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Searching for hidden sectors in multiparticle production at the LHC

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Abstract

Most signatures of new physics in colliders have been studied so far on the transverse plane with respect to the beam direction where background is much reduced. In this work we study the impact of a hidden sector beyond the Standard Model (SM) on inclusive longitudinal (pseudo)rapidity correlations and moments of the multiplicity distributions, with special emphasis in the LHC results.

1. Introduction

Most signatures and signals of new phenomena are expected to be found at colliders in hard events on the transverse plane with respect to the beam direction (i.e. at high transverse momentum $p_\perp$), where background is much reduced.

Conversely, in this work we focus on rather diffuse soft signals in pp interactions at the LHC. For example, a non-standard state of matter from a Hidden Sector (HS) would alter observables related to (pseudo)rapidity particle correlations and factorial moments of multiplicity distributions [1].

1.1. Hidden Sectors

Among possible HS scenarios, hidden valley models are extensions of the SM when a new gauge group is added to the theory, leading to new bound states (v-particles) with relatively low masses for some values of the model parameters. Such scenarios [2] are physically motivated by top-down models including string theory constructions. Their experimental consequences and signatures for LHC experiments have been already studied [3]. For example, v-hadrons might promptly decay back to SM fermions contributing to the parton shower hadronizing to final-state particles.

2. Multi-step cascade: HS as an extra contribution

Following the old ideas of multiperipheral model [4, 5], we will assume that particles are produced by *clusters* from the interaction of two active partons of the colliding protons yielding a fragmenting string. We treat this possibility as resulting from a 2-step process and apply the same expression as an independent superposition of sources [6]:

$$P^{(2)}(n) = \sum_{N_c} P(N_c) \sum_{n_i} \prod_{i=1}^{N_c} P^{(1)}(n_i) \qquad (1)$$

where n and N_c denote the number of (charged) particles and clusters, respectively; $P^{(k)}(n)$ stands for a probability distribution where the superscript denotes a k-step cascade, such that $\sum_{n=1}^{\infty} P^{(k)}(n) = 1$.

However, more than two partons can interact and more than one string can be produced per event, with a probability distribution $P(N_s)$ depending on the nature of the interacting bodies and cms energy. We consider this possibility as deriving from a 3-step process:

$$P^{(3)}(n) = \sum_{N_s} P(N_s) \sum_{n_j} \prod_{j=1}^{N_s} P^{(2)}(n_j) \qquad (2)$$

where N_s denotes the number of strings per collision.

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Finally, we consider that a new stage in the parton cascade can be produced as a consequence of a HS on top of the parton shower, as motivated in the Introduction. We shall refer to this scenario as a 4-step process:

$$P^{(4)}(n) = \sum_{N_h} P(N_h) \sum_{n_k} \prod_{k=1}^{N_h} P^{(3)}(n_k) \tag{3}$$

where N_h denotes the number of hidden sources per collision and $P(N_h)$ its probability distribution.

3. Correlation functions

3.1. Two-particle rapidity correlations

Rapidity inclusive 2-particle correlation function for inelastic collisions are defined as

$$C_2(y_1, y_2) = \rho_2(y_1, y_2) - \rho(y_1)\,\rho(y_2) \tag{4}$$

where y_i denotes the longitudinal rapidity of particle i.

Above we have introduced the single and 2-particle (charged particle) rapidity densities:

$$\rho(y) = \frac{1}{\sigma_{in}} \int d^2p_\perp \frac{d^3\sigma}{dy d^2p_\perp}$$

$$\rho_2(y_1, y_2) = \frac{1}{\sigma_{in}} \int d^2p_{\perp 1} d^2p_{\perp 2} \frac{d^6\sigma}{dy_1 d^2p_{\perp 1} dy_2 d^2p_{\perp 2}}$$

where σ_{in} refers to the inelastic cross section; the normalizations are obtained by integration over the selected rapidity range.

$$\int dy\, \rho(y) = \langle n \rangle \; ; \int dy_1 dy_2\, \rho_2(y_1, y_2) = \langle n(n-1) \rangle$$

$$\int dy_1 dy_2\, C_2(y_1, y_2) = D^2 - \langle n \rangle \tag{5}$$

where $D^2 = \langle n^2 \rangle - \langle n \rangle^2$ is the variance of the charged emitted particles. For a Poisson distribution $D^2 = \langle n \rangle$, or equivalently the integral (5) is equal to zero corresponding to independent emission.

Quite generally, the inclusive correlation function is split into two terms:

$$C_2(y_1, y_2) = C_2^{LR}(y_1, y_2) + C_2^{SR}(y_1, y_2) \tag{6}$$

where the *short-range* (SR) term C_2^{SR} is generally assumed to be more sensitive to dynamical correlations, while C_2^{LR} stands for *long-range* (LR) correlations, usually arising from mixing different topologies in events [7].

4. Factorial Moments in multiplicity distibutions

Factorial moments of multiplicity distributions [8, 9] are obtained from integration of the inclusive correlation functions. Normalized factorial moment of rank q can be defined as:

$$F_q = \frac{\langle n(n-1)\cdots(n-q+1) \rangle}{\langle n \rangle^q} \; ; \quad q = 2, 3, \cdots \tag{7}$$

Such factorial moments represent any correlation between the emitted particles in events. Cumulant K_q represent genuine q-particle correlations not reducible to the product of lower order correlations. They are defined through

$$K_q = \sum_{m=0}^{q-1} C_{q-1}^m K_{q-m} F_m \tag{8}$$

where $C_{q-1}^m = 1/mB(q, m)$ and $B(q, m)$ denotes the beta function.

It is also convenient to define the ratio

$$H_q = \frac{H_q}{F_q} \tag{9}$$

which appear in a natural manner as solutions of QCD equations for the generating functions of multiplicity distributions [10]. QCD predicts that H_q moments should oscillate (change of sign) as a function of the rank q in e^+e^- annihilations. This prediction has been confirmed experimentally, also for pp and heavy ion collisions. In the latter cases, such oscillations are ascribed to a multicomponent structrure of the multiparticle production process (see [8] for a review and references therein). This interpretation is quite relevant in our study since a new (HS) component could show up in the production mechanism.

4.1. Cluster concept in hadronic production

According to, e.g., Ref.[11], a cluster if formed of all particles originating directly or indirectly from a (e.g. quark or gluon) common “ancestor”.

Equations expressing the factorial moments as functions of parameters of the multiplicity distributions of clusters, strings (and eventually hidden sources) are modified as more steps are introduced into the description of the parton cascade. In particular, we will denote by a superindex in the factorial moments, $F_q^{(j)}$, $K_q^{(j)}$ and $H_q^{(j)}$ with $j = 2, 3, 4$, for two-, three- and four-step cascades, corresponding to a single-string, multi-string and hidden-source scenario, respectively.

Of course, as more steps become included in description of the parton cascade, more parameters (encoding the complexity of the underlying soft hadronic dynamics) need to be introduced (see appendix). Nevertheless, the situation improves dramatically once a special type of distribution (for each step) is considered (e.g. Poisson: $F_q = 1, \forall q$).

4.2. Negative Binomial Distribution

The negative binomial distribution (NBD) deserves a special attention since it has been used to describe quite successfully experimental multiplicities in a wide variety of proceses and over a large energy range. The NBD is given by

$$P_n = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)}\left(\frac{\langle n\rangle}{k}\right)^n\left(1+\frac{\langle n\rangle}{k}\right)^{-n-k} \tag{10}$$

where k is a parameter with the physical meaning of the number of independent sources. Moreover

$$\frac{1}{k} = \frac{D^2 - \langle n\rangle}{\langle n\rangle^2} \tag{11}$$

The Poisson distribution is obtained in the limit $k \to \infty$.

5. Two-step cascade: single string

Let us first assume in the simplest (2-step) scenario that clusters are produced from a single string per collision, yielding final state particles (mainly hadrons).

5.1. Two-particle correlations

Let us introduce the rapidity 2-cluster correlation function

$$C_2^{(c)}(y_1,y_2) = \rho_2^{(c)}(y_1,y_2) - \rho^{(c)}(y_1)\,\rho^{(c)}(y_2) \tag{12}$$

with $\rho_2^{(c)}(y_1,y_2)$ and $\rho^{(c)}(y)$ refering to cluster densities in rapidity space, satisfying the usual normalization condition

$$\int dy_{c1}dy_{c2}\; C_2^{(c)}(y_{c1},y_{c2}) = D_c^2 - \langle N_c\rangle \tag{13}$$

where $D_c^2 = \langle N_c^2\rangle - \langle N_c\rangle^2$ denotes here the dispersion of the cluster distribution per collision.

The 2-particle correlation function in Eq.(4) can be split into two pieces following Eq.(6):

$$C_2(y_1,y_2) = C_2^{LR}(y_1,y_2) \;+\; \langle N_c\rangle\, C_2^{(1)}(y_1,y_2) \tag{14}$$

where the piece $C_2^{(1)}$ corresponding to 2-particle correlations "inside" a *single* cluster, reads

$$C_2^{(1)}(y_1,y_2) = \rho_2^{(1)}(y_1,y_2) - \rho^{(1)}(y_1)\,\rho^{(1)}(y_2) \tag{15}$$

Note the assumption that $C_s^{(1)}(y_1,y_2)$ has no explicit dependence on cluster rapidity, as indeed one expects an overall dependence on the $|y_1 - y_2|$ difference.

Confining our attention to a rapidity interval within the central region (where single spectra are approximately constant) we finally get

$$C_2(y_1,y_2) = (\overline{\rho}^{(1)})^2\, D_c^2 \;+\; \langle N_c\rangle\, C_2^{(1)}(y_1,y_2) \tag{16}$$

where $\overline{\rho}_1^{(1)}$ stands for the average (charged) particle density from a single cluster decay, obeying the relation $\overline{\rho} = \langle N_c\rangle \cdot \overline{\rho}^{(1)}$ with $\overline{\rho}$ denoting the average one-particle density in pp collisions.

From Eqs.(14) and (16) one can identify

$$C_2^{LR} = D_c^2\,(\overline{\rho}^{(1)})^2;\;\; C_2^{SR} = \langle N_c\rangle\, C_2^{(1)}(y_1,y_2) \tag{17}$$

5.2. Factorial moments

From Eq.(16) it is easy to obtain

$$F_2^{(2)} = F_2^{(c)} + \frac{F_2^{(1)}}{\langle N_c\rangle} \tag{18}$$

where $F_2^{(c)} = \langle N_c(N_c-1)\rangle/\langle N_c\rangle^2$ and $F_2^{(1)} = \langle n_1(n_1-1)\rangle/\langle n_1\rangle^2$, and $\langle n_1\rangle$ stands for the average particle multiplicity per single cluster decay.

In the Appendix we present several expressions for $F_{3,4}^{(j)}$ leaving higher rank (i.e. $q \geq 5$) $F_q^{(j)}$ moments for a forthcoming longer paper. Let us also mention that $F_2^{(j)}$ often determines the values of higher rank moments in different approaches, and this is, in fact, the method to be followed in this work.

6. Three-step cascade: multiple strings

Let us develop to some extent some expressions for correlations and scaled factorial moments for a 3-step partonic cascade.

6.1. Two-particle correlations

As in the previous 2-step cascade, by integration over rapidities of intermediate sources of particles in the central rapidity region, the SR part of the 2-particle correlation function can be identified now as:

$$C_2^{SR}(y_1,y_2) = \langle\, N_c\rangle\, C_2^{(1)}(y_1,y_2) \tag{19}$$

The LG part reads:

$$C_2^{LR}/(\overline{\rho}^{(1)})^2 \;=\; \langle N_c^s\rangle^2\, D_s^2 \;+\; \langle N_s\rangle\, D_c^2 \tag{20}$$

with $D_s^2 = \langle N_s^2\rangle - \langle N_s\rangle^2$. When setting $\langle N_s\rangle = 1$ and $D_s^2 = 0$ Eq.(16) is quickly recovered. Also note that when passing from a 2-step to a 3-step scenario the LR contribution tends to increase.

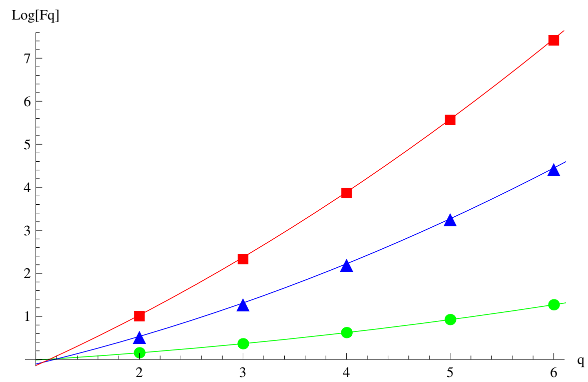

Figure 1: Expectations for the normalized moments $\ln F_q^{(2,3,4)}$ versus the rank q according to: Green circles: 2-step cascade with $\langle N_c\rangle = 16$; Blue triangles: 3-step cascade with $\langle N_s\rangle = 2, \langle N_c\rangle = 16$: Red squares: 4-step cascade with $\langle N_h\rangle = 1, \langle N_s\rangle = 2, \langle N_c\rangle = 16$. All $F_q^{(h,s,c,1)} \simeq 1$. Solid lines correspond to parabolic fits to points.

6.2. Factorial moments

$$F_2^{(3)} = F_2^{(s)} + \frac{F_2^{(c)}}{\langle N_s\rangle} + \frac{F_2^{(1)}}{\langle N_c\rangle} \tag{21}$$

where $\langle N_c\rangle = \langle N_s\rangle \cdot \langle N_c^s\rangle$; the scaled moment of the distribution of fragmenting strings and clusters are here respectively defined as $F_2^{(s)} = \langle N_s(N_s-1)\rangle/\langle N_s\rangle^2$ and $F_2^{(c)} = \langle N_c^s(N_c^s-1)\rangle/\langle N_c^s\rangle^2$.

7. Four-step cascade: hidden sources

As previously argued, we model a possible HS contribution to the standard parton cascade as a new stage yielding a 4-step scenario, thereby modifying rapidity correlations and factorial moments as we study below focusing on the central rapidity region of pp collisions.

7.1. Two-particle correlations

The SR part reads again

$$C_2^{SR}(y_1, y_2) = \langle N_c\rangle\, C_2^{(1)}(y_1, y_2)$$

while the LR part becomes:

$$C_2^{LR}/(\bar{\rho}^{(1)})^2 = \langle N_s\rangle^2 D_h^2 + \langle N_h\rangle \langle N_c^s\rangle^2 D_s^2 + \langle N_s\rangle D_c^2$$

where now $\langle N_s\rangle = \langle N_h\rangle \cdot \langle N_s^h\rangle$, $\langle N_c\rangle = \langle N_s\rangle \cdot \langle N_c^s\rangle$, and D_h denotes the dispersion of the hidden source distribution.

7.2. Factorial moments

$$F_2^{(4)} = F_2^{(h)} + \frac{F_2^{(s)}}{\langle N_h\rangle} + \frac{F_2^{(c)}}{\langle N_s\rangle} + \frac{F_2^{(1)}}{\langle N_c\rangle} \tag{22}$$

with $F_2^{(s)} = \langle N_s^h(N_s^h-1)\rangle/\langle N_s^h\rangle^2$ and $F_2^{(h)} = \langle N_h(N_h - 1)\rangle/\langle N_h\rangle^2$.

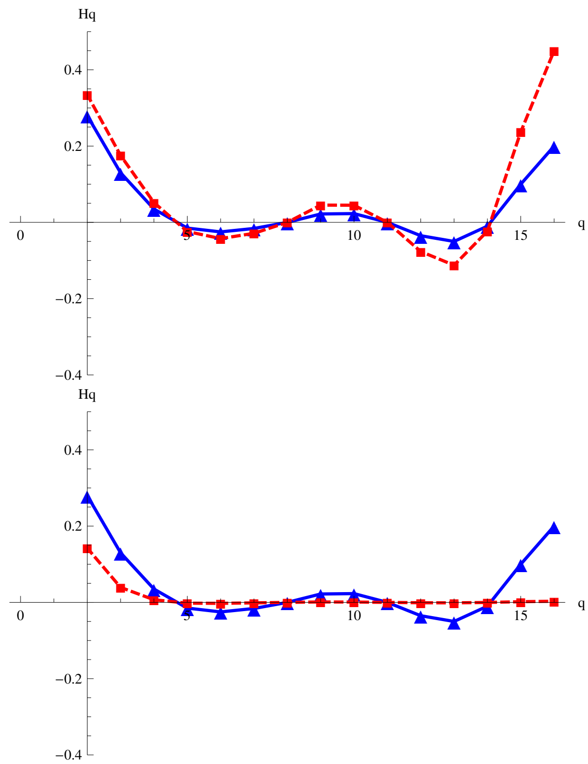

Figure 2: Expectations for the ratios $H_q^{(3,4)} = K_q^{(3,4)}/F_q^{(3,4)}$. Upper panel: $\langle N_h\rangle = 1$; Lower panel: $\langle N_h\rangle = 8$; Blue triangles and solid line: conventional parton cascade according to Ref.[6]. Red squares and dashed line: hidden source(s) on top of the parton cascade.

8. HS-cascade versus a conventional cascade

In this work we advocate that a new stage at the onset of the partonic cascade can appreciably change the multiplicity distribution of final-state particles in pp collisions. To this aim we implement such a new stage in the conventional cascade described as a 3-step (multi-string) process thereby becoming a 4-step cascade.

Let us remark that, keeping fixed the number of steps of the parton cascade, a higher number of sources (e.g. $\langle N_c\rangle$) should lead to smaller $F_q^{(2,3,4)}$ moments, in accordance with the dilution effect [12, 13].

However, increasing the number of steps in the cascade leads quite generally (except for large values of $\langle N_h\rangle$) to an increase of the $F_q^{(2,3,4)}$ moments, as can be seen in Fig.1 where the green (triangle), blue (square) and red (circle) points represent 2-, 3- and 4-step cascades, respectively. It is worth pointing out that larger values of $\langle N_h\rangle$ would considerably lower the values of $F_q^{(4)}$, even below the 2-step and 3-step cascades.

8.1. Analysis of $H_q^{(3,4)}$ oscillations

Let us get started by wrtiting the expression for the H_q ratios corresponding to a NBD

$$H_q^{(3,4)} = \frac{\Gamma(q)\,\Gamma(k_{eff}^{(3,4)}+1)}{\Gamma(k_{eff}^{(3,4)}+q)} = k_{eff}^{(3,4)}\ B(q,k_{eff}^{(3,4)}) \quad (23)$$

where an *effective* $k_{eff}^{(3,4)}$ parameter is obtained from

$$\frac{1}{k_{eff}^{(3,4)}} = F_2^{(3,4)} - 1 \quad (24)$$

This approach relies on the fact that the resulting final-state multiplicity distribution is itself a NBD whose k parameter determines the $H_q^{(3,4)}$ ratios. In a forthcoming paper, we will calculate $H_q^{(3,4)}$ from specific expressions of $F_q^{(3,4)}$; some of which are shown in the appendix.

Assuming Poisson-like distributions at all intermediate steps of the cascade ($F_q^{(h,s,c,1)} \simeq 1$), one gets for the 2-step scenario $k_{eff}^{(2)} \lesssim \langle N_c \rangle$.

For the 3-step scenario one obtains

$$k_{eff}^{(3)} = \frac{\langle N_s \rangle \langle N_c \rangle}{\langle N_c \rangle + 1} \quad (25)$$

If $\langle N_c \rangle >> 1$, $k_{eff}^{(3)} \lesssim \langle N_s \rangle$.

Finally we get for the 4-step scenario

$$k_{eff}^{(4)} = \frac{\langle N_h \rangle \langle N_s \rangle \langle N_c \rangle}{\langle N_h \rangle \langle N_s \rangle + \langle N_h \rangle \langle N_c \rangle + \langle N_s \rangle \langle N_c \rangle + 1} \quad (26)$$

If besides $\langle N_h \rangle << \langle N_s \rangle < \langle N_c \rangle$ then $k_{eff}^{(4)} \lesssim \langle N_h \rangle$.

8.2. Results

Let us define a rescaling factor r_q for each value of q as the ratio

$$r_q = \frac{H_q^{(4)}}{H_q^{(3)}} \quad (27)$$

which should allow us to calculate $H_q^{(4)}$ from the a 3-step conventional cascade that we assume to coincide with the expectations for pp collisions at the LHC (14 TeV) obtained in Ref.[6].

Thus, squares and the dashed line (in red) of Fig.2 (upper and lower panels) correspond to a hidden source in a 4-step cascade, with $N_h = 1$, $N_s^h = 2$, $N_c^s = 8$, and $N_h = 8$, $N_s^h = 2$, $N_c^s = 8$, respectively. Triangles and the solid line (in blue) correspond to the points from the above-mentioned Ref.[6].

Admittedly, this is a rough approach in order to compare $H_q^{(3,4)}$ oscillations, i.e. multiparticle production *with* and *without* a HS. As already advocated in this paper, the key idea is that a (statistically significant) deviation one from each other might prove to be useful to unravel a possible signal of a hidden sector participating in the parton cascade.

9. Summary and final remarks

In this work we advocate that a new stage of matter (stemming from a hidden sector beyond the SM) on top of the partonic cascade leading to multiparticle final states in pp collisions at the LHC can have an influence on rapidity correlations and factorial moments (and their ratios) of multiplicity distributions.

On the one hand, as a general trend one should expect that a HS yields longer (in rapidity) and stronger 2-particle correlations among emitted charged particles.

On the other hand, depending on the number of hidden sources two different behaviours of the oscillation pattern of $H_q^{(4)}$ moments can be distinguished:

- For a small number of hidden sources (e.g. $\langle N_h \rangle = 1$), the oscillation amplitudes become considerably larger than in a conventional cascade as obtained in [6]. See figure 2 (upper panel).
- For a larger number of hidden sources (e.g. $\langle N_h \rangle = 8$), the oscillation amplitudes are considerably smaller than in a conventional cascade as obtained in [6]. See figure 2 (lower panel). Let us note that the crossing point shifts to a smaller q value for higher $\langle N_h \rangle$.

Let us point out that that there could be combinations of parameters such that $H_q^{(3,4)}$ oscillations would become practically indistinguishable from each other.

Moreover, both (conventional and HS) process will be present together in the collected sample of events, although specific cuts (such as high multiplicity, flavour tagging, etc) should be applied on events to enrich the possible new physics contribution.

Another caveat is in order. The behaviour of such oscillations is very sensitive, e.g., to multiplicity cuts on events [8]. Therefore, to avoid a bias one should be careful to compare samples at different energies with the same cuts applied on events.

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Appendix A. Factorial moments for 2-, 3- and 4-step scenarios

In this appendix we only show expressions for factorial moments $F_3^{(j)}$ and $F_4^{(j)}$ (where $j = 2,3,4$ denotes 2-, 3- or 4-step scenarios, respectively) and leave higher rank factorial moments for a long paper in preparation.

2-step cascade

$$F_3^{(2)} = F_3^{(c)} + 3\,\frac{F_2^{(c)}}{\langle N_c\rangle}F_2^{(1)} + \frac{F_3^{(1)}}{\langle N_c\rangle^2}$$

$$F_4^{(2)} = F_4^{(c)} + \frac{F_4^{(1)}}{\langle N_c\rangle^3} + 4\,\frac{F_2^{(c)}F_3^{(1)}}{\langle N_c\rangle^2} + 6\,\frac{F_3^{(c)}F_2^{(1)}}{\langle N_c\rangle} + 3\,\frac{F_2^{(c)}F_2^{(1)2}}{\langle N_c\rangle^2}$$

3-step cascade

$$F_3^{(3)} = F_3^{(s)} + \frac{F_3^{(c)}}{\langle N_s\rangle^2} + 3\left[\frac{F_2^{(s)}F_2^{(c)}}{\langle N_s\rangle} + \frac{F_2^{(s)}F_2^{(1)}}{\langle N_c\rangle} + \frac{F_2^{(c)}F_2^{(1)}}{\langle N_s\rangle\langle N_c\rangle}\right] + \frac{F_3^{(1)}}{\langle N_c\rangle^2}$$

$$F_4^{(3)} = F_4^{(s)} + \frac{F_4^{(c)}}{\langle N_s\rangle^3} + \frac{F_4^{(1)}}{\langle N_c\rangle^3} + 4\left[\frac{F_2^{(s)}F_3^{(c)}}{\langle N_s\rangle^2} + \frac{F_2^{(s)}F_3^{(1)}}{\langle N_c\rangle^2} + \frac{F_2^{(c)}F_3^{(1)}}{\langle N_s\rangle\langle N_c\rangle^2}\right] + 6\left[\frac{F_3^{(s)}F_2^{(c)}}{\langle N_s\rangle} + \frac{F_3^{(s)}F_2^{(1)}}{\langle N_c\rangle} + \frac{F_3^{(c)}F_2^{(1)}}{\langle N_s\rangle^2\langle N_c\rangle}\right] + 3\left[\frac{F_2^{(s)}F_2^{(c)2}}{\langle N_s\rangle^2} + \frac{F_2^{(s)}F_2^{(1)2}}{\langle N_c\rangle^2} + \frac{F_2^{(c)}F_2^{(1)2}}{\langle N_s\rangle\langle N_c\rangle^2}\right] + 18\,\frac{F_2^{(s)}F_2^{(c)}F_2^{(1)}}{\langle N_s\rangle\langle N_c\rangle}$$

4-step cascade

$$F_3^{(4)} = F_3^{(h)} + \frac{F_3^{(s)}}{\langle N_h\rangle^2} + \frac{F_3^{(c)}}{\langle N_s\rangle^2} + \frac{F_3^{(1)}}{\langle N_c\rangle^2} + 3\left[\frac{F_2^{(s)}F_2^{(1)}}{\langle N_h\rangle\langle N_c\rangle} + \frac{F_2^{(s)}F_2^{(c)}}{\langle N_h\rangle\langle N_c\rangle} + \frac{F_2^{(c)}F_2^{(1)}}{\langle N_s\rangle\langle N_c\rangle} + \frac{F_2^{(h)}F_2^{(1)}}{\langle N_c\rangle} + \frac{F_2^{(h)}F_2^{(c)}}{\langle N_s\rangle} + \frac{F_2^{(h)}F_2^{(s)}}{\langle N_h\rangle}\right]$$

$$F_4^{(4)} = F_4^{(h)} + \frac{F_4^{(s)}}{\langle N_h\rangle^3} + \frac{F_4^{(c)}}{\langle N_s\rangle^3} + \frac{F_4^{(1)}}{\langle N_c\rangle^3} + 4\left[\frac{F_2^{(s)}F_3^{(c)}}{\langle N_h\rangle\langle N_s\rangle^2} + \frac{F_2^{(s)}F_3^{(1)}}{\langle N_h\rangle\langle N_c\rangle^2} + \frac{F_2^{(c)}F_3^{(1)}}{\langle N_s\rangle\langle N_c\rangle^2} + \frac{F_2^{(h)}F_3^{(s)}}{\langle N_h\rangle^2} + \frac{F_2^{(h)}F_3^{(c)}}{\langle N_s\rangle^2} + \frac{F_2^{(h)}F_3^{(1)}}{\langle N_c\rangle^2}\right] + 6\left[\frac{F_3^{(s)}F_2^{(c)}}{\langle N_h\rangle^2\langle N_s\rangle} + \frac{F_3^{(s)}F_2^{(1)}}{\langle N_h\rangle^2\langle N_c\rangle} + \frac{F_3^{(c)}F_2^{(1)}}{\langle N_s\rangle^2\langle N_c\rangle} + \frac{F_3^{(h)}F_2^{(c)}}{\langle N_s\rangle} + \frac{F_3^{(h)}F_2^{(1)}}{\langle N_c\rangle} + \frac{F_3^{(h)}F_2^{(s)}}{\langle N_h\rangle}\right] + 3\left[\frac{F_2^{(s)}F_2^{(1)2}}{\langle N_h\rangle\langle N_c\rangle^2} + \frac{F_2^{(s)}F_2^{(1)2}}{\langle N_h\rangle\langle N_s\rangle^2} + \frac{F_2^{(c)}F_2^{(1)2}}{\langle N_s\rangle\langle N_c\rangle^2} + \frac{F_2^{(h)}F_2^{(1)2}}{\langle N_c\rangle^2} + \frac{F_2^{(h)}F_2^{(c)2}}{\langle N_s\rangle^2} + \frac{F_2^{(h)}F_2^{(s)2}}{\langle N_h\rangle^2}\right] + 18\left[\frac{F_2^{(s)}F_2^{(c)}F_2^{(1)}}{\langle N_h\rangle\langle N_s\rangle\langle N_c\rangle} + \frac{F_2^{(h)}F_2^{(c)}F_2^{(1)}}{\langle N_s\rangle\langle N_c\rangle} + \frac{F_2^{(h)}F_2^{(s)}F_2^{(c)}}{\langle N_h\rangle\langle N_s\rangle} + \frac{F_2^{(h)}F_2^{(s)}F_2^{(1)}}{\langle N_h\rangle\langle N_c\rangle}\right]$$

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