Beyond the Standard Model

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Abstract
A Brief review on the physics beyond the Standard Model.

1 Quest of BSM

Although the standard model of elementary particles(SM) describes the high energy phenomena very well, particle physicists have been attracted by the physics beyond the Standard Model (BSM). There are very good reasons about this;

1. The SM Higgs sector is not natural.
2. There is no dark matter candidate in the SM.
3. Origin of three gauge interactions is not understood in the SM.
4. Cosmological observations suggest an inflation period in the early universe. The non-zero baryon number of our universe is not consistent with the inflation picture unless a new interaction is introduced.

The Higgs boson candidate was discovered recently. The study of the Higgs boson nature is extremely important for the BSM study.

The Higgs boson is a spin 0 particle, and the structure of the radiative correction is quite different from those of fermions and gauge bosons. The correction of the Higgs boson mass is proportional to the cut-off scale, called "quadratic divergence". If the cut-off scale is high, the correction becomes unacceptably large compared with the on-shell mass of the Higgs boson. This is often called a "fine turning problem". Note that such quadratic divergence does not appear in the radiative correction to the fermion and gauge boson masses. They are protected by the chiral and gauge symmetries, respectively.

The problem can be solved if there are an intermediate scale where new particles appears, and the radiative correction from the new particles compensates the SM radiative correction. The scale is probably much less than $\mathcal{O}(100)$ TeV, where the ratio between the SM radiative correction and the Higgs vev is more than 1000. The turning of the factor 1000 may sound unnatural, but it is much better than the scale among other parameters, such as Planck scale to the order of electroweak symmetry breaking, or the large difference among Yukawa couplings.

An idea to introduce a new particle that couples to the Higgs boson to cancel one loop level quadratic correction, is not successful, because such accidental cancellation does not hold all order in the perturbation theory. One needs new symmetry to cancel the quadratic divergence in the SM by a new physics contribution. The known ideas to achieve the reduction of quadratic divergence are the following;

1. **Supersymmetry**: Extend the SM so that the theory has "supersymmetry". Supersymmetry is the symmetry between bosons and fermions, which allows the divergence of Higgs boson mass controlled by "chiral symmetry" of fermions. Due to the cancellation among various diagrams involving SM particles and their superpartners (SUSY particles), there are no quadratic divergence to the Higgs bosons mass in this theory.

2. **Dynamical symmetry breaking**: In this theory, a new strong interaction causes the spontaneous gauge symmetry breaking of the SM. The Higgs doublet is a Nambu-Goldstone boson of the symmetry breaking and bound states of fermions charged under the strong interaction, corresponding

to the pions in the QCD. The Higgs boson does not exist above the symmetry breaking scale, so there are no problem of quadratic divergence.

3. **Extra dimension** Although we recognize that we live in the four dimensional space-time, we might live in more than the five dimension space time where the extra dimensions are compactified. The true Planck scale may be much closer to electroweak scale in such a theory, or the fundamental parameters in the Higgs sector is of the order of Planck scale in the higher dimensional theory but looks small in the effective four dimensional theory. In some class of the model the Higgs boson may be a part of gauge boson in the 5th dimension so that the divergence of the Higgs mass parameters is controlled by the gauge symmetry.

Those models are constrained strongly by precision measurements. Currently there are no measurements with significant deviation from the SM predictions. In the SM theory, one can predict various observables from a few fundamental parameters: the gauge couplings $g_i(i = 1, 2, 3)$, and the Higgs vacuum expectation value (vev) v. By measuring the deviations from the SM predictions, we can set constrains on the new physics. Especially, the S and T parameters which parametrize the new physics contributions to the gauge two point functions are sensitive to all particles that couple to the gauge bosons. Measurements of flavor changing neutral current (FCNC) constrain the existence of flavor off-diagonal interactions. Very precisely measured parameters sometimes exhibit significant deviations from the SM predictions. Currently muon anomalous magnetic moment deviates from the SM prediction by more than $3\,\sigma$. It is sensitive to the new physics that couples to muon.

The quadratic divergence of the Higgs sector exists if the divergence is estimated by the momentum cut off Λ, the upper bound of the various loop integral appearing in the radiative correction in the mass. We have to keep it in mind that the quadratic divergence does not depend on the external momentum, therefore it is a regularization dependent object. Especially in dimensional regularization, quadratic divergence is trivially zero. Then, is there any reason that we should take the fine turning problem seriously?

The fine turning argument based on momentum cut-off is justified in the case that the theory has large symmetry at some higher energy scale. For example, in the supersymmetric model, the regularization must respect to supersymmetry and one cannot subtract all quadratic divergence. To this end, the Higgs sector receives radiative corrections proportional to the SUSY scale (superpartner mass scale) under correct regularization. In the limit that superpartners are much heavier than SM particles, the low energy theory looks like the SM with the momentum cutoff at the SUSY scale. Fine turning arguments hold for the theories with an intermediated scale above which a new symmetry emerges.

There is another indication of the existence of new physics between the weak scale and the Planck scale. We may consider the Higgs potential at large field value in the SM and study the stability. The potential is a function of the top and Higgs masses, and current top and Higgs mass measurements favor metastable Higgs potential. There is not any reason that the Higgs vev should fall in such a metastable point, and this also suggests that additional particles that couple to the Higgs sector change the shape of the potential.

Another strong indication of new physics is the existence of dark matter in our Universe. Global fit of the cosmological observation favors the existence of stable, neutral particle, dark matter, which accounts for 27% of the total energy of our Universe. The existence of the dark matter is also confirmed by various observations of the stellar objects. Rotation curve of the stars of the galaxy indicates that galaxies are dominated by the non-luminous component. The is also a technique to measure the matters extended beyond the galaxy scale using gravitational lensing.

Our universe is 1.38×10^{10} years old, roughly 10^{17} s $\leftrightarrow 10^{-43}$ GeV^{-1}. The dark matter life time must be at least of the oder of the age of the Universe to remain in the current Universe. [1] On the other

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[1] In order to avoid the constraints coming from cosmic ray observations, the lifetime of the dark matter in our Universe must be significantly longer than the age of the Universe.

hand, a particle with mass m (GeV) with interaction suppressed by $1/M_{pl}$ has a decay width of order of $g^2 (m/1 \text{ GeV})^3 10^{-38}$ GeV. Namely the lifetime, $\tau \sim g^{-2} 10^{14} \text{s}/(m/1 \text{ GeV})^3$, would be much shorter than the life of our Universe ($\sim 4.3 \times 10^{17}$ s), where g is the coupling of the decay vertex. To account for the lifetime of the dark matter in our universe, its decay must be very strongly suppressed, or forbidden.

For the case of the SM particles, existence of stable particles is ensured by the symmetry. Electron is the lightest charged particle and electronic charge is conserved by the gauge symmetry. Proton is the lightest bound state of quarks. There are no interaction to break proton in the SM, because number of quark is conserved for interaction with the gauge bosons or the Higgs boson, and direct interaction with electron is forbidden by the gauge symmetry. It is possible to conserve the Baryon number 1/3 to the quarks in the SM, and this reflects the fact that proton is stable. To consider the particle model involving the stable (or long-lived) dark matter, we must introduce new symmetry to protect the dark matter from decaying.

Another puzzle of the SM is the hyper-charge assignments of the fermions. In the first glance, it is not easy to find the rules to assign the charge to the SM matters. But, it fits very nicely to the representation of a $SU(5)$ group, where $SU(3) \times SU(2) \times U(1)$ generators are embedded as

$$T^a_{SU(3)} = \begin{pmatrix} \lambda^a & 0 \\ 0 & 0 \end{pmatrix} \quad T^i_{SU(2)} = \begin{pmatrix} 0 & 0 \\ 0 & \sigma^i \end{pmatrix} \quad T_{U(1)} = \begin{pmatrix} -\frac{1}{3}\mathbf{1}_3 & 0 \\ 0 & \frac{1}{2}\mathbf{1}_2 \end{pmatrix}. \tag{1}$$

Here, λ^a and σ^i are the $SU(3)$ and $SU(2)$ generators, $\mathbf{1}_3$ and $\mathbf{1}_2$ are 3×3 or 2×2 unit matrix, and $T_{SU(3)}, T_{SU(2)}, T_{U(1)}$ satisfy the commutation relations of $SU(3)$, $SU(2)$, and $U(1)$ generators. Under this generator assignment, $\mathbf{5}^*$ and $\mathbf{10}$ representations of $SU(5)$ have a charge assignment as

$$\mathbf{5}^* = \begin{pmatrix} (\mathbf{3}^*, \mathbf{1})_{1/3} \\ (\mathbf{1}, \mathbf{2})_{-1/2} \end{pmatrix}, \tag{2}$$

while $\mathbf{10}$ representation is decomposed into $(\mathbf{3}, \mathbf{2})_{1/6} \oplus (\mathbf{3}^*, \mathbf{1})_{-2/3} \oplus (\mathbf{1}, \mathbf{1})_1$ which reside in the 5×5 antisymmetric matrix as

$$\mathbf{10} = \begin{pmatrix} (\mathbf{3}^*, \mathbf{1})_{-2/3} & (\mathbf{3}, \mathbf{2})_{1/6} \\ * & (\mathbf{1}, \mathbf{1})_1 \end{pmatrix}. \tag{3}$$

This suggests that $SU(3) \times SU(2) \times U(1)$ symmetry of the SM can be unified into the $SU(5)$ gauge symmetry. To realize this, the SM three gauge couplings must unify at the short distance, so that the $SU(5)$ symmetry is recovered above that scale. The gauge couplings at the short distance is calculated by utilizing the SM renormalization group equations from the low energy inputs. They do not unify for the particle content of the SM, therefore to realize the idea of GUT, new set of particles are needed. We will see a successful gauge coupling unification is realized in the Supersymmetric model in the next section.

2 Supersymmetry

Supersymmety is the symmetry exchanging bosons into fermion, and fermions into bosons. The generators of the supersymmetric transformation satisfy the following anti-commutation relations

$$\left\{ Q^\alpha, \bar{Q}_{\dot{\beta}} \right\} = 2\sigma^\mu_{\alpha, \dot{\beta}} P_\mu \tag{4}$$

Here Q is a spin 1/2 and mass dimension 1/2 operator and α and $\dot{\beta}$ ($= 1, 2$) are the spin indices of chiral and anti-chiral fermions, and $\sigma^\mu = (1, \sigma^i)$ is the Pauli matrices.

This anti-commutation relation can be reduced for any massive eigenstate $|a\rangle$ by taking the rest frame $P^\mu |a\rangle = m_a \delta_{0\mu} |a\rangle$ as follows:

$$\left\{ Q^\alpha, \bar{Q}_{\dot{\beta}} \right\} = 2\delta_{\alpha, \dot{\beta}} m_a. \tag{5}$$

Table 1: Particle content of the Minimal Supersymmetric Standard Model.

represenations	quark	squark
$(3,2)_{1/6}$	$q_L = (u,d)_L$	$\tilde{q}_L = (\tilde{u}_L, \tilde{d}_L)$
$(3^*,1)_{-2/3}$	u_R^c	$(\tilde{u}_R)^c$
$(3^*,1)_{1/3}$	$(d_R)^c$	$(\tilde{d}_R)^c$
	lepton	slepton
$(1,2)_{1/2}$	$l_L = (\nu,e)_L$	$\tilde{q}_L = (\tilde{\nu}_L, \tilde{e}_L)$
$(1,1)_1$	$(e_R)^c$	$(\tilde{e}_R)^c$
	Higgsino	Higgs
$(1,2)_{-1/2}$	$(\tilde{H}_1^0, \tilde{H}_1^-)$	(H_1^0, H_1^-)
$(1,2)_{1/2}$	$(\tilde{H}_2^+, \tilde{H}_2^0)$	(H_2^+, H_2^0)
	spin 1/2	spin 1
$(8,1)_0$	$\tilde{G}$ (gluino)	G^μ
$(1,3)_0$	$\tilde{W}$ (wino)	W^μ
$(1,1)_0$	$\tilde{B}$ (bino)	B^μ

The relation is same as that of a two-fermion system in quantum mechanics. One can construct an irreducible representation of this algebra starting from a state which annihilates any $\bar{Q}_i$. Suppose the state is spin 0, $|0\rangle$, all possible states are generated as follows;

$$|0\rangle \to Q_1|0\rangle, Q_2|0\rangle \to Q_1 Q_2|0\rangle. \tag{6}$$

Because $Q_1 Q_1 = Q_2 Q_2 = 0$, no more state can be obtained by multiplying the generator Q_i. Two spin 0 states and two spin 1/2 states are obtained. These states form a SUSY multiplet, and the spin 0 states are the superpartners of the spin 1/2 states and vise versa. Because this multiplet contains spin 1/2 states, we can regard this as a matter multiplet.

Starting from a spin 1/2 state annihilating $\bar{Q}$ one gets two spin 1/2 fermion states, a spin 1 massive bosonic states and a spin 0 bosonic state, namely 4 fermion degrees of freedom and 4 bosonic degrees of freedom. This may be regarded as two chiral fermions, one massive gauge boson and one massive Higgs boson. Repeating similar analysis to the massless particles, one obtains states with helicity $h = \lambda$ and $\lambda + 1/2$. If $\lambda = 1/2$, a massless gauge boson and its superpartner fermion make a supersymmetric multiplet. The number of bosonic degrees of freedom is the same as that of fermionic degrees of freedom in this theory.

All states in the above multiplet have the same mass, which looks irrelevant for describing real particles, but it is known that such mass degeneracy is removed by spontaneous supersymmetry breaking. Supersymmetry breaking is discussed in the next section.

The minimal supersymmetric standard model (MSSM) is an extension of the SM that has a supersymmetry in the limit where all particle masses are ignored. The model is thought to be an effective theory of a fully supersymmetic theory. Due to the spontaneous supersymmetry breaking of the full theory, the superparters of the SM particles receive a mass much higher than the SM particles. A superpartner of a fermion is called sfermion and it is a spin 0 particle. A superpartner of a gauge boson is called gaugino and has spin 1/2. A Higgs boson superpartner is called a higgsino and has spin 1/2. The particle content of the MSSM is given in Table 2. The SM particles and their superpartners have same charge, because the generator of supersymmetric transformation Q commutes with the SM $SU(3) \times SU(2) \times U(1)$ transformation. The number of Higgs doublets is two in the MSSM because one should add two Higgsinos, chiral fermions with charge $(1,2)_{\pm 1/2}$ in the SM because of a condition of anomaly cancellation.

As one can see from Table 2, the number of particles are doubled in the MSSM. The supersymmetry specifies all dimensionless couplings of interactions of new particles, such as four point interaction of

scalers and Yukawa couplings, while mass parameters of superpartners are undetermined. To understand the coupling relations, one needs to understand the supersymmetric field theory. In this lecture, I do not have enough time to talk about it in detail, so I just sketch the important elements.

Fields in the same supersymmetric matter multiplet can be arranged in a "chiral superfield" which is a function of coordinate x, θ and $\bar{\theta}$ a grassmanian Lorentz spinors with mass dimension $-1/2$,

$$\Phi(x, \theta, \bar{\theta}) = \phi(y) + \sqrt{2}\psi(y)\theta + F(y)\theta\theta, \tag{7}$$

where $y^\mu = x^\mu - i\theta\sigma^\mu\theta$. Note that by redefining the coordinate from x to y, Φ becomes a function of y and θ, and $\bar{\theta}$ does not appear. There are only three fields ϕ, F and ψ appearing as the component fields of Φ. When θ is zero, $\Phi(x) = \phi(x)$, therefore Φ is an extension of the scalar field of non-supersymmetric theory. On the other hand, $\Phi(y, \theta)$ represents both fermonic and bosonic fields simultaneously.

Φ is dimension 1, so that $\dim(\phi) = 1$ and $\dim(\psi) = 3/2$. F is then spin 0 and dim 2 field. The only $dim < 4$ kinetic term of F is FF^*, therefore F is not dynamical. The product of a chiral superfield is also a chiral superfield depending only y and θ. On the other hand, $\Phi\bar{\Phi}'$ is not a chiral superfield as it has the terms proportional to $\bar{\theta}$.

Just as operator P^μ, translation in coordinate space x is expressed as $\partial/\partial x$, supersymmetric transformation Q is a translation in the θ and $\bar{\theta}$ space. Namely, in the coordinate representation it is expressed as

$$S_\alpha = \frac{\partial}{\partial\theta^\alpha} + i(\sigma^\mu\partial_\mu\bar{\theta}). \tag{8}$$

The second term is needed to satisfy the SUSY algebra give in Eq. 4. With this transformation, each field transform as

$$\begin{aligned}
\delta_{SUSY}\phi &= \sqrt{2}\alpha\psi, \\
\delta_{SUSY}\psi &= -i\sqrt{2}\partial_\mu\sigma^\mu\phi\bar{\alpha} + \sqrt{2}F\alpha, \\
\delta_{SUSY}F &= -i\sqrt{2}\bar{\alpha}\partial_\mu\bar{\sigma}^\mu\psi,
\end{aligned} \tag{9}$$

where α and $\bar{\alpha}$ are transformation parameters. Under this transformation kinetic term

$$\mathcal{L}_{kin} = \partial_\mu\phi\partial^\mu\phi^* + i\bar{\psi}\sigma^\mu\partial_\mu\psi + F^*F \tag{10}$$

is invariant.

There are a few things worth paying attention. First The $\delta_{SUSY}F$ is total derivative. Because the product of chiral superfields is also a superfield, the $\theta\theta$ component $\mathcal{F}$ transforms as $\mathcal{F} = \partial_\mu J^\mu$, namely $\mathcal{F}$ can be interaction terms which are invariant under supersymmetric transformation. For example, $\Phi_1\Phi_2\Phi_3$ gives F term

$$L_{Yukawa} = F_1\phi_2\phi_3 + F_2\phi_1\phi_3 + F_3\phi_1\phi_2 - \psi_1\psi_2\phi_3 - \psi_2\psi_3\phi_1 - \psi_3\psi_1\phi_2. \tag{11}$$

The interaction contains Yukawa interaction term $y_{ijk}\psi_i\psi_j\phi_k$ which is symmetric under the exchange of i, j, k, and also the scalar potential terms proportional to $y_{ijk}F_i\phi_j\phi_k$ Combined with kinetic term FF^*, interactions of four point scalar fields proportional to y^2 is generated. The similar relations also holds for supersymmetric gauge interactions. The interaction between gaugino-fermion- sfermion is proportional the gauge coupling g, and there are scalar four point interactions proportional to g^2. While many scalar and fermion partners are introduced, there are no new dimensionless coupling introduced.

In addition to the F term, $\theta\theta\bar{\theta}\bar{\theta}$ term of general field product, $\mathcal{D}$ is supersymmetric. For example, supersymmetric kinetic term is $\theta\theta\bar{\theta}\bar{\theta}$ term of $\Phi\bar{\Phi}$.

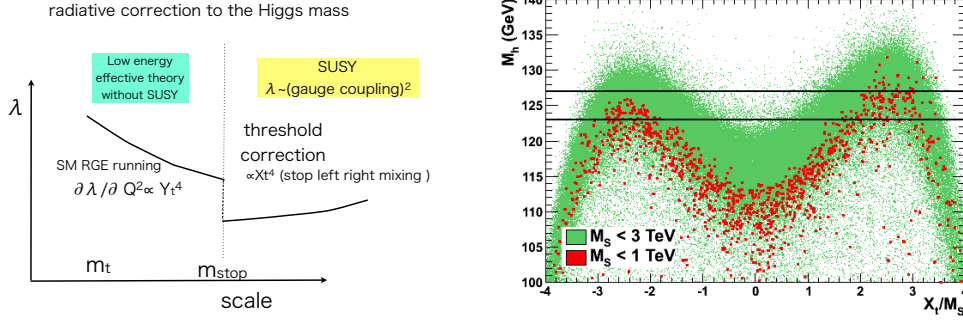We now address some important features of supersymmetric models.

Fig. 1: Left: the running of Higgs four point coupling changes at the scale of m_{stop}. Right: Maximal value of the Higgs boson mass as a function of X_t/M_{SUSY} when all the other parameter are scanned. From arXive 1311.0720.

– There are no quadratic divergence in the theory. The quadratic divergence coming from the top loop is canceled by the stop loop generated by the Higgs-Higgs-stop-stop four point interaction. Both of them are proportional to y^2. The Higgs four point coupling is proportional to the square of the gauge coupling, and quadratic divergence arising from the diagram is canceled by the gauge and gaugino-higgsino loops. This is because scalar particles are now in a same multiplet with the fermion, and the mass of the fermion is only logarithmically divergent. The fine-turning in the Higgs sector is now significantly reduced.

– Because the Higgs four point coupling is a gauge coupling, the Planck scale Higgs four point coupling is always positive, therefore significantly less in danger of running into metastable vacuum At low energy the Higgs mass is upper bounded by the Z boson mass in tree level, and radiative corrections proportional to the $(mt^4/m_W^2)\log(m_{\tilde{t}}/m_t)$ appear in the Higgs boson mass formulae. This correction is interpreted as the running of the Higgs boson four point coupling from the stop mass scale to the top mass scale under the SM renormalization group equation, because below the stop mass scale, the theory is effectively the SM. In addition there are contribution proportional to the fourth power of stop left-right mixing X_t. See Fig. 1 (left) for the RGE interpretation of the radiative corrections to the Higgs mass. In this theory, the Higgs boson mass is calculated from the scalar top mass and its mixing, therefore the SUSY scale is predicted from the Higgs boson mass. In other words, the measured Higgs boson mass gives a strong constraint to the SUSY mass scale and mixing. See Fig. 1 (right).

– In the SM, one cannot write an interaction violating baryon and lepton numbers due to the gauge invariance. This is no longer true because Higgsino and lepton doublets have same quantum numbers. The product of superfields W whose $\theta\theta$ terms is the SM Yukawa interactions

$$W = -y_e H_1 \cdot E^c L - y_d H_1 \cdot D^c Q - y_u H_2 \cdot U^c Q - \mu H_1 \cdot H_2, \tag{12}$$

where $Q = \tilde{q}_L + \theta q_L...$, $U^c = \tilde{u}_R^c + \theta u_R^c$...... are the superfields whose bosonic component is a sfermion and a fermionic component is quarks or leptons. However, $\theta\theta$ term of W'

$$W' = \epsilon_L LLE^c + \epsilon_{BL} LQD^c + \epsilon_B U^c D^c D^c + \epsilon_{LH} LH_2 \tag{13}$$

is not forbidden by the gauge symmetry, because H_1 and L have a same quantum numbers, and $UDD = \epsilon_{abc} U^a D^b D^c$ is a gauge singlet. The interactions violate lepton and/or baryon numbers and should be forbidden.

The symmetry that forbids L and B violating terms is called the conserved R-parity. In the MSSM R-parity may be assigned to the superfield and coordinate θ as follows,

$$R(L) = R(E) = R(Q) = R(U) = R(D) = -1, R(H) = 1, R(\theta) = -1. \tag{14}$$
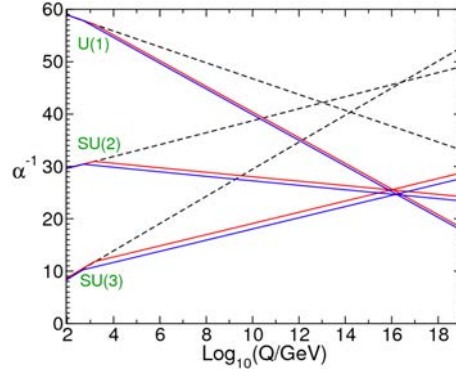
Fig. 2: Two loop renormalization group evolution of the gauge couplings in the SM (dashed lines) and MSSM (solid lines) from "A Supersymmetry primer" hep-ph/9709356.

In this assignment, all the SM particles have $R = 1$ and all superpartners have $R = -1$, and $R(W|_{\theta\theta}) = 1$, and $R(W'|_{\theta\theta}) = -1$. The interaction term from W multiplicatively conserves R parity, namely, product of R parity of all particles involved in a vertex is one. Namely, $R = -1$ particle decays into the final states which contains odd number of $R = -1$ particles. If two $R = 1$ particle collides, the final state contains even number of $R = -1$ particles. By requiring multiplicatively conserved R parity, the lightest supersymmetric particle (LSP) becomes stable. The LSP can be a dark matter candidate.

– Gauge coupling: In the supersymmetric model, the number of particles is doubled and running of the gauge couplings would be modified above the SUSY particle mass scale. The gauge couplings unify at the GUT scale much better than that of the SM as can be seen in Fig. 2. This means "supersymmetric GUT" is consistent with experimental data, though there are still some fine turning issues when we consider the Higgs sector violating GUT symmetry.

3 Origin of SUSY breaking

As we have mentioned already, the MSSM is not a complete theory, because it requires a mechanism to break the supersymmetry somewhere outside the MSSM. A general set up of the SUSY breaking models are the following; there are hidden sector H, and fields Z_i in the sector H break the supersymmetry spontaneously. This hidden sector couples to our sector indirectly though a messenger sector. The particles in the messenger sector have a mass scale M.

The spontaneous symmetry breaking is realized for the vacuums which do not annihilate with the supersymmetric generator Q and $\bar{Q}$. If such a vacuum exists, there are some fermions ψ whose supersymmetric transformation $\delta_{SUSY}\psi = \{Q, \psi\}$ has non-zero vev, namely $\langle 0|\delta_{SUSY}\phi|0\rangle = -\sqrt{2}\langle 0|F|0\rangle \neq 0$. Some of the superfields in the Hidden section must have non-zero F terms in our setup.

If F term of Z has non zero vev, $\langle Z \rangle = \langle F_Z \rangle\theta\theta$, various mass terms are induced in the low energy effectively. A simple example is $\theta\theta\bar{\theta}\bar{\theta}$ term of $Z\bar{Z}\Phi\bar{\Phi}/M^2$, which may be induced through the messenger interactions. After the symmetry breaking the term $(\langle F \rangle^2/M^2)\phi\phi^*$ is the effective SUSY breaking mass term of the scaler boson ϕ.

There are already severe constraints to the interaction of the messenger sector to the MSSM sector. These constrains come from the flavor changing neutral currents such as K^0-$\bar{K}^0$ mixing. The constraints typically require

$$\left[\frac{10\text{TeV}}{m_{\tilde{q},\tilde{g}}}\right]^2 \left[\frac{\Delta m^2_{\tilde{q}_{12}}/m^2}{0.1}\right]^2 < 1, \tag{15}$$

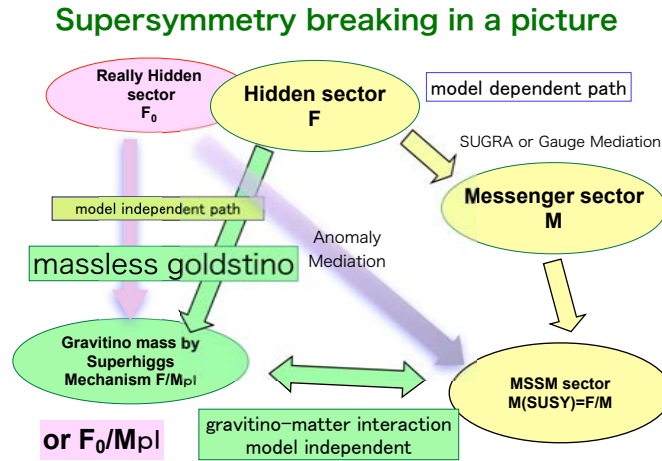## Supersymmetry breaking in a picture

Fig. 3: Relation between the MSSM sector and SUSY breaking sector.

where $m^2_{\tilde{q}_{12}}$ is a mixing parameter of the first and second generation squark, and m^2 is diagonal squark masses. The SUSY breaking sector H therefore must couple to the MSSM matter sector universally.

Several mechanisms have been proposed to assure the universality of the soft scalar masses. The supergravity model uses the gravity interaction as the messenger mechanism, on the other hand, gauge mediation models uses some vector-like matters charged under the SM gauge groups as the messenger fields. Even if there are no direct couplings between the MSSM and SUSY breaking sectors, there are mediation mechanism through the superconformal anomaly, and the model utilizing this is called anomaly mediation model.

It is difficult to access the HIdden sector directly. The SUSY breaking of the total theory F_0 and mass of the gravitino(super partner of graviton) $m_{3/2}$ is related as $m_{3/2} = F_0/M_{pl}$. The gravitino could be the LSP, in that case the next lightest SUSY particle(NLSP) is long-lived. The NLSP can be detected directly at the collider, the decay lifetime provide the information of hidden sector SUSY breaking. If gravitino is not the LSP, the gravitino can be long-lived and may have impact on big-bang neucleosynthesis. See Fig. 3.

The mediation mechanism sets the sparticle mass parameters at the mediation scale, and on-shell masses of the SUSY particles are obtained by running the RGE equation of the masses down to the low energy scale. If the boundary condition is universal at M_{GUT}, squark and gluino masses are much heavier than those of electroweakly interacting superpartners such as sleptons, wino, bino and Higgsinos. The square of Higgs mass parameter is driven to be negative at the weak scale, and Higgsino mass parameter μ compensates it so that the Higgs vev is the correct value. The cancellation between μ and SUSY breaking parameters at the weak scale is a measure of the fine turning in the Higgs sector. See Fig. 4.

4 Collider search of supersymmetric particles

So far, a proton-proton collider at CERN, the Large Hadron Collider (LHC), has collected ~ 30 fb^{-1} of integrated luminosity for each experiment at 7 to 8 TeV. It will start operation again from 2015 aiming for 300 fb^{-1} at 13 TeV.

A proton is a composite particle and quarks and gluons in the proton are the elementary particles that are involved in the high energy scatting process. The momentum of the quarks and gluons are parallel to the beam direction but the absolute values are not fixed. Therefore the collision system is boosted to one of the beam directions. The production cross section is generally the highest near the threshold. It
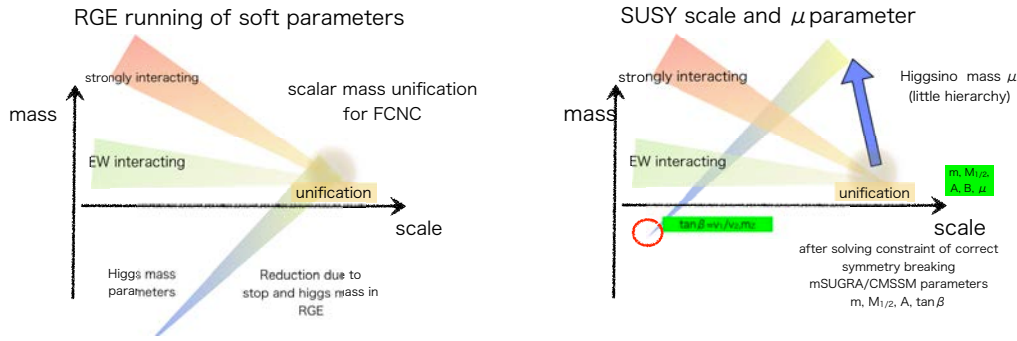
Fig. 4: Relation between the MSSM sector and SUSY breaking sector.
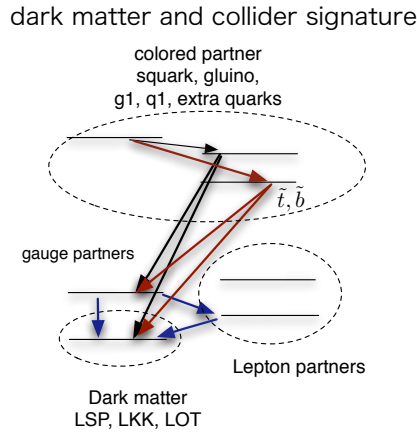

Fig. 5: The decay pattern of squark and gluino produced at the LHC, and particles emitted from the cascade decay chain. The particles in the Little Higgs model with T parity or universal extra dimension model may also give a similar signature.

reduces gradually with the increase of the parton collision energy $\sqrt{s}$. The quarks and gluons in the final state are fragmented and hadronized into hadrons, forming the jets. Electroweakly interacting particles W, Z, γ, leptons and neutrinos are also produced from various production processes.

Colored superpaticles are copiously produced at the hadron collider. Due to the conserved R-parity of the MSSM, superpartners are produced in pairs, each superpartner decays to the final state involving another superpartner, and at the end of the cascade decay, the LSP appears. The LSP is stable. Due to the cosmological constraints, it is neutral and color-singlet, and escapes detection. If the mass difference between the superpartners are large, the decay product tends to have high p_T. In such a case, the LSP, which cannot be detected directly, is also relativistic (See Fig. 5). The sum of LSP momentum transverse to the beam direction is balanced against other visible particles. Namely, significant missing transverse momentum $\mathbf{P}_{Tmiss}$ defined as

$$\mathbf{P}_{Tmiss} = -\sum_i \mathbf{p_T}_{\text{jet}}^i + \sum_j \mathbf{p_T}_1^j, \qquad (16)$$

is a signature of SUSY particle production. Another important quantity is the sum of absolute values of
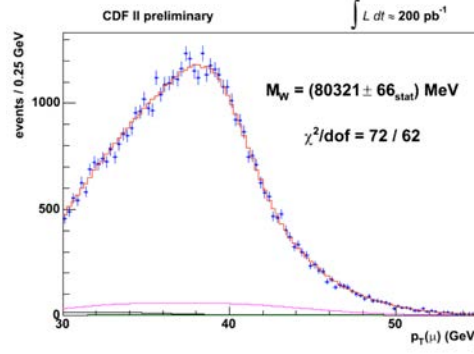
Fig. 6: Distribution of p_T^μ from the W boson decay measured at CDF experiment at Tevatron.
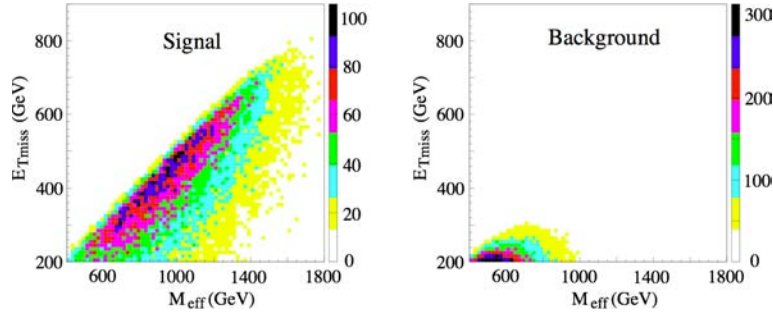

Fig. 7: Distribution of m_{eff} and E_{Tmiss} of the top partner pair production at the LHC followed by the decay into top and stable neutral gauge boson (left) compared with the $t\bar{t}$ distribution.

the transverse momentum

$$H_T = \sum_i p^i_{\text{Tjet}} + \sum_j p^j_{\text{Tl}}, \tag{17}$$

or the effective mass

$$m_{eff} = \sum_i p^i_{Tjet} + \sum_j p^j_{Tl} + E_{Tmiss}, \tag{18}$$

where E_{Tmiss} is the absolute value of missing transverse momentum.

The m_{eff} distribution peaks at the sum of the produced particles at the hard process. To observe this fact, let us first consider the p_T distribution of leptons from W boson decay produced at CDF experiment at Tevatron, a $p\bar{p}$ collider at 1.8 TeV. The distribution peaks at 40 GeV, which is a half of the W boson mass. See Fig. 6. The feature is easily understood when we calculate the p_T distribution of spherically decaying W boson boosted to the beam direction,

$$f(x)dx = \frac{2}{\sqrt{1-x^2}}dx, \tag{19}$$

where $p_T = (m_W/2)\sin\theta = xm_W/2$; The distribution strongly peaks at $p_T = m_W/2$ ($\sin\theta = 1$) and the structure remains even though W bosons are boosted transversely in the realistic situation, because the production cross section is largest near the threshold. The fact applies to all production processes at the hadron collider; the sum of the p_T of the decay products peaks near the parent's mass. When heavy particles are produced in pairs, the sum of the p_T of the decay products peaks at the sum of the produced particle masses.

Fig. 7 compares the distributions of T_-T_- and $t\bar{t}$ pair productions. Here a hypothetical particles T_- is assumed to decay into t and B_H, and B_H is a neutral stable massive $U(1)$ gauge boson. The signal
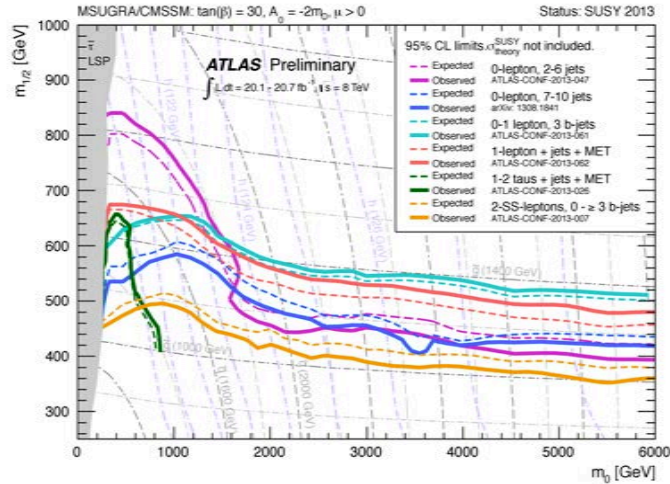
Fig. 8: Latest mass limit of the MSSM squarks and gluinos shown as a function of GUT scale gaugino mass and scaler mass. Presented in SUSY2013.

contains $t\bar{t}$ and existence of two B_H's is observed by the missing transverse momentum of the events, namely, the signal is similar to that of superpartner pair production. The signal production cross section is $\mathcal{O}(1)$ pb, while the $t\bar{t}$ production cross section is huge at the LHC, around 800 pb. If the distribution overlaps significantly, the signal is very difficult to be observed. However, the signal m_{eff} distribution peaks around 1 TeV and missing momentum as close as half of the M_{eff}, while the background peaks around $m_{eff} \sim 400$ GeV and $E_{Tmiss} \ll M_{eff}/2$. Because of this distribution differences, the T_- signature with the production cross section much less than 1 pb may be observed at the LHC.

So far we have been talking about "inclusive" quantity. They are defined using all objects in an event. We may also select jets or leptons with special features and use kinematical information to separate signals and backgrounds. Let us consider events with one lepton and some missing momentum. The event with one lepton + multiple jets + missing momentum is an important signature of superpartner production. However, events involving W boson also produce such signatures. However, the events with W boson can be reduced significantly if we require that m_T of a lepton and missing p_T is above 100 GeV where m_T is defined as

$$m_T = \sqrt{2p_T^l E^{Tmiss}(1 - \cos(\Delta\phi(l, p_T)))} \tag{20}$$

The cut significantly reduces the background from the W boson production to the SUSY process.

The current bound of the SUSY process is obtained after successful reduction of background using the above kinematical variable. The understanding of background distribution is quite important, especially the cross section of W,Z, $t\bar{t}$ with multiple jets must be correctly calculated. The techniques to obtain multiple jets amplitudes with parton shower has been established only this century, and current SUSY searches at the LHC is benefitted by those techniques greatly. The current limit typically excludes squark with mass 1.8 TeV and gluino with mass less than 1.4 TeV, if the mass splitting between the LSP and colored SUSY particles are large enough. See Fig. refsusylimit for the latest limits.

5   Dynamical symmetry breaking and BSM

Supersymmetry is not a unique solution of the hierarchy problem. Another important class of solutions is dynamical symmetry breaking models. When a global symmetry is broken spontaneously, a massless scalar modes (Nambu-Goldstone boson) appears, even if the theory does not have an elementary Higgs boson. An important example is chiral symmetry breaking in QCD. The QCD Lagrangian has $SU(2)_L \times SU(2)_R$ symmetry when quark masses are ignored. The symmetry is spontaneously broken to $SU(2)_V$

dynamically, and the Goldstone boson of the symmetry breaking are pions $\pi \sim \bar{q}_i \gamma_5 q'$, and $\langle \bar{q}q \rangle$ has non-zero vev.

The pion has the same charge as the Goldstone boson in the Higgs sector. Therefore, it is natural to consider scale up of the mechanism. The model involves a set of new quarks Q with EW charges, but couple to different asymptotic free gauge interactions whose couplings blow up at the scale of EW symmetry breaking. If $\bar{Q}Q$ condense, the light $\bar{Q}\gamma_5 Q$ states work as the Goldstone bosons of the EW symmetry breaking. This class of the model called Technicolor model. The model has no quadratic divergence because the massless bound states only appear in the low energy effective theory.

This is an interesting and beautiful idea, but is not consistent with precision EW observations. At LEP, gauge boson two point functions were precisely measured. Especially the parameter called S, receives non decoupling contribution from $SU(2)$ doublets Q which is colored in the new strong interactions, and also necessary charged under $SU(2) \times U(1)$ symmetry in the SM to break the gauge symmetry. Their contribution appears constructively to the gauge two point functions, and therefore the model is tightly constrained. In addition, these models tend to predict a heavy Higgs boson inconsistent with the data.

Another class of models called "composite Higgs models" allows a Higgs boson which is light but non-elementary. In these models, the Higgs doublet itself is a pseudo Goldstone boson of some dynamical symmetry breaking. Though the mechanism of dynamical symmetry breaking is not specified, the smallness of the mass of the Higgs boson is thought to be ensured by the global symmetry of the theory. The model requires extension of the top sector because the top Yukawa coupling violates the desired global symmetry strongly. The extended top sector is a target of extensive ATLAS and CMS searches.

6 Extra dimension models

In the Extra dimension models the space has more than three dimensions but the additional space dimension is compactified with a small size R so that we could not recognized it easily. When the extra dimension is flat, the fields in the extra dimension may satisfy the periodic boundary condition such as

$$\phi(x, y) = \phi(x, y + R), \tag{21}$$

where x represents four dimensional space time, while y is the fifth dimension. Under this boundary condition, the wave function is expressed as

$$\psi(x, y) = \psi'(x) \exp(ip_5 y), \tag{22}$$

where $p_5 R = 2\pi n$ (n is an integer). This leads to an equation of motion of a free particle propagating in the the fifth dimension,

$$E_n^2 = p^2 + p_5^2 = p^2 + (2\pi)^2 \left(\frac{n}{R}\right)^2. \tag{23}$$

Namely, the model predicts an infinite tower of particles of the four dimensional effective theory, which corresponds to different values of the discrete momenta in the fifth direction.

The coupling of the fifth dimension related with the couplings in the four dimensional effective theory in non-trivial manner. A simple example is the gauge coupling of the fifth dimensional theory and the four dimensional effective theory,

$$\int d^4 x dx_5 \frac{1}{g_5^2} F_{\mu\nu} F^{\mu\nu} \rightarrow \int d^4 x \frac{1}{g_4^2} F_{\mu\nu} F^{\mu\nu}, \tag{24}$$

where $g_4 = g_5/\sqrt{R}$. Larger the size of the fifth dimension is, g_4 becomes small. This is also true for gravitational interactions. The four dimensional gravitational interaction may be small because the size

of extra dimension is large. The Large extra dimension model tried to solve the fine tuning problem by making true Phanck scale in the higher dimensional theory much smaller than the M_{pl}.

The extra dimension may not be flat. In the RS model, the fifth dimension has non-trivial metric as follows:

$$ds^2 = e^{-2\sigma(\phi)}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\phi^2, \tag{25}$$

where $\phi = 0$ and π is the boundary of the fifth dimension. The gravity action in the bulk is expressed as

$$S_{gravity} = \int d^4x \int_{-\pi}^{\pi} d\phi \sqrt{-G} - \Lambda + 2M^3 R, \tag{26}$$

when the $\sigma(\phi)$ is expressed as

$$\sigma(\phi) = r_c |\phi| \sqrt{\frac{-\Lambda}{24M^3}}, \tag{27}$$

provided appropriate fine tuning of the boundary actions.

The geometry allows us to control the masses of SM particles. If the Higgs boson is at $\phi = \pi$ boundary (which is called visible brane), the kinetic term is expressed as

$$S_{vis} = \int d^4x \sqrt{-\bar{g}_{vis}} e^{-4kr_c\pi} \times \left\{ g_{vis}^{\mu\nu} e^{2kr_c\pi} D_\mu H^\dagger D_\nu H - \lambda(|H|^2 - v_0^2)^2 \right\}. \tag{28}$$

The mass term receives the suppression factor of $e^{-kr_c\pi}$ after rescaling the Higgs field so that they have canonical kinetic terms. By adjusting parameters one can easily obtain the mass of the SM particle of the oder of the EW scale while all parameters of the fundamental fifth dimensional Lagrangian are of the order of M_{pl} without fine turning.

The model predicts towers of KK particles with mass of the order of $\Lambda_\phi = \sqrt{6}M_{pl}e^{-kr_c\pi}$ for the particles living in the fifth dimension(bulk). A popular set up of the model is that all the SM fermions are the zero mode of the particles living in the bulk, and the Higgs boson lives in the IR brane. Mass term of the fifth dimensional Lagrangian of the SM model matters control the profile of the fields in the bulk. One can adjust the mass so that light (heavy) quarks and lepton have small (large) overlap with the IR brane so that Yukawa couplings in the four dimensional effective Lagrangian is realized without introducing too much hierarchy among the interactions between the Higgs boson and the bulk fermions. There are on-going search of the KK gauge bosons and KK fermions at the LHC, however, FCNC constraints require $\Lambda_\phi > 10$ TeV already, and it is unlikely that these new particles will be found at the LHC.

7 Suggested reading

To those who is interested in Supersymmetry, a good review for start with is S. P. Martin, "A Supersymmetry primer," In *Kane, G.L. (ed.): Perspectives on supersymmetry II* 1-153 [hep-ph/9709356]. For A review of composite Higgs model, I suggest R. Contino, "The Higgs as a Composite Nambu-Goldstone Boson," arXiv:1005.4269 [hep-ph].