ZEMAX SIMULATIONS OF DIFFRACTION AND TRANSITION RADIATION

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Abstract

Diffraction Radiation (DR) and Transition Radiation (TR) are produced when a relativistic charged particle moves in the vicinity of a medium or through a medium respectively. The target atoms are polarised by the electric field of the charged particle, which then oscillate thus emitting radiation with a very broad spectrum. The spatial-spectral properties of DR/TR are sensitive to various electron beam parameters. Several projects aim to measure the transverse (vertical) beam size using DR or TR. This paper reports on how numerical simulations using Zemax can be used to study such a system.

INTRODUCTION

Optical system design is no longer a skill reserved for a few professionals. With readily available commercial optical design software, these tools are accessible to the general optical engineering community. The Zemax Optical Design Program is such a comprehensive software tool [1]. It integrates all the features required to conceptualise, design, optimise, analyse and tolerance virtually any optical system. It is widely used in the optics industry as a standard design tool.

Geometrical ray tracing is an incomplete description of light propagation. Strictly speaking, the propagation of light is a coherent process. As a wavefront travels through free space or optical medium, the wavefront coherently interferes with itself. Modelling this coherent propagation comprises the domain of physical optics. Physical Optics Propagation (POP) is the capability of Zemax which uses diffraction calculations to propagate a wavefront through an optical system surface by surface. The coherent nature of light is fully accounted for by this capability. When using POP, the wavefront is modelled using an array of points. Each point in the array stores complex amplitude information about the beam. The array is user-definable in terms of its dimension, sampling and aspect ratio.

To propagate the beam from one surface to another, either a Fresnel diffraction propagation or an angular spectrum propagation algorithm is used. Zemax automatically chooses the algorithm that yields the highest numerical accuracy. The diffraction propagation algorithms yield correct results for any propagation distance, for any arbitrary beam and can account for any surface aperture, including user defined apertures. When the wavefront reaches an optical surface (e.g. a lens), it is decomposed into rays in order to simulate aberrations and diffraction coming out through the lens, then rays are recomposed into wavefront at the exit of the lens; therefore both, aberrations and diffraction through optical lines, are simulated.

In POP mode, a custom electric field source provided in a C file and compiled as a DLL can be used as an input to Zemax. In this way, the simulation of any source of light is possible (e.g. TR, DR, synchrotron radiation (SR))

MOTIVATION

The theories used to describe both, optical transition and diffraction radiation (OTR and ODR), are highly simplified and use many assumptions, e.g. free-floating targets and single-electron pass. In order to take account of realistic setups using real optical elements such as finite-size lenses, filters, targets and other apertures, simulations using Zemax should be implemented. As a first step, simulations have to agree with the analytical expressions, given the same assumptions used to derive those. Once agreement is shown, the simulations provide a powerful tool to study experimental setups regarding their diffraction limitations, possible misalignments and beam size effects.

Using the target as the radiation source, the initial electric field is either defined in a user-defined two-dimensional matrix in binary or text format or computed with a Windows Dynamic Link Library (DLL). When using POP, the wavefront is modelled using an array of discretely sampled points. For each point in the array, the complex amplitude of the beam is stored. The entire array is then propagated in free space between optical surfaces. At each optical surface, a transfer function is computed which propagates the the beam matrix from one side of the surface to the other. In this paper, only simulations in free-space are presented.

THEORY

OTR

Following the approach for calculating TR from a particle obliquely passing through a boundary between vacuum and an ideal conductor [2] and applying an ultra-relativistic

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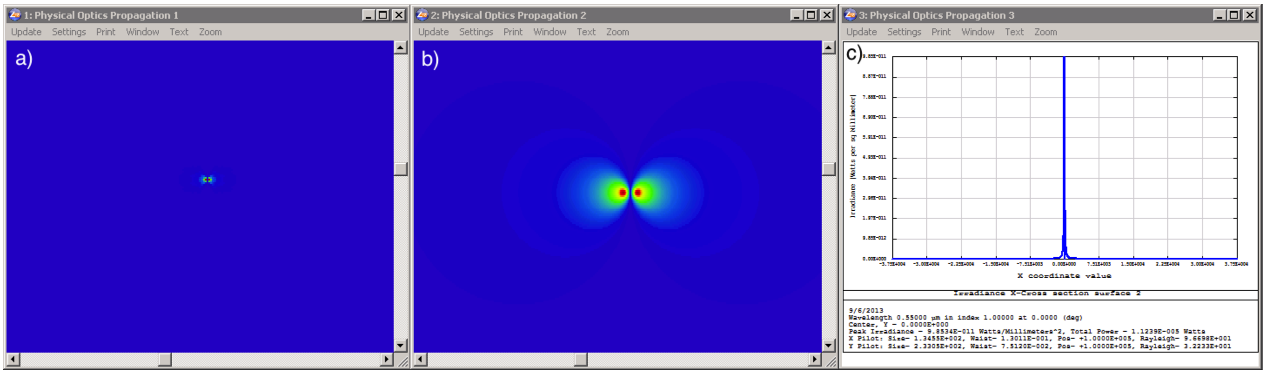

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Y Pilot: Size= 2.3305E+002, Waist= 7.5120E-002, Pos= +1.0000E+005, Rayleigh= 3.2233E+001
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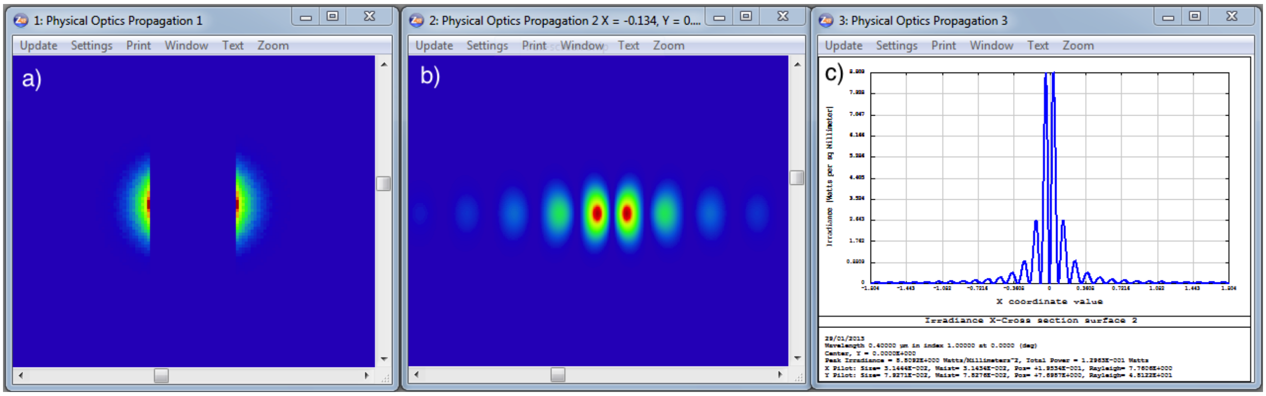
Figure 1: OTR Zemax output: source (a), detector plane (b) and horizontal cross-section of the detector plane (c).



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Y Pilot: Size= 7.9271E-002, Waist= 7.8276E-002, Pos= +7.6957E+000, Rayleigh= 4.8122E+001
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Figure 2: ODR Zemax output: source (a), detector plane (b) and horizontal cross-section of the detector plane (c).

approximation $(\theta_x, \theta_y, \gamma^{-1} \ll 1)$, the following equation is obtained for the angular distribution of intensity [3]:

$$\frac{d^2W_{TR}}{d\omega d\Omega} = \frac{\alpha}{\pi^2} \frac{{\theta_x}^2 + {\theta_y}^2}{\left(\gamma^{-2} + {\theta_x}^2 + {\theta_y}^2\right)} \tag{1}$$

Here θ_x and θ_y are the radiation observation angles measured either from the mirror reflection direction or from the particle trajectory, α is the fine structure constant. Eq. 1 is TR in the case of normal incidence, i.e. with no target tilt.

ODR

The ODR model considers the case when a charged particle moves through a slit between two tilted semi-planes i.e. only DR produced from the target is considered. In the case of a horizontal slit, the vertical polarisation component is sensitive to beam size [4]. Eq. 2 gives the expression for the ODR vertical polarisation component convoluted with a Gaussian distribution [4], where α is the fine-structure constant, γ is the Lorentz factor, θ_0 is the tilt angle of the target, $t_{x,y} = \gamma\theta_{x,y}$ where $\theta_{x,y}$ are the radiation angles measured from the mirror reflection direction, λ is the observation wavelength, σ_y is the rms vertical beam size, a is the target aperture size, a_x is the offset of the beam centre with respect to the centre of the slit and $\psi = \arctan\left(\frac{t_y}{\sqrt{1+{t_x}^2}}\right)$. This model is applicable when the transition radiation contribution from the tails of the Gaussian distribution is negligible, which means approximately $a \geq 4\sigma_y$.

$$\frac{d^2W_y^{slit}}{d\omega d\Omega} = \frac{\alpha\gamma^2}{2\pi^2} \frac{\exp\left(-\frac{2\pi a \sin\theta_0}{\gamma\lambda}\sqrt{1+{t_x}^2}\right)}{1+{t_x}^2+{t_y}^2} \times \left\{\exp\left[\frac{8\pi^2{\sigma_y}^2}{\lambda^2\gamma^2}\left(1+{t_x}^2\right)\right]\cosh\left(\frac{4\pi\overline{a}_x}{\gamma\lambda}\sqrt{1+{t_x}^2}\right) - \cos\left(\frac{2\pi a \sin\theta_0}{\gamma\lambda}t_y + 2\psi\right)\right\} \tag{2}$$

Generally, DR intensity is inversely proportional to the aperture size and the sensitivity to beam size is inversely proportional to the observation wavelength. The sensitivity to beam size is dependent on the visibility (I_{min}/I_{max}) of the DR angular distribution, where I_{min} is the minimum intensity taken at the centre of the distribution between the two main lobes. Therefore the maximum and minimum intensities of the DR angular distribution must be measured accurately [4].

SIMULATIONS

Since this paper only deals with a free-space propagation not involving any optical elements, the setup is straightforward. For either simulation, a source is defined using a DLL. The field is then propagated in free space. Finally, the detector plane is placed at a distance far enough to fulfil the far-field requirement for either radiation type, i.e. the distance to the observation point must obey the following condition $L \gg \frac{\gamma^2\lambda}{2\pi}$ [5]. Figure 1(a) shows the intensity at the source, created by a single electron passing through an ideal conductor ($\gamma = 2500$, $\lambda = 550$ nm). Figures 1(b) and (c) show the intensity at the detector plane in the far-field. Figure 2(a) shows the intensity at the source, created by a single electron passing through a 1-mm vertical slit ($\gamma = 4110$, $\lambda = 400$ nm). Figures 2(b) and (c) show the intensity at the detector plane in the far-field.

OTR

At the Accelerator Test Facility (ATF), the OTR monitor uses an observation wavelength of $\lambda = 550$ nm for a beam Lorentz factor of $\gamma = 2500$ [6, 7]. Figures 3 and 4 show the Zemax simulated OTR irradiance horizontal cross section at the source for a wavelength of $\lambda = 550$ nm with varying energy and for a Lorentz factor of $\gamma = 2500$ with varying wavelength respectively. After the field used to calculate the distributions in Fig. 3 and 4 is propagated in free space using Zemax, it can be compared with analytical theory.

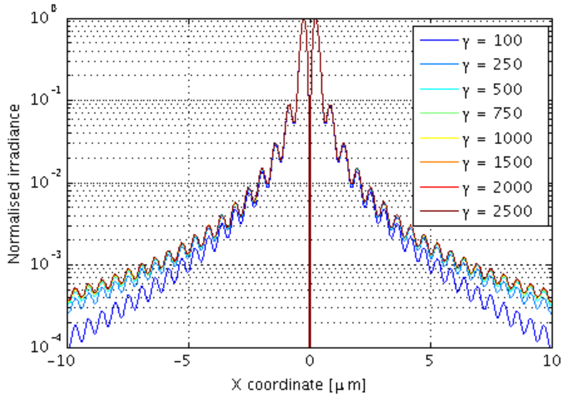

Figure 3: OTR irradiance horizontal cross section at the source for $\lambda = 550$.

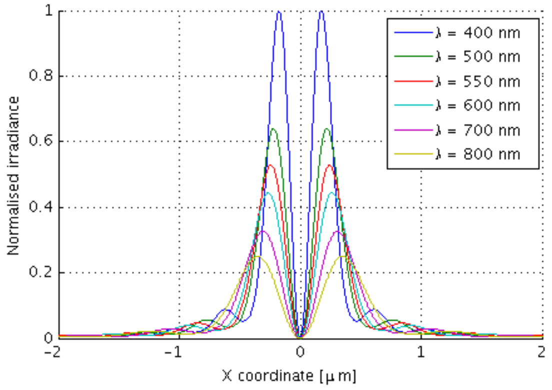

Figure 4: OTR irradiance horizontal cross section at the source for $\gamma = 2500$.

Figure 5 compares the theoretical angular distribution with the Zemax simulation. A nearly perfect agreement can be observed. The far-field requirement for OTR is $L \gg \frac{\gamma^2\lambda}{2\pi} = 0.55$ m, therefore distance between source and detector plane for the simulation was set to 100 m. The size of the source was $r_{max} = 10 \cdot \frac{\gamma\lambda}{2\pi} = 2.188$ mm and the aperture size in the theoretical model was set to $a = 1$ nm.

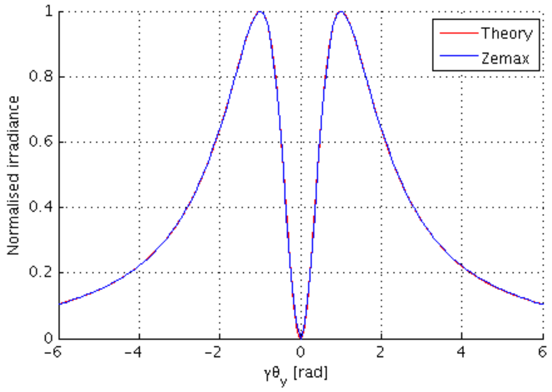

Figure 5: Comparison of the theoretical OTR angular distribution in far-field condition with the Zemax simulation.

Figure 6 shows the effect on the angular distribution when moving the detector plane from the near-field into the far-field. The distribution was again simulated for an observation wavelength of $\lambda = 550$ nm for a beam Lorentz factor of $\gamma = 2500$, at three different distances from the source - $\frac{\gamma^2\lambda}{2\pi}$, $2 \cdot \frac{\gamma^2\lambda}{2\pi}$ and $10 \cdot \frac{\gamma^2\lambda}{2\pi}$. This figure is in excellent agreement with analytical calculations [5].

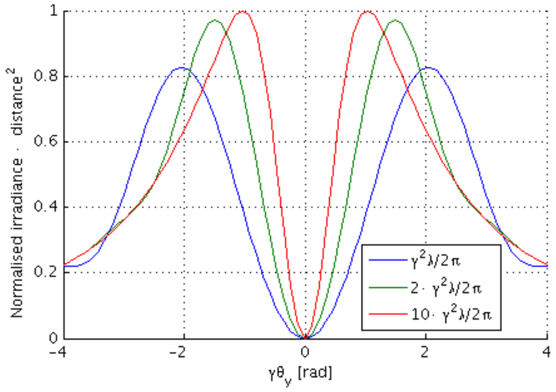

Figure 6: Zemax simulation of the OTR angular distribution for different distances from the source.

ODR

After using the field at the target surface and taking out the part corresponding to a slit, as it is shown in Figure 2(a), the ODR Zemax simulations can be compared with analytical theory. For an observation wavelength of $\lambda = 400$ nm and a Lorentz factor of $\gamma = 4110$, which corresponds to the experimental conditions at the ODR monitor at the Cornell Electron Storage Ring (CesrTA) [8], the distribution at the detector plane can be found in Figure 7, using a slit width of 1 mm and an incident target angle of $\theta_0 = 70°$. Again, nearly perfect agreement can be observed. The far-field condition for the given parameters is fulfilled for a distance $L \gg \frac{\gamma^2\lambda}{2\pi} = 1.08$ m, as described by Eq. 2. The distance between source and detector plane was set to 100 m, therefore angular distribution is fully defined. The size of the source was $r_{max} = 10 \cdot \frac{\gamma\lambda}{2\pi} = 2.617$ mm.

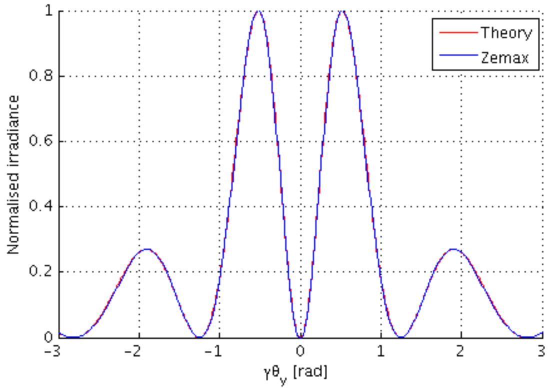

Figure 7: Comparison of the theoretical ODR angular distribution in far-field condition with the Zemax simulation.

As in the OTR case, Fig. 8 shows the effect on the angular distribution when moving the detector plane from the near-field into the far-field, this time using a slit width of 300 μm and an incident target angle of $\theta_0 = 90°$. The distribution was simulated for an observation wavelength of $\lambda = 400$ nm for a beam Lorentz factor of $\gamma = 4110$, again at three different distances from the source - $\frac{\gamma^2\lambda}{2\pi}$, $2 \cdot \frac{\gamma^2\lambda}{2\pi}$ and $10 \cdot \frac{\gamma^2\lambda}{2\pi}$.

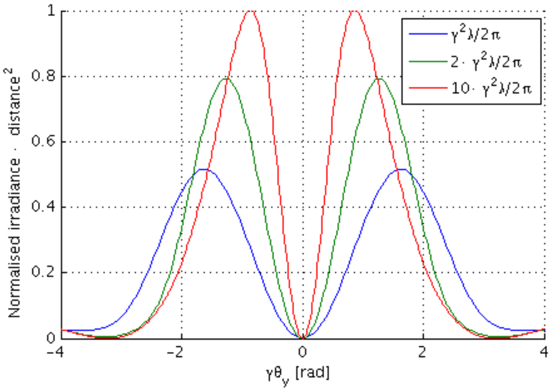

Figure 8: Zemax simulation of the ODR angular distribution for different distances from the source.

CONCLUSIONS AND OUTLOOK

For OTR and ODR, it has been shown, that with assumptions similar to theoretical boundary conditions, Zemax simulations agree with the analytical expressions. The next steps are comparing analytical equations for angular distributions with Zemax simulations for a finite beam size, which will involve the need for convolution or a Monte Carlo type extension. After this, the software will have been proven useful for studies of any type of optical system using OTR or ODR. It will enable simulations of all misalignment errors and optimisation of an optical system to be implemented in a real diagnostic station.

ACKNOWLEDGMENTS

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