UMN-TH-608/87
CERN-TH.4762/87
May 1987

On the Viability of Rank Six Superstring Models

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Abstract

We consider the possibility of breaking a rank six superstring model to the rank four standard model. In particular, we point out the difficulties in generating two vacuum expectation values for the two standard model singlets contained in the 27 of E_6. Although one expectation value is compatible with low energy phenomenology, a vev for ν^c is problematic because of the absence of large neutrino masses and/or flavor changing neutral currents. We show that even simple models containing extra fields from incomplete multiplets or E_6 singlets do not resolve these problems.

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I. Introduction

Recently, there has been much excitement about the possibility that some superstring theory is the fundamental theory of nature[1]. In particular, much activity has centered on the ten dimensional $E_8 \times E_8'$ heterotic superstring,[2] and its compactification to four dimensions[3]. Unfortunately, the extraction of four dimensional physics from the ten dimensional superstring is poorly understood at present. In principle, one constructs manifolds,[3), 4] or orbifolds,[5] satisfying the string equations of motion for the six compactified dimensions, and analyzes their physical implications. But even the simplest manifold compactifications correspond to deformations[6] of three complex-dimensional Ricci-flat Kahler manifolds (Calabi-Yau manifolds) of which very few are even implicitly known. In view of this, we restrict ourselves to consideration of the general features that realistic compactifications of this type must exhibit. In particular, we will discuss some general problems which would afflict wide classes of superstring models based on Calabi-Yau compactifications if these models possessed gauge groups of rank six.

In rank six superstring models there would of necessity be two steps of gauge symmetry breaking beyond that of the standard model. We will review the structure of this class of models, and the possibility of gauge symmetry breaking. Next we note some of the problems that arise if some of the gauge symmetry breaking occurs at an intermediate mass scale[7]. We then consider in detail the dynamical problem of generating the two extra scalar vevs[8] (vacuum expectation values) required for gauge symmetry breaking in these models. We will examine increasingly elaborate variants of these models, incorporating assumptions on the generation structure of superpotential couplings, the appearance of (extra) incomplete chiral multiplets,[4), 9)] and the appearance of gauge singlets,[10), 11)] in order to assess their problems and prospects.

The construction of the class of models we wish to consider, begins with the requirement that an $N = 1$ supersymmetry survives in the effective four dimensional theory[3)]. This aids solution of the ten dimensional equations of motion, and the residual supersymmetry may be useful in imposing the gauge hierarchy between the natural string and compactification scale ($O(M_c)$), and the scale of electroweak symmetry breaking ($O(M_W)$). Compactifications to $M_4 \times K$, with M_4 a maximally symmetric space-time and K a compact six dimensional internal space, which preserve four dimensional supersymmetry, requires K to have SU(3) holonomy. If the spin connection on K is torsion free, then K must be a perturbation of a Calabi-Yau space where $O(\alpha'^3)$ string corrections deform the metric away from Ricci-flatness[6)]. If, for the $E_8 \times E_8'$ heterotic string one now sets the observable sector (E_8) gauge connection equal to the spin connection, to satisfy the ten-dimensional equations of motion, then the observable sector gauge group will be explicitly broken to E_6. For non-simply connected K one may further break the E_6 by gauge potentials winding around the "holes" in K (the "Hosotani mechanism")[12), 5)]. The resulting non-zero Wilson line integrals, for homotopically non-trivial loops, act like symmetry breaking fields in the adjoint representation of E_6. The resulting low energy gauge group will have rank 6 if $\pi_1(K)$ is Abelian, but may instead have rank 5 for non-Abelian $\pi_1(K)$[4)]. It is possible, in principle, to obtain four dimensional gauge groups of rank 4 or 5 by compactifying on manifolds with differing spin and gauge connection,[10), 13)] by compactifying on orbifolds,[5)] or by the direct construction of four dimensional string theories[14] with augmented internal degrees of freedom. These alternatives will not be analyzed further here, although they are all worthy of independent consideration.

The observable sector chiral matter multiplets come from the zero modes of the E_6 gauge multiplet compactified on K[3]. There will in general be a number, N_g, of generations of the $\underline{27}$ dimensional representation of broken E_6, plus some number, δ, of paired subsets of the $\underline{27} + \overline{\underline{27}}$ representations, which form representations of the low energy gauge group. The precise representation content of the paired subsets depends on the details of the Wilson-line symmetry breaking[12] to the low energy group. The content of the $\underline{27}$ representation, (labeled by quantum numbers under $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)'$, with Y′ the linear combination of $Y_L + Y_R$, orthogonal to Y, in the $SU(3)_c \times SU(3)_L \times SU(3)_R$ decomposition of E_6) is:

$$Q\,(\underline{3}, \underline{2}, \tfrac{1}{3}, \tfrac{1}{3}),\ d^c(\bar{\underline{3}}, \underline{1}, \tfrac{2}{3}, -\tfrac{1}{6}),\ u^c(\bar{\underline{3}}, 1, -\tfrac{4}{3}, \tfrac{1}{3}),$$

$$L(\underline{1}, \underline{2}, -1, -\tfrac{1}{6}),\ e^c(\underline{1}, \underline{1}, 2, \tfrac{1}{3}),\ H(\underline{1}, \underline{2}, 1, -\tfrac{2}{3}),\ \bar{H}(\underline{1}, \underline{2}, -1, -\tfrac{1}{6}),$$

$$D(\underline{3}, \underline{1}, -\tfrac{2}{3}, -\tfrac{2}{3}),\ D^c(\bar{\underline{3}}, \underline{1}, \tfrac{2}{3}, -\tfrac{1}{6}),\ \nu^c(\underline{1}, \underline{1}, 0, \tfrac{5}{6}),\ N(\underline{1}, \underline{1}, 0, \tfrac{5}{6}).$$

In addition, there may arise in the compactification some gauge singlet chiral supermultiplets S_i, which are octets of the SU(3) holonomy[10]. Though present at the classical level as massless states they can, in general, acquire $O(M_c)$ mass due to world sheet instanton effects[15]. When present they can have gauge invariant superpotential couplings to themselves or to conjugate pieces of the $\underline{27} + \overline{\underline{27}}$ incomplete multiplets.

The effective four dimensional superpotential interactions of the chiral multiplets result from gauge interactions of the E_6 gauge supermultiplet zero modes on K. So all the trilinear terms in the superpotential must be such that they would appear in the E_6 invarient trilinear term $\underline{27}^3$. The allowed couplings between the various states, only bear the constraints arising from the low energy gauge group however, and not the full E_6 relations. The most general set of allowed trilinear superpotential couplings of our multiplets is thus:

$$W = \lambda_o HL\nu^c + \lambda_1 \bar{H}Le^c + \lambda_2 \bar{H}Qd^c + \lambda_3 HQu^c$$

$$+ \lambda_4 N\bar{H}H + \lambda_5 ND^cD + \lambda_6 DQQ + \lambda_7 D^cu^cd^c$$

$$+ \lambda_8 D^cQL + \lambda_9 Du^ce^c + \lambda_{10} Dd^c\nu^c \qquad (1)$$

where each coupling λ has three, generation, indices. These couplings are determined by the topology, complex structure and geometry of the manifold K[16]. Thus, in these theories, the spectrum of light states and their gauge and superpotential interactions are completely determined by the compactification manifold, K, and the E_6 vacuum gauge potentials on it.

II. Intermediate Scales

We will briefly review arguments against intermediate scales[7]. But, because it may be possible to carefully construct such models, we will comment on the implications of a large ν^c vev in the cases below.

In principle, an intermediate scale can arise[17), 18)] when, in addition to the cubic superpotential, one adds non-renormalizable interactions of the form

$$W \simeq \frac{1}{M_c}(\underline{27})^2(\overline{\underline{27}})^2 + \frac{1}{M_c^2}\left((\underline{27})^4(\overline{\underline{27}}) + (\overline{\underline{27}})^4(\underline{27})\right) + \ldots \quad (2)$$

If one includes only the quartic term in (2) a typical form for the scalar potential becomes

$$V = \tilde{m}^2\,\psi^2 + \frac{1}{M_c^2}\,\psi^6 \quad (3)$$

where ψ might be the scalar partner of ν^c and $\tilde{m}^2$ ($\lesssim 0(1\text{TeV})$) is due to soft supersymmetry breaking effects. We have neglected D-term contributions to the potential which vanish along directions such as $\langle\psi\rangle = \langle\bar{\psi}\rangle$. If $\tilde{m}^2$ becomes negative due to renormalization group evolution, a vacuum expectation value for ψ is generated with $\langle\psi\rangle \sim M_I = (\tilde{m}M_c)^{1/2} \sim 10^{10}$ GeV.

Although this procedure for generating an intermediate scale is easy to describe, it is much more difficult to achieve in practice[7)]. A major constraint on the value of M_I, comes from cosmological considerations of the dynamical generation of a vacuum value[19)]. Independent of the sign of the quadratic term in (3), its magnitude (being related to supersymmetry breaking) is constrained to be $\lesssim 0(1$ TeV), and hence thermal effects in the early Universe prevent the transition from occurring until $T \sim \tilde{m}$. When the difference in vacuum energy densities between $\langle\psi\rangle = 0$ and $\langle\psi\rangle = M_I$, $\Delta V \sim$

$\tilde{m}^2 M_I^2$, is released, the entropy generated in the transition may wash away any existing baryon asymmetry. To avoid this, one needs to bound $M_I \lesssim 10^7$ GeV.

This bound can be relaxed if the baryon asymmetry is produced at low temperatures[7]. Affleck and Dine[20] have proposed a mechanism using the relaxation of large vacuum values of squarks and sleptons when $T \sim \tilde{m}$. Their scenario takes place naturally in the context of superstring models[21]. In this case the bound on M_I becomes[7] $M_I \lesssim 10^{12}$ GeV. Other mechanisms for producing a baryon asymmetry by the late out-of-equilibrium decays of the field ψ have been proposed[22] but requires many specific ad hoc assumptions about couplings in order to both keep baryon violating interaction present and protons stable.

Proton decay itself typically requires the masses of particles mediating baryon number violating interactions to be $M_I \sim O(10^{16}$ GeV) although a discrete symmetry as simple as Z_2 can be imposed (found?) to eliminate all baryon number violating interactions[23), 24)]. Such symmetries are necessary in models without intermediate scales as well.

Another potential problem for intermediate scales with $M_I \lesssim O(10^{16}$ GeV) is that strong coupling effects begin to dominate if there are large numbers of additional fields beyond the 3 $\underline{27}$'s. The appearance of such fields is common to many models. If large vevs are generated, then higher order terms in the superpotential may lead to unacceptably large masses for our light fields. For example, a quintic such as $\frac{1}{M_c^2} HL\nu^c \nu^c \bar{\nu}^c$ could lead to an HL mixing mass $\sim \langle \nu^c \rangle^3 / M_c^2$. Such effects were typically found in the effective rank 5 model of ref. 25.

Finally, a last objection to intermediate scale models is the generation of a negative mass^2 in the potential (3) by running the renormalization group equations[7), 26)]. It has been shown that although not impossible, obtaining a negative mass^2 again requires strong assumptions on the couplings of the fields ψ. Based on these arguments we feel that intermediate scales do not offer a simple resolution to either problems concerning breaking rank six models or problems concerning neutrino masses.

III. Generation of $\langle\nu^c\rangle = y \neq 0$

Now let us turn to the dynamical problem of generating the vev's necessary to break a rank six model to the rank four standard model gauge group[8)]. This will require vevs for both an N scalar and a ν^c scalar. We review the difficulties in generating a ν^c scalar vev in the simplest superstring models[8)] and then show how the problems encountered persist in varients of these models. For a model with N_g generations, plus possibly some incomplete multiplets, there will be $N_g + \delta$ copies of the N and ν^c scalars (δ is, for each scalar, the number of them from split multiplets). We can however (and will) transform to a basis in which only one N field and one ν^c field receive a vev (we denote these vevs $\langle 0|N|0\rangle = x$ and $\langle 0|\nu^c|0\rangle = y$).

The superpotential couplings of the N and ν^c are:

$$W_N = \lambda_4 \, N \bar{H} H + \lambda_5 \, N D^c D \tag{4a}$$

$$W_{\nu^c} = \lambda_0 \, H L \nu^c + \lambda_{10} \, D d^c \nu^c \tag{4b}$$

The $N \bar{H} H$ and $N D^c D$ Yukawa couplings in (4a) can drive $m_N^2 < 0$ via the renormalization group equation. This reduces the rank of the gauge group by one; it also provides the $\bar{H}$ H mixing for the Higgs responsible for standard model gauge symmetry breaking, which requires λ_4 <N> to be $\lesssim$ O(TeV).

It is, on the other hand, difficult to arrange $<0|\nu^c|0> = y \neq 0$, which would be required for symmetry breaking in a rank six model.

First we note that there is no linear term in the Taylor expansion of effective potential for ν^c which could drive the ν^c vev. This is because ν^c appears only in $HL\nu^c$ and $Dd^c\nu^c$ couplings, and we expect $<0|L|0> = 0 = <0|D|0> = <0|d^c|0>$ to avoid lepton number violation or breaking of SU(3) colour. (Later, we will consider the possibility that the Higgs is actually a linear combination of $\bar{H}$ and L.) So to have $<0|\nu^c|0> = y \neq 0$ we must arrange $m^2_{\nu^c} < 0$. The difficulty in arranging $y \neq 0$ now arises from the problems in arranging a superpotential coupling which could act in the R.G.E. to drive $m^2_{\nu^c} < 0$.

We examine the difficulties with such superpotential terms in classes of models of increasing complexity. To start, let us recapitulate the arguments of reference (8) for the case where the matter multiplets arise in $\underline{27}$'s (including perhaps some bits from split multiplets with the corresponding $\overline{\underline{27}}$ bits appearing also), and we refrain from considering convenient, generation dependent zeros in the Yukawa couplings. There are two possible ν^c superpotential terms in Equation (4b). The $\lambda_0 H\, L\, \nu^c$ term gives the fermion bilinear λ_0 H $(L\nu^c)$, where ( ) denotes the Lorentz scalar fermion bilinear. With a Higgs vev $<0|H|0> = v = 0(100\text{ GeV})$ we get a Dirac neutrino mass $m_\nu = \lambda_0 v$. Since there is no ν^c bilinear coupling in the superpotential to serve as a source of Majorana mass for the right handed neutrino there is no see-saw mechanism operative, and the Dirac mass terms will be the physical neutrino masses. (We return to the question of the "see-saw" mechanism in the presence of singlets below.) Using the particle and astrophysical limits $m_\nu < 0$ (100 ev) we see that $\lambda_0 = m_\nu/v \lesssim 10^{-9}$; so the λ_0 coupling, if not entirely absent from the superpotential, is certainly too small to drive $m^2_{\nu^c} < 0$, to produce $\langle 0|\nu^c|0\rangle \neq 0$.

The $\lambda_{10} D d^c \nu^c$ superpotential coupling is in general phenomenologically acceptable, unless it appears in conjunction with $\lambda_6 DQQ$ or $\lambda_7 D^c u^c d^c$ which would induce proton decay. With this coupling, a ν^c vev would produce mass mixing between the d and D quarks $\lambda_{10} y(Dd^c)$. Suppressing generation indices, and omitting the possibility of a (lepton number violating) ν vev, the charge -1/3 quark mass matrix would become[8), 9)]:

$$(d^c,\ D^c) \begin{pmatrix} \lambda_2 \bar{v} & \lambda_{10} y \\ 0 & \lambda_5 x \end{pmatrix} \begin{pmatrix} d \\ D \end{pmatrix} \qquad + \text{h.c.} \tag{5}$$

which we diagonalize by a bi-unitary transformations

$$\begin{pmatrix} m_< & 0 \\ 0 & m_> \end{pmatrix} = \begin{pmatrix} C_R & S_R \\ -S_R & C_R \end{pmatrix} \begin{pmatrix} \lambda_2 \bar{v} & \lambda_{10} y \\ 0 & \lambda_5 x \end{pmatrix} \begin{pmatrix} C_L & S_L \\ -S_L & C_L \end{pmatrix} \tag{6}$$

where $C_{L,R} = \cos(\theta_{L,R})$; $S_{L,R} = \sin(\theta_{L,R})$.
The mass eigenvalues are[8, 9)]

$$m_< = C_R\,(\lambda_2 \bar{v}\, C_L - \lambda_{10}\, y\, S_L) - S_R\, \lambda_5\, x\, S_L \tag{7a}$$

$$m_{>} = -S_R\ (\lambda_L \bar{v}\ S_L + \lambda_{10}\ y\ C_L) + C_R\ \lambda_5\ x\ C_L \tag{7b}$$

and the mixing angles are:

$$\tan\ \theta_L \simeq y\ \lambda_2 \lambda_{10} \bar{v}/(\lambda_5^2 x^2 + \lambda_{10}^2 y^2) \tag{8a}$$

$$\tan\ \theta_R \simeq -\lambda_{10} y/\lambda_5 x \tag{8b}$$

where we assume $\lambda_2 \bar{v} << (\lambda_5 x^2 + \lambda_{10} y^2)^{1/2}$ as required by phenomenology ($m_Z{}'$ $>> m_Z$ etc.). Then the approximate mass eigenvalues are[8, 9]

$$m_{d_{phys}} = m_{>} \simeq \lambda_2 \bar{v} \lambda_5 x/(\lambda_5^2 x^2 + \lambda_{10}^2 y^2)^{1/2} \tag{9a}$$

$$m_{D_{phys}} = m_{<} \simeq (\lambda_5^2 x^2 + \lambda_{10}^2 y^2)^{1/2} \tag{9b}$$

where we identify the known charge -1/3 quarks with the light eigenvalue(s), so $m_{<} \sim O(1\ \text{GeV})$. Now the same $\bar{H}$ vev $<0|\bar{H}|0> = \bar{v}$ which supplies charge -1/3 quark masses also gives leptons mass $\lambda_1 \bar{v}$ which is again of order 1 GeV. If the superpotential couplings of leptons and charge -1/3 quarks to $\bar{H}$ are comparable at the compactification scale, then after renormalization group evolution to low energies they should agree to within a factor of order three. Certainly we expect $\lambda_2 \bar{v} \lesssim 10(\lambda_1 \bar{v})$. Then, since physical charge -1/3 quarks are at least as heavy as leptons,

$$m_{d_{phys}} \simeq \frac{\lambda_2 \bar{v}\ \lambda_5 x}{(\lambda_5^2 x^2 + \lambda_{10}^2 y^2)^{1/2}} \gtrsim \lambda_1 \bar{v} = m_L \tag{10}$$

we have that $\lambda_{10}\, y \lesssim 10(\lambda_5\, x)$ (i.e., for comparable superpotential couplings $y \lesssim 10\, x \simeq O(\text{few TeV})$).

In the absence of generation dependent zeros imposed on the superpotential couplings, the $D \leftrightarrow d$ mixing will occur among all generations of light charge -1/3 quarks. Since D_L and d_L have different weak isospin and hypercharge this induces flavour changing neutral current couplings of the Z^0 boson of the form $\sqrt{g_2^2 + g'^2}\ \theta_L^2\ \bar{s}_L\ \gamma^\mu d_L Z_\mu$. Similarly for $x \lesssim 1$ TeV there will be a Z' boson in the few hundred GeV mass range whose couplings to D_R and d_R differ, so that D_R/d_R mixing will induce flavour changing couplings such as $g' \sin^2\theta_m\ \bar{s}_R\ \gamma^\mu d_R\ Z'_\mu$, where θ_m is the smaller of θ_R and $\bar{\theta}_R$ ($\sin\bar{\theta}_R = |\sin(\pi/2 - \theta_R)|$). An analysis[24] of experimental limits on flavour changing neutral processes so induced indicates that $\theta_L < 10^{-2}$; $\theta_m < 2 \times 10^{-2}$. But for small $\bar{\theta}_R \simeq \lambda_5\, x/(\lambda_{10}\, y) < 2 \times 10^{-2}$ we have $\lambda_{10} y > 50\ \lambda_5 x$, which is inconsistent with our constraint from charge -1/3 quark masses $\lambda_{10} y \lesssim 10(\lambda_5 x)$. If on the other hand we have small $\theta_R < 2 \times 10^{-2}$ then $\lambda_{10}\, y < (1/50)\ \lambda_5 x$, and for $x \le 1$ TeV, $\lambda_5 x = O(\text{few} \times 10^2\ \text{GeV})$, and not too light a Z" boson mass ($y \gg 200$ GeV), we have too small a λ_{10} to drive $m^2_{\nu^c} < 0$ through R.G.E. evolution.

The preceding argument precluded a ν^c vev by bounding it (or more correctly $\lambda_{10} < \nu^c >$) from below by limits on flavour changing neutral currents, and from above by the requirement that it not "see-saw" the charge -1/3 quarks to too light a mass. One might inquire whether it is possible to get by the latter bound simply by increasing the direct $\lambda_2\ \bar{H}Qd^c$ coupling responsible for the Dirac d mass, so maintaining the charge -1/3 quark masses even in the presence of the "see-saw". To do this however, would exacerbate an incipient problem with flavour changing neutral currents which appears in any Calabi-Yau based superstring model without convenient generation dependence. Since the 27 matter multiplets in these models include a pair of Higgs doublets H_i (and $\bar{H}_i$) for each generation ($i = 1, \ldots, N_g$) then, as well as the linear combination of the H_i and ($\bar{H}_i$) which receive vevs and give mass to the quarks and leptons, there are N_g-1 orthogonal combinations of H_i (and $\bar{H}_i$) whose couplings are not in general flavour diagonal. Analysis of the limits on flavour changing interactions which would be induced by exchange of these extra Higgs indicates that (for typical Higgs masses 0(100 GeV)) those extra Higgs must have flavour off-diagonal couplings[24] $\lesssim 3 \times 10^{-5}$, and imaginary parts of their flavour off-diagonal couplings $\lesssim 10^{-6}$. This is already uncomfortably smaller than the (flavour diagonal) coupling of the Higgs whose vev is responsible for charge -1/3 quark masses, which must range up to $\gtrsim 10^{-2}$. To increase the coupling of the mass generating H (and $\bar{H}$) would only worsen the fine tuning required. Worse yet, since the $\lambda_{10}\, Dd^c\nu^c$ couplings are in general not diagonal in the bases of charge -1/3 fermions in which the $N\, D^cD$ and $\bar{H}\, Qd^c$ couplings are diagonal, incorporating the λ_{10} coupling with the full generation structure in the mass matrix will mean the coupling of the mass generating $\bar{H}$ Higgs will no longer be diagonal in the basis of charge -1/3 quark mass eigenstates, further complicating the flavour violation problem. In summary, to generate a ν^c vev from the $Dd^c\, \nu^c$ superpotential term would require strong, generation structure in both the $\lambda_2\bar{H}Qd^c$ and $\lambda_{10}\, Dd^c\nu^c$ couplings to avoid problems with charge -1/3 quark masses, or flavour changing neutral currents.

Another possibility that may have been overlooked is the possibility of $\bar{H}$ - L mixing[9]. These fields have the same quantum numbers under the low energy group and perhaps if instead of quark and lepton masses arising from vevs of $\bar{H}$, it was some linear combination of $\bar{H}$ and L which picks up a vev. In general we can define a linear combination $\bar{H} = a\bar{\mathcal{H}} + b\mathcal{L}$ and $L = c\bar{\mathcal{H}} + d\mathcal{L}$ with the constraints $a^2 + b^2 = c^2 + d^2 = 1$, where $\bar{H}$ and L are defined such that $\langle\mathcal{L}\rangle = 0$ and $\langle\bar{\mathcal{H}}\rangle = \bar{v}$. Rewriting the superpotential (1) in terms of $\bar{\mathcal{H}}$ and $\mathcal{L}$, we can obtain a number of constraints on possible values of a, b, c, d and the couplings . First of all, the coefficient for $H\bar{\mathcal{H}}$ mixing is now $\lambda_0 cy + \lambda_4\, ax \lesssim 1$ TeV. Small Dirac neutrino masses require $\lambda_0 d \lesssim O(10^{-9})$ and $\lambda_4 b \lesssim O(10^{-9})$. The absence of (lepton number violating) $H\mathcal{L}$ mixing requires $\lambda_4 bx + \lambda_0 dy \lesssim 1$ GeV and the absence of explicit lepton number violating interactions from $\mathcal{L}\mathcal{L}e^c$ requires $\lambda_1 bd < O(10^{-2})$. It is straightforward to check that in order to satisfy all of these constraints either $a \sim 1$, $b \sim 0$, $c \sim 0$, $d \sim 1$ which is the original case we considered or $a \sim 0$, $b \sim 1$, $c \sim 1$ and $c \sim 0$ which corresponds to a rotation under* $SU(2)_N$ of $d^c \leftrightarrow D^c$, $\nu^c \leftrightarrow N$ and $\bar{H} \leftrightarrow L$. But all this amounts to is a redefinition of the names of our matter fields. Thus without explicitly violating lepton number conservation we still cannot provide a large Yukawa coupling to ν^c so as to generate $y \neq 0$.

IV. Incomplete Multiplets

Having seen that intrinsic generation structure in the Yukawa couplings must be an essential feature of attempts to use them to drive $m^2_{\nu^c} < 0$ to

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*$SU(2)_N$ is the SU(2) subgroup of $SU(3)_R$ which commutes with the electric charge operator.

generate a ν^c vev, we now examine some circumstances under which this might plausibly occur. The fields appearing in the candidate couplings include ν^c (in $HL\nu^c$) and d_R (in $Dd^c\nu^c$), in the respective cases of interest. Since these fields appear in the standard model in three generations, with the expected standard model properties, their use in superpotential couplings in a generation dependent way seems problematic. An obvious possibility in this regard would be to obtain the requisite fields for the superpotential term used to drive $m^2_{\nu^c} < 0$ from members of "incomplete multiplet" pieces[4)]of the $\underline{27} + \overline{\underline{27}}$, plus possibly gauge singlets. In this section we will discuss the case of extra fields from the $\underline{27} + \overline{\underline{27}}$ only; in the subsequent section we consider the extra possibilities which arise if E_6 gauge singlets also survive as light fields.

The incomplete multiplets may arise when $b_{1,1} \geq 1$, in which case there will be vector like pairs of some components of the $\underline{27} + \overline{\underline{27}}$. Precisely which components survive depends on the details of the flux breaking mechanism. If one is considering a Calabi-Yau manifold K/G, where G is a discrete isometry group of a simply connected manifold K (thus $\pi_1(K/G) = G$) then the flux breaking occurs when the Wilson lines around non-contractible loops on K/G take non trivial values in the gauge group (i.e. Wilson line operators map $G \to \bar{G} \subset E_6$). Then the components of the $\underline{27} + \overline{\underline{27}}$ incomplete multiplets which survive as massless states are those invariant under the combined action[4)] of $G + \bar{G}$.

For the case when $b_{1,1} = 1$ the $\overline{\underline{27}}$ states are invariant[4)] under G and hence remain light if they are $\bar{G}$ invariant. For a given $\bar{G}$ then both the flux broken gauge group (that subgroup of E_6 whose elements commute with $\bar{G}$)

and the light components of the incomplete multiplet are determined. The cases of gauge symmetry breaking of E_6 by Wilson lines, which leave light doublets when $b_{1,1} = 1$ have been enumerated[9), 27)]. If we demand that the gauge group after flux breaking not mediate proton decay through its vector interactions (since our cosmological arguments indicate further gauge symmetry breaking by fields in the incomplete multiplets cannot take place at a large enough scale to break the baryon and lepton number violating gauge interactions sufficiently to ensure proton stability) this leaves three cases. However all of them will exhibit an $SU(2)_R$ gauge group even after breaking by the ν^c vev. When renormalized down to O(1TeV), where it is broken by the N vev, the T^3_R piece of $SU(2)_R$ will appear in the weak hypercharge and its coupling strength will result in much too large a value of $\sin^2\theta_w$. So in the $b_{1,1} = 1$ case one does not obtain viable models with sets of $\{Dd^c\nu^c\}$ or $\{H\ L\ \nu^c\}$ appearing in incomplete multiplets.

For the case where $b_{1,1} > 1$ it is no longer necessarily the case that the $\underline{27} + \overline{\underline{27}}$ multiplets are singlets under G, so invariance under $G + \bar{G}$ as required to keep them light no longer implies $\bar{G}$ invariance. If the $\underline{27} + \overline{\underline{27}}$ are G invariant (which is always true for at least on $\underline{27} + \overline{\underline{27}}$ pair in the models) then we are back to the cases considered, except that now the several generations of ν^c and N multiplets from the incomplete sectors could have vevs which cannot all align, allowing the gauge group to be broken directly to the standard model. However, if we wish to use this to avoid the problems with $SU(2)_R$ noted earlier, then we need to arrange the gauge breaking at an intermediate scale. This leads to the cosmological problems noted earlier, and also often causes particle physics problems associated with Higgs masses and mixings, baryon number and lepton number conservation, and particle mass generation.

In the case where $b_{1,1} > 1$ and the $\underline{27} + \overline{\underline{27}}$ multiplets are not singlets under G, it is not a priori possible to say what components will survive for a given pattern of gauge symmetry breaking. These models then would have to be considered on a case by case basis, although they certainly would have to obey the arguments we have advanced concerning intermediate scales, as well as the requirement of producing correct particle physics phenomenology.

V. Singlets and Neutrino Masses

Another possibility[10)] for generating a ν^c vev involves the introduction of E_6 singlets, S, (octets of SU(3) holonomy) which couple to $\underline{27}\ \overline{\underline{27}}$

$$W_s = S(\underline{27})\ (\overline{\underline{27}}) \tag{11}$$

This coupling has also been studied because of its possibilities for generating a see-saw mechanism in order to obtain low mass neutrinos and thereby escape from the constraints on the cubic superpotential coupling λ_0. Indeed, if a see-saw can be accomplished, and instead of $\lambda_0 < 10^{-9}$ we have $\lambda_0 \sim 1$, our arguments based on the constraints on the $Dd^c\nu^c$ coupling do not exclude the possibility of generating a non-zero value for y. Equation (11) offers two possibilities: either 1) ν^c and $\bar{\nu}^c$ obtain a vev and drive a see-saw through a $S\nu^c\ \bar{\nu}^c$ coupling or 2) a vev for the singlet S is generated. Clearly both of these possibilities require using fields from incomplete multiples with $b_{1,1} \geq 1$. Acceptable embeddings of the discrete group G into K allow for this possibility (a single ν^c and $\bar{\nu}^c$ pair) with a minimal rank 6 gauge group $SU(3)_c \times SU(2)_L \times U(1)^3$.

The standard see-saw mechanism[28)] involves two neutrino states which we can call ν_L and η_L (η_L is equivalent to our ν^c). If Dirac masses are available for both ν_L and η_L and in addition a Majorana mass term is present for η_L we get a neutrino mass matrix of the form

$$(\nu_L \quad \eta_L)^* \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \begin{pmatrix} \nu_L \\ \eta_L \end{pmatrix} \tag{12}$$

where m is the Dirac mass and M is the Majorana mass. If $M \gg m$, the two mass eigenvalues are M, m^2/M and with a typical mass $m \sim O(1)$ GeV the light eigenvalue can be driven small enough if $M \gtrsim 10^9$ GeV.

In the superstring case[10, 11)], we can consider first the possibility that a pair ν_I^c and $\bar{\nu}_I^c$ obtain a vaccum expectation value, of order M and have a coupling $\eta_L \bar{\nu}_I^c S$. Our notation is such that ν_I^c and $\bar{\nu}_I^c$ come from an incomplete multiplet and η_L are the remaining ν^c's (the three linear combinations which do not receive a vev). We can then consider 3 x 3 neutrino mass matrix (actually 7 x 7, it is understood that there are three copies of ν_L and η_L)

$$(\nu_L \; \eta_L \; S)^* \begin{pmatrix} 0 & m & 0 \\ m & 0 & M \\ 0 & M & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ \eta_L \\ S \end{pmatrix} \tag{13}$$

where again m corresponds to the mass term arising from $HL\eta_L$. The mass eigenvalues in this case are easily found yielding one massless eigenstate and two states with mass $(m^2 + M^2)^{1/2} \sim M$. In writing down (13) we have neglected the coupling $\nu_I^c \bar{\nu}_I^c S$. If present this would lead to a 5 x 5 mass matrix of the form.

$$(\nu_L \; \eta_L \; \nu_I^c \; \bar{\nu}_I^c \; S)^* \begin{pmatrix} 0 & m & m' & 0 & 0 \\ m & 0 & 0 & 0 & M' \\ m' & 0 & 0 & 0 & M \\ 0 & 0 & 0 & 0 & M \\ 0 & M' & M & M & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ \eta_L \\ \nu_I^c \\ \bar{\nu}_I^c \\ S \end{pmatrix} \tag{14}$$

where we have allowed for the possibility that the Dirac masses m due to $HL\eta_L$ and m' due to $HL\nu_I$ are different and the M due to $\eta_L \bar{\nu}_I^c S$ and M' due to $\nu_I^c \bar{\nu}_I^c S$ are also different. This matrix has eigenvalues 0, m, m', M, M'. Although there is a zero mass eigenvalue, one can check that it does not involve ν_L but rather η_L, ν_I^c and $\bar{\nu}_I^c$ and thus does not solve our problem.

A more serious difficulty for this mechanism is the presence of a Majorana mass term for the singlet S. As we mentioned earlier, world sheet instanton effects generate masses $\mu \sim M_c$ for these singlets so that the appropriate mass matrix becomes (when the $\nu_I^c \; \bar{\nu}_I^c$ S term is neglected)

$$(\nu_L \; \eta_L \; S)^* \begin{pmatrix} 0 & m & 0 \\ m & 0 & M \\ 0 & M & \mu \end{pmatrix} \begin{pmatrix} \nu_L \\ \eta_L \\ S \end{pmatrix} \tag{15}$$

The mass eigenvalues in this case become μ, M^2/μ and $\mu m^2/M^2$ when $m << M < \mu$. (Note for smaller M, the eigenvalues become μ, m and m). In this case a see-saw is possible if $M \gtrsim 10^{14}$ GeV corresponding to a large intermediate scale for which the cosmological problems previously discussed would be present. One can verify that including both a non-zero mass term for the singlet and the $\nu_I^c\ \bar{\nu}_I^c$ S coupling does not help in producing an effective see-saw mechanism. There is again the same zero mass eigenvalue and the remaining eigenvalues become, $\mu, M^2/\mu, m, m'$ and again unfortunately the M^2/μ eigenvalue does not involve ν_L. Thus the presence of the singlet mass term upsets the possibility of a see saw mechanism as a means for achieving a non-zero value for $\langle\nu^c\rangle$.

Our second alternative was to give the singlet S a vacuum expectation value, because $M_S \sim \mu \sim M_c$ we would expect $M_S \sim \langle S\rangle \sim M_c$ as well. In general we obtain the following mass matrix

$$(\nu_L\ \eta_L\ \nu_I^c\ \bar{\nu}_I^c\ S)^* \begin{pmatrix} 0 & m & m' & 0 & 0 \\ m & 0 & 0 & \langle S\rangle & M' \\ m' & 0 & 0 & \langle S'\rangle & M \\ 0 & \langle S\rangle & \langle S'\rangle & 0 & M \\ 0 & M' & M & M & \mu \end{pmatrix} \begin{pmatrix} \nu_L \\ \eta_L \\ \nu_I^c \\ \bar{\nu}_I^c \\ S \end{pmatrix} \qquad (16)$$

If, for the moment, we restrict our attention to the case when $M = M' = 0$, we find two mass eigenstates $\sim \langle S\rangle, \langle S'\rangle$ one mass eigenstate μ, and two mass eigenstates m, m'. We also find that, if $\mu = 0$, it would be possible to have two massless eigenstates if we are prepared to arrange a cancellation so that $m, m' \sim 1$ GeV while $m - m' \lesssim 10^{-8}$ GeV. (Note that if one is contented with such a cancellation, the restriction on λ_0 is again lifted allowing for the possibility for generating a ν^c (η_L) vev.)

In the most general case when μ, <S>, <S$'$>, M, M$'$, m, m$'$ all are non-zero and we assume $\mu \sim \langle S\rangle \sim \langle S'\rangle \sim M_c$ the five eigenvalues are, $m^2\mu/M^2$, M^2/μ, μ, <S>, and <S$'$>. The first eigenvalue is sufficiently small if $M \sim 10^{14}$ GeV and, as before, the cosmological problems follow. The second eigenvalue can be made small if $M \lesssim 10^5$ GeV but again this eigenstate does not involve ν_L. We conclude therefore that a see-saw mechanism cannot simply supply a low mass ν_L so as to allow λ_0 large enough to generate a non-zero value for ν^c.

Summary and Conclusions

We have considered general features of rank 6 models that may arise from the compactification of the heterotic superstring. These models contain two neutral scalars in each 27 generation of chiral supermultiplets. We have examined the implications of a vev for one of these neutral scalar fields, the conjugate neutrino ν^c, since such a vev is required for symmetry breaking of the rank 6 gauge group. After reviewing cosmological and phenamenological arguments against having such symmetry breaking occur at an intermediate scale we examined the dynamical problem of generating a ν^c vev. In order for renormalization group evolution to drive $m^2_{\nu^c} < 0$, one of the gauge invariant ν^c superpotential couplings ($HL\nu^c$ on $Dd^c\nu^c$) must be substantial. The $HL\nu^c$ coupling produces a Dirac mass for the weak doublett neutrino and hence must be small in the absence of a see-saw mechanism. The $Dd^c\nu^c$ coupling produces flavor changing interactions and D-d mixing when

$\langle\nu^c\rangle = y \neq 0$. Limits on these effects rule out a ν^c vev in models without generation dependent suppression of couplings. We have shown that these conclusions are not altered by possible H - L mixing. Nor have we found that one can obtain combinations of incomplete multiplets to use in these superpotential terms when $b_{1,1} = 1$. We finally considered the possibility that gauge singlets combined with $(\nu^c + \bar{\nu}^c)$'s from incomplete multiplets could provide a neutrino mass see-saw and allow the use of the $HL\nu^c$ term. We found that in the general case of mass mixing this does not solve the neutrino mass problem (we were sometimes able to obtain light non-zero mass states; however, these were not primarily the weak doublet neutrino).

Acknowledgements

We would like to thank John Ellis for a critical reading of this manuscript. One of us (BAC) would like to thank the University of Minnesota and the CERN Theory Division for their kind hospitality during the completion of this work. His research was supported by NSERC Canada under grant A8863. The work of KAO and DBR was supported in part by DOE grant DE-AC02-83ER-40105.

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