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ON THE POSSIBILITY OF AVOIDING SINGULARITIES BY DILATON EMISSION

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ABSTRACT

We analyze the classical equations of supergravity theories containing a dilaton field, investigating the possibility that dilaton emission may prevent the formation of singularities. An initial cosmological singularity can be avoided in a no-scale supergravity theory if there is a non-zero charge density associated with the R symmetry current. However, this is only possible if some fields have negative metric initially, which may indicate a breakdown of the classical equations. A similar situation seems to occur with the Schwarzschild singularity.

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Classical general relativity predicts singularities in both cosmological and static configurations, and their existence disquiets both cosmologists and physicists. Much effort has been devoted to the search for cosmologies without initial singularities, or which explain the present flatness and homogeneity of the Universe on scales larger than the apparent particle horizon. Static singularities are generally surrounded by event horizons, but these bring new problems at the quantum level. The loss of information across an event horizon may cause a pure quantum state to evolve into a mixed state[1]. If this occurs on a microscopic level, it would signal a conflict between our current formulations of gravity and quantum mechanics.

We believe for other reasons, notably to embed it in a renormalizable or even finite theory, that Einsteinian relativity must be modified at distances of order the Planck length. Such a modification would not alter the physics of macroscopic event horizons, such as those surrounding stellar mass black holes, but could modify or remove microscopic event horizons and singularities. The appropriate framework for discussing any such modifications in four dimensions is presumably N = 1 supergravity. The particular class of supergravity theories which is most likely to be relevant is that of no-scale supergravity models[2]. These yield effective potentials which are positive semidefinite[*] with at least one flat direction corresponding to a dilaton, and seem to emerge naturally from compactification of the superstring[3].

Some form of superstring is presumably the Theory of Everything (T.O.E.), and many physicists hope and expect that it is finite, has no singularities, and reconciles gravity with quantum mechanics. The precise mechanism by which singularities are avoided is a matter of debate. It may involve the opening up of extra dimensions, or the intrinsic non-locality of particles in string theory, or higher-order modifications to Einstein's equations, or the emission of dilatons. The first two effects show up together at the Planck scale, and their investigation requires a fuller understanding of string dynamics which goes beyond the scope of this article. The last two effects can in principle be studied using an effective low-energy four-dimensional field theory inspired by the superstring. The specific subject of this article is the possibility[4] that dilaton emission might avert cosmological or Schwarzschild singularities. We study this question in the framework of no-scale supergravity, both when it is coupled generally to

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*) Generic supergravity models yield effective potentials with "holes" of depth $-O(1)m_P^4$.

conventional matter and with the special additional features suggested by the superstring[3].

We first investigate the classical cosmological solutions of no-scale supergravity, and find that the initial cosmological singularity can be avoided if there is a non-zero charge density associated with the R symmetry current. However, this is only possible if either the graviton or the dilaton acquires a negative metric in some region of space-time. We give a general argument that a static, singularity-free initial condition is impossible even in the presence of matter if all the fields have positive metric. Then we make a similar study of the possible avoidance of the Schwarzschild singularity, which exhibits some similar features, although we are not able to reach such a definite conclusion. Finally, in the light of these negative results, we comment on the possibility that superstrings avoid singularities. We think that the most likely mechanism for this is the non-locality of the superstring.

The bosonic part of the general N = 1 supergravity Lagrangian with a dilaton field can take the following form[2]:

$$\mathcal{L} = -\frac{1}{2}R + |D_\mu \phi|^2 + \frac{1}{6}R|\phi|^2 - 3A_\mu^2 + \mathcal{L}_M \qquad (1)$$

where R is the curvature scalar, $\mathcal{L}_M$ the matter Lagrangian and $D_\mu\phi \equiv (\partial_\mu - iA_\mu)\phi$ with A_μ the auxiliary supergravity spin-1 field corresponding to the R symmetry current, which is given from the equations of motion by:

$$A_\mu = \frac{i}{2}\,\frac{\phi^+ \overleftrightarrow{\partial_\mu} \phi}{3-\phi^+\phi} + \cdots \qquad (2)$$

where the dots stand for the contribution of $\mathcal{L}_M$. Here ϕ is the complex scalar field of the dilaton multiplet and the physical dilaton ϕ_D is given by

$$\phi_D = -\sqrt{\frac{3}{2}}\;\ln\left[\frac{3-\phi^+\phi}{(\sqrt{3}-\phi)(\sqrt{3}-\phi^+)} + \cdots\right] \qquad (3)$$

For pedagogical reasons we will first consider the simplest SU(1,1)/U(1) case and then we will discuss the more general case. Writing $\phi = \Delta e^{-i\theta}$, the Lagrangian (1) becomes

$$\mathcal{L} = -\frac{1}{2}\left(1-\frac{\Delta^2}{3}\right)R + (\partial\Delta)^2 + \Delta^2(\partial_\mu\theta + A_\mu)^2 - 3A_\mu^2 + \mathcal{L}_M \qquad (4)$$

and leads to the following equations of motion:

$$\Delta^2 \left(A_\mu + \partial_\mu \theta\right) - 3 A_\mu = 0 \tag{5a}$$

$$A_{\mu;}{}^{\mu} = \frac{1}{6} \frac{\delta \mathcal{L}_M}{\delta \theta} \tag{5b}$$

$$\Box \Delta - \frac{\Delta}{6} \mathbb{R} + \frac{9}{\Delta^3} A_\mu^2 = -\frac{1}{2} \frac{\delta \mathcal{L}_M}{\delta \Delta} \tag{5c}$$

$$\frac{1}{2}\left(1 - \frac{\Delta^2}{3}\right)\left(\mathbb{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathbb{R}\right) - \left(\delta^\alpha_\mu \delta^\beta_\nu - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta}\right)\left[\partial_\alpha \Delta\, \partial_\beta \Delta - 3\left(1 - \frac{3}{\Delta^2}\right) A_\alpha A_\beta\right]$$
$$- \frac{1}{6}\left[g_{\mu\nu} \Box \Delta^2 - (\Delta^2)_{;\mu\nu}\right] = \frac{1}{2} T^M_{\mu\nu} \tag{5d}$$

where Eq. (5a) corresponds to the variation with respect to A_μ, (5b) to Θ, (5c) to Δ, (5d) to $g_{\mu\nu}$, and (5a) has been used for the derivation of (5b-d). The semicolons stand for covariant differentiation and $\Box$ for the covariant box. The divergence of (Sd) vanishes as a result of Th$^{M\mu\nu}$; $\nu = 0$ and the other field equations. Taking the trace of (5d) and combining it with (5c), one finds

$$\mathbb{R} + 6 A_\mu^2 = \Delta \frac{\delta \mathcal{L}_M}{\delta \Delta} + T^{M}{}_{\mu}{}^{\mu} \; . \tag{5e}$$

Equation (5b) expresses the conservation of the R symmetry current which implies that $\delta \mathcal{L}_M / \delta\theta = 0$. On the other hand, if Δ is a dilaton field one has $\Delta(\delta \mathcal{L} / \delta\Delta) = -T^\mu_\mu$ and thus Eq. (5e) gives the vanishing of (the bosonic part of) the supercurvature.

Cosmological singularity avoidance?

We first examine cosmological solutions of Eqs. (5) in the presence of a non-trivial gravitational background which has the Robertson-Walker line element

$$ds^2 = dt^2 - R^2(t)\, d\vec{x}^{\,2} \tag{6}$$

parametrized by the scale factor R(t). Setting $A_a = 0 (a=1,2,3)$, the R current conservation [Eq. (5e)] then gives

$$A_0 = \frac{c}{R^3} \tag{7}$$

where c is a constant which represents the total U(1) R charge of the Universe. Substituting Eq. (7) into Eq. (5e) (vanishing of the supercurvature) one finds:

$$\dot{H} + 2H^2 = A_0^2 = \frac{c^2}{R^6} \tag{8}$$

where $H = \dot{R}/R$ is the Hubble parameter. The general solution to Eq. (8) is

$$H^2 = \frac{d^2}{R^4} - \frac{c^2}{R^6} \tag{9}$$

which gives a non-zero minimum value for the scale factor, $R > R_{min}$ with $R_{min} = |c/d|$ where d is an integration constant. So, for a non-vanishing total R charge ($c = 0$) the Universe is non-singular for all times. Notice that for large times ($R \to \infty$) the extra term becomes irrelevant and one recovers the known result. At R_{min}, $\dot{R} = 0$ and $\ddot{R}$ is positive; in fact, $\dot{H} = RHH' = \frac{1}{2}R(H^2)'\,|_{R=R_{min}} > 0$ where the prime denotes derivatives with respect to R. The positivity of $\ddot{R}$ corresponds to a violation of the energy condition $\mathbb{R}_{oo} > 0$ (since $\mathbb{R}_{oo} = -3(\ddot{R}/R)$ which is a necessary, but in general not sufficient, condition for singularity avoidance[5]. In our case it is satisfied at initial times which correspond to $R < \sqrt{3/2}\, R_{min}$.

Although the situation seems to be promising up to now, the trouble comes from the two other independent equations which can be obtained by combining (5c) and the "oo" component of (5d). One obtains

$$3H^2 = (\dot{\Delta} + H\Delta)^2 + 3\left(\frac{3}{\Delta^2} - 1\right)A_0^2 + \rho \tag{10a}$$

$$\dot{\rho} + 4H\rho = \left(\frac{\dot{\Delta}}{\Delta} + H\right)(\rho - 3p) \tag{10b}$$

where we have parametrized the matter energy-momentum tensor as usual

$$T^M_{\mu\nu} = -p\,g_{\mu\nu} + (\rho + p)u_\mu u_\nu \quad ; \quad u_0 = 1 \;,\; u_i = 0 \tag{11}$$

in terms of the energy density ρ and pressure p. When $R = R_{min}$, Eq. (10a) becomes

$$0 = \dot{\Delta}^2 + 3\left(\frac{3}{\Delta^2} - 1\right)A_0^2 + \rho \tag{12}$$

which can be satisfied only if $\Delta^2 > 3$. But $\Delta^2 > 3$ is a forbidden region because it corresponds to a change of the sign of the Einstein-Hilbert action, gives to the graviton a negative metric [see Eq. (4)]. Note that in the "unitary" representation, i.e., when we normalize the coefficient of $\not{R}$ to one by a field redefinition of the metric, the point $\Delta^2 = 3$ corresponds to the initial singularity problem. The only possible way to avoid this problem would be if initially at $R = R_{min}$: $\Delta^2 = 3$, $\dot{\Delta} = 0$ and $p = 0$, a situation which corresponds to the conformally-invariant phase of the theory.

To see if this is a possible set of initial conditions, we have to examine Eq. (10b). Consider first the case $p = 3p$. From (10b) one finds

$$\rho = \frac{c_\rho}{R^4} \tag{13a}$$

where c_ρ is an integration constant. Eq. (13) shows that (12) cannot be satisfied unless $\Delta^2 > 3$. In the general case, $\rho \neq 3p$, the solution of (10b) becomes

$$\ln R\Delta = \int_{R_{min}}^{R} \frac{d(\rho R^4)}{(\rho - 3p)R^4} + \ln\left(R_{min}\sqrt{3}\right) \tag{13b}$$

where the integration constant has been chosen such that $\Delta^2 \to 3$ as $R \to R_{min}$, $H \to 0$. Eq. (13b) can be compatible with (10a) only when $\rho \to 0$ in the same limit, but then we must avoid a potential divergence as $R \to R_{min}$ in the integral of (13b). In this limit the integral becomes $\int d\rho/(\rho-3p)$ which shows that we want

$$(\rho - 3p) \sim \rho^\alpha \quad \text{with} \quad \alpha < 1 \tag{14}$$

But (14) does not correspond to a physical situation because as $p \to 0$ one has $|3p| \gg \rho$ in this limit.

It is straightforward to check that the above results remain unchanged in the more general case [e.g., the SU(N,1)/SU(N)×(1) model[2)]], the only inputs being R symmetry current conservation and the existence of a dilaton field. For completeness, in this work we will present the analysis in one more example corresponding to the effective N = 1 supergravity Lagrangian which one finds by compactifying the ten-dimensional heterotic superstring on a Calabi-Yau manifold[3)]. This theory contains one additional gauge singlet field, the so-called S field, which adds to the Lagrangian (4) the extra piece:

$$\mathcal{L}_S = \left(1 - \frac{\Delta^2}{3}\right) \frac{1}{S_R} \left[(\partial S_R)^2 + (\partial S_I)^2 \right] \tag{4$_S$}$$

where we have defined $S_R \equiv \text{Re}S$ and $S_I \equiv \text{Im}S$. Eqs. (5a,b) remain unchanged while (8) and (10a) become:

$$\dot{H} + 2H^2 = A_0^2 - \frac{1}{3S_R}\left(\dot{S}_R^2 + \dot{S}_I^2\right) \tag{8$_S$}$$

$$3H^2 = \left(\dot{\Delta} + H\Delta\right)^2 + 3\left(\frac{3}{\Delta^2} - 1\right) A_0^2 + \left(1 - \frac{\Delta^2}{3}\right) \frac{1}{S_R}\left(\dot{S}_R^2 + \dot{S}_I^2\right) . \tag{10a$_S$}$$

For simplicity, in the above equations we have neglected the contribution of the matter energy-momentum tensor ($T^M_{\mu\nu} = 0$). We finally have two additional equations:

$$\partial_0 \left[R^3 \frac{\dot{S}_I}{S_R} (3 - \Delta^2) \right] = 0 \tag{15a}$$

$$\frac{2}{R^3} \partial_0 \left[R^3 (3 - \Delta^2) \frac{\dot{S}_R}{S_R} \right] + \frac{1}{S_R^2} (3 - \Delta^2) \dot{S}_I^2 = 0 \tag{15b}$$

which are obtained from the variation of the action with respect to S_I (15a) and S_R (15b). From (5b) we obtain the solution (7) as before, while (15a) gives

$$R^3 (3 - \Delta^2) \frac{\dot{S}_I}{S_R} = f \tag{16}$$

where f is a constant. We will present the simplest case $f = 0$, the general case $f \neq 0$ being left to the reader. Eq. (15b) then gives

$$\frac{\dot{S}_R}{S_R} = \frac{g}{R^3 (3 - \Delta^2)} \tag{17}$$

with g another integration constant. Eq. (8)$_S$ can now be rewritten as

$$\dot{H} + 2H^2 = A_0^2 - \frac{1}{3} S_R \frac{g^2}{R^6 (3 - \Delta^2)^2} \tag{18}$$

To get violation of the energy condition $\dot{H} > 0$ when $H = 0$, we need

$$c^2 > \frac{1}{3} \frac{g^2}{(3-\Delta^2)^2} S_R \tag{19a}$$

which is satisfied either when

$$S_R < 0 \tag{19b}$$

or

$$\Delta^2 < 3 - \frac{1}{\sqrt{3}} \left|\frac{g}{c}\right| S_R^{1/2} \quad , \; S_R > 0 \tag{19c}$$

But Eq. $(10a)_S$ can be rewritten as

$$3H^2 = (\dot{\Delta} + H\Delta)^2 + (3-\Delta^2)\left[\frac{3}{\Delta^2} A_\circ^2 + \frac{1}{3} \frac{\dot{S}_R^2}{S_R}\right] \tag{20}$$

which shows that to get $H = 0$ we need

either $\quad S_R < 0 \qquad$ (21a)

or $\quad (3-\Delta^2) < 0 \qquad$ (21b)

Equations (19c) and (21b) are incompatible, while for $S_R < 0$ the S_R acquires a negative kinetic energy term which leads to the same conclusion as before: thus avoidance of the initial singularity is possible only when one of the fields has a negative metric in some region of space-time or if one adds a matter system which satisfies the unphysical condition (4).

Schwarzschild Singularity Avoidance?

We now examine the black hole type of classical solution described by a static, spherically symmetric metric corresponding to the line element[6)]

$$ds^2 = B(r)\,dt^2 - A(r)\,dr^2 - r^2\,d\theta^2 - r^2 \sin^2\theta\, d\phi^2 \tag{22}$$

which depends on two independent functions A(r) and B(r). Examination of the time-dependent spherically symmetric case is beyond us at present. We expect in any case that any dynamical process of dilaton emission would lead to a static solution of the type we now discuss. Substituting the ansatz (22) in the equations of motion (5), in the absence of $\mathcal{L}_M$ we find that the solution of (5b) reads

$$A_r = \frac{c}{r^2}\left(\frac{A}{B}\right)^{1/2} \tag{23}$$

while (5c,d) give

$$\frac{B''}{B} - \frac{1}{2}\frac{B'}{B}\left(\frac{A'}{A}+\frac{B'}{B}\right) + \frac{2}{r}\left(\frac{B'}{B}-\frac{A'}{A}\right) - \frac{2}{r^2}(A-1) - 6A_r^2 = 0 \tag{24a}$$

$$\left(1-\frac{\omega}{3}\right)\left[\frac{1}{2}\frac{B''}{B} - \frac{1}{4}\frac{B'}{B}\left(\frac{A'}{A}+\frac{B'}{B}\right) + \frac{1}{r}\frac{B'}{B}\right] - \frac{c^2}{r^4}\left(\frac{9}{\omega}-\omega\right)\frac{A}{B} - \frac{1}{4}\frac{\omega'^2}{\omega} + \frac{1}{3}\omega'' + \frac{1}{6}\left(-\frac{A'}{A}+\frac{4}{r}\right)\omega' = 0 \tag{24b}$$

$$\left(1-\frac{\omega}{3}\right)\left[\frac{A}{r^2} - \frac{1}{2r}\left(\frac{B'}{B}-\frac{A'}{A}\right) - \frac{1}{r^2}\right] + \frac{c^2}{r^4}\left(\frac{9}{\omega}-\omega\right)\frac{A}{B} + \frac{1}{4}\frac{\omega'^2}{\omega} - \frac{1}{3}\omega'' - \frac{1}{6}\left(\frac{B'}{B}-\frac{A'}{A}+\frac{2}{r}\right)\omega' = 0 \tag{24c}$$

$$\left(1-\frac{\omega}{3}\right)\left[-\frac{1}{2}\frac{B''}{B} + \frac{1}{4}\frac{B'}{B}\left(\frac{A'}{A}+\frac{B'}{B}\right) + \frac{1}{r}\frac{A'}{A}\right] - \frac{c^2}{r^4}\frac{(3-\omega)^2}{\omega}\frac{A}{B} - \frac{1}{4}\frac{\omega'^2}{\omega} - \frac{1}{6}\left(\frac{B'}{B}+\frac{4}{r}\right)\omega' = 0 \tag{24d}$$

where the prime stands for the derivative with respect to r and we have defined $\omega \equiv \Delta^2$. Equations (24b, c and d) correspond to the tt, $\theta\theta$ and rr components of (5d) respectively.

After some algebraic manipulations and defining $x \equiv 1-\omega/3 = 1-\Delta^2/3$, Eqs. (24a-d) lead the following three independent relations:

$$\frac{1}{2}\frac{B''}{B} - \frac{1}{4}\frac{B'}{B}\left(\frac{A'}{A}+\frac{B'}{B}\right) - \frac{1}{r}\left(\frac{A'}{A}-\frac{B'}{B}\right) - \frac{A-1}{r^2} - 3\frac{c^2}{r^4}\frac{A}{B} = 0 \tag{25a}$$

$$\left[\frac{1}{2r}\left(3\frac{A'}{A}-\frac{B'}{B}\right) + 2\frac{A-1}{r^2} + 3\frac{c^2}{r^4}\frac{A}{B}\right]x + \frac{1}{2}\left(\frac{B'}{B}-\frac{2}{r}\right)x' = 0 \tag{25a}$$

$$\left[\frac{1}{2r}\left(\frac{A'}{A}+\frac{B'}{B}\right) + 3\frac{c^2}{r^4}\frac{A}{B}\right]x + \frac{1}{2}\left(2\frac{B'}{B}-\frac{A'}{A}+\frac{6}{r}\right)x' + x'' = 0 \; . \tag{25c}$$

In this case the violation of the energy condition can be written in the form

$$\frac{1}{r}\frac{A'}{A} + \frac{A-1}{r^2} + 3\frac{c^2}{r^4}\frac{A}{B} < 0 \quad . \tag{26}$$

We note that many of the terms in Eqs. (25) change sign with x, but we have not been able to establish a direct relation between violation of the energy condition and negativity of the kinetic terms: $x = 1-(\Delta^2/3) < 0$. However, we expect on general grounds and by analogy with the previous cosmological case that the central singularity can be avoided only if the kinetic terms become negative. The standard singularity theorems in General Relativity are based on various energy conditions[5], e.g., $\mathbb{R}_{ab}k^a k^b \geq 0$ for all null vectors k^a. From Eq. (5d) it follows that if k^a is a null vector

$$\begin{aligned}\frac{1}{2}\left(1-\frac{\Delta^2}{3}\right)\mathbb{R}_{\mu\nu}k^\mu k^\nu = \left(k^\mu\partial_\mu\Delta\right)^2 + \frac{9}{\Delta^2}\left(1-\frac{\Delta^2}{3}\right)\left(A_\mu k^\mu\right)^2 \\ -\frac{1}{6}(\Delta^2)_{;\mu\nu}k^\mu k^\nu + \frac{1}{2}T^M_{\mu\nu}k^\mu k^\nu\end{aligned} \tag{27}$$

Thus this null convergence condition will be satisfied in general if $(1-\Delta^2/3) > 0$, because to violate it the third term on the right-hand side would have to become sufficiently negative to overcome the other three terms. However, if $(1-\Delta^2/3) < 0$ this energy condition can easily be violated, and we cannot apply the singularity theorem in general. Numerical analysis of the Eqs. (25) which may resolve this question is underway[7].

The above results indicate that singularities are not avoided by modifications to the classical Einstein equations at distances larger than the Planck length L_p. Presumably it is at distances $\lesssim L_p$ that the superstring must succeed in avoiding singularities. At this scale, one must take into account the effects of the infinite tower of massive states, which is connected with the non-locality of the theory.

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