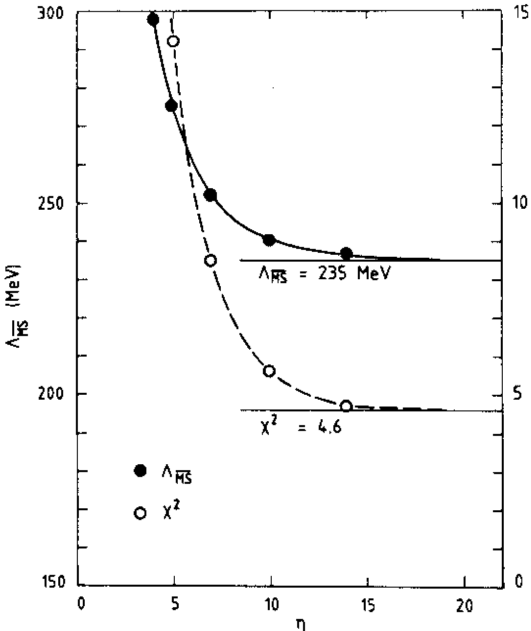

FIG. 3

Fig.3. Results for $\Lambda_{\overline{MS}}$ (closed points, left-hand scale) of flavour singlet fits with the method of ref./11/ to $F_2(x,Q^2)$ as a function of the power η of a gluon momentum distribution $xG(x) = {}^1/_2(\eta + 1)(1-x)^\eta$. The open points show the corresponding χ^2 (right-hand scale) for the comparison of measured and predicted scaling violations. The horizontal lines are the results of the nonsinglet fits discussed in the text.

In conclusion, we find that the scaling violations observed in our high statistics measurement of the nucleon structure function $F_2(x,Q^2)$ at $Q^2 > 25$ GeV^2 are in excellent agreement with predictions of perturbative QCD. In the range $0.275 \leqslant x \leqslant 0.75$ the data are well described by a flavour nonsinglet approximation and therefore favour a soft gluon distribution. From next to leading order fits, we find a strong coupling constant $\alpha_s = 0.160 \pm 0.003$(stat.) $\pm$ 0.010(syst.) at $Q^2 = 100$ GeV^2.

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Бенвенути А.С. и др. E1-87-699
Проверка КХД и определение Λ из нарушения скейлинга в нуклонной структурной функции $F_2(x,Q^2)$ при высоких Q^2

Проведено сравнение с предсказаниями пертурбативной КХД нарушения скейлинга в нуклонной структурной функции $F_2(x,Q^2)$, измеренной с высокой статистикой в глубоконеупругом рассеянии мюонов на углеродной мишени. Наблюдено прекрасное согласие данных во всей области измеренных переменных x и Q^2 с численными решениями эволюционных уравнений Альтарели–Паризи. В несинглетном приближении в следующем порядке к лидирующему определен масштабный параметр КХД $\Lambda_{\overline{MS}}$ = 230±20 /стат./ ± 60 /сист./ МэВ. Синглетный фит данных предпочитает мягкое глюонное распределение.

Препринт Объединенного института ядерных исследований. Дубна 1987

Benvenuti A.C. et al. E1-87-699
Test of QCD and a Measurement of Λ from Scaling Violations in the Nucleon Structure Function $F_2(x, Q^2)$ at High Q^2

Scaling violations in the nucleon structure function $F_2(x, Q^2)$ measured with high statistics in deep inelastic scattering of muons on a carbon target are compared to predictions of perturbative QCD. Excellent agreement is observed with numerical solutions of the Altarelli-Parisi evolution equations over the entire x and Q^2 range of the data. In a next-to-leading order nonsinglet approximation, the QCD mass scale parameter $\Lambda_{\overline{MS}}$ is determined to be 230±20 (stat.) ± 60 (syst.) MeV. A singlet fit to the data favours a soft gluon distribution.

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