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V.I. Fal'ko, I.S. Shapiro

THE TRIPLET PAIRING IN NUCLEI

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* Institute for Solid State Physics, Chernogolovka

Summary

The classification of phases of triplet superfluidity is offered for the case of strong spin-orbital coupling. The two-gap character of quasiparticle spectrum is shown . Some new collective modes in nuclear spectra are discussed. The asymptotic formulas for nuclear moment of inertia are obtained in high spin region. The theoretical results are compared with experimental data.

Introduction

Triplet cooper pairing (spin of the pair S=1) in nuclei was not so far investigated *). Meanwhile, the properties of nucleon interactions don't exclude it. That is why it is interesting to find out, what physical effects triplet pairing in nuclei could induce.

The three groups of qualitative consequences of triplet pairing draw the peculiar attention:

(1) the enrichement of spectra of collective excitations of nuclei owing to the variety of phases of triplet pairing;

(2) the anisotropy of triplet superfluid. It is remarkable that the anisotropy is not connected with ad hoc introdced nonspherical effective potential;

(3) the two-gap structure of quasiparticle excitation spectrum for some superfluid phases.

The theory of triplet pairing was developed in connection with superfluidity of He^3 /3/. In this liquid the spin-orbital coupling is weak and in the first approximation could be neglected **). In nuclear matter the spin-orbital coupling is strong and it is the reason for differences between superfluid phases of these two liquids.

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*) The possibility of triplet pairing in neutron stars was discussed in /1/ and some other works (see ref. in /1/). The first suggestion about superfluidity in neutron stars was in ref. /2/.

**) We call spin-orbital coupling not only the spin-orbital interactions, but also tensor forces.

In nuclear physics we are interrested not only in ground state (which corresponds to the minimum of free eenergy) but also in excited (quasistationary) states, which could correspond to different superfluid phases. In section 1 we represent the whole phase analysis for the case of triplet pairing with strong spin-orbital coupling.

The wavefunctions of nuclei in different phases must be "strongly orthogonal" (for example, the radiative transitions between them must be hindered). Hence we obtain some nonintercombining bands in nuclear spectra. In section 2 we investigate the level of the strong orthogonality. We construct the wavefunctions by means of so called quasispin method.

The next section3 is devoted to quasiparticle excitations. The phases with two-gap spectra are enumerated there and the structure of two-gap multiplets for even nuclei is described. The multipolarity of electromagnetic transitions within multiplet components is established too.

In section 4 we consider the rotational spectra of nuclei with triplet (or nontrivial singlet) pairing. We obtain formulas for high-spin behavior of nuclear moment of inertia in different phases and make attempt to describe some experimental data.

In section 5 we discuss the order parameter boundary conditions.

We want to emphasize that in present paper we don't consider the concrete microscopic nuclear models which make conditions for triplet pairing. We understand our aim in picking out the possible consequences of hypothetical triplet pairing in nuclei.

1. The phase analysis

First, we remind briefly some basic points of the theory, though all this is represented in reviews /3/ mentioned above.

The matrix element

$$\langle N | \hat{\Psi}_\alpha (\vec{r}_1) \hat{\Psi}_\beta (\vec{r}_2) | N+2 \rangle = f_{\alpha\beta} (\vec{r}, \vec{\xi}) \qquad (1.1)$$

is called the wave function of cooper pair. Here N is the number of identical fermions (neutrons or protons), $\hat{\Psi}(\vec{r})$ are the fermion annihilation operators, α and β are spinor indices; $\vec{\xi}$ and r are the coordinates of relative motion and center of mass coordinates of particles in pair:

$$\vec{\xi} = r_1 - r_2 \qquad ; \qquad r = (r_1 + r_2)/2$$

To extract the scalar (S = 0) and vector (S = 1) parts in pair wavefunction we must represent $f = (f_{\alpha\beta})$ as a sum of Pauli matrixes

$$f = a(\vec{r}, \vec{\xi}) i\sigma_2 + \vec{b}(\vec{r}, \vec{\xi}) i\sigma_2 \vec{\sigma} \qquad (1.2)$$

with

$$a(\vec{r}, -\vec{\xi}) = a(\vec{r}, \vec{\xi}) \qquad ; \qquad \vec{b}(\vec{r}, -\vec{\xi}) = -\vec{b}(\vec{r}, \vec{\xi}) \qquad (1.3)$$

Only the states with odd (even) orbital momentum L are possible in the case of triplet (singlet) pairing. Later we shall be restricted with triplet p-pairing (L = 1).

If spin-orbital coupling is strong, orbital and spin angular momenta must be combined into total angular momentum of the pair **J**, which can take values 0, 1, 2. So, the cooper pair's state is characterized with angular momentum J and it's projection on some axis. The superfluid state is the bose-condensate of cooper pairs, and each phase is defined by the choice of J and M . The phases, differing in sign of M only, are equivalent (because $\vec{b}$ have equal rights with $\vec{b}^*$). Such qualitative consideration gives us 6 superfluid phases /5/. The concrete form of wavefunctions for each phases will be constructed under formalism proposed later.

Under fixed orbital quantum number L it is expediently to extract angular variables $\hat{\vec{\rho}} = \vec{\rho}/\rho$. In phenomenological analysis we can ignore an incidental ρ dependence and rewright

$$\vec{b}_j(\vec{r},\vec{\rho}) = B_{ij}(\vec{r})\,\hat{\rho}_i \qquad ; \qquad i\ ,\ j = 1,\ 2,\ 3 \tag{1.4}$$

The complex tensor B is called order parameter (OP). The energy of the system $\mathcal{E} = F[B]$ is a functional on OP. In strong spin-orbital coupling limit F is invariant under two kinds of OP's transformations. Firstly, F stays unchangeable under SO(3) rotations of OP. At second, F is real value and then it stays unchangeable under U(1) transformations $B \to Be^{i\Phi}$. So we can wright the whole symmetry group of F as $G = SO(3) \otimes U(1)$:

$$F[gB] = F[B] \quad , \quad g \in G$$

Two OPs B and B' , which could not be transformed one into another with some G-transformation ($B' \neq gB$), obviously describe different phases. Particularly, irreducible tensor $B^J_{i,j}$ with different J describe different phases.

The degeneration space gB (all OPs belonging to one phase) is equivalent to the co-space G/H of group G with maximal stable subgroup H of tensor B :

$$H\ B = B \tag{1.5}$$

The subgroup H consists of discrete and continuous parts. The continuous one may be constructed on

$$h_z = I_z^J + iM\,\partial/\partial\phi \qquad (1.6)$$

generators only. The operator I_z^J is the generator of rotations around axis $\hat{z}$,and $i\partial/\partial\phi$ is the generator of U(1) transformation $B \to Be^{i\phi}$.

The OP's structure we obtain after resolving the equation

$$h_z B = 0, \qquad (1.7)$$

which immediately follows from (1.5). We can resolve (1.7) for integer $M = -J, \ldots, J$ only, because $I_z^J B^{JM} = MB^{JM}$.

The OP in (J , M) phase remains unchangeable under rotations around axis z with simultaneous multiplication all B-components by $e^{iM\phi}$. Because an arbitrariness in direction of $\hat{z}$ the (J , M) and (J ,-M) phases are equivalent. So we see that the formal analysis leads us to the same results as the preliminary consideration.

Resolving equation (1.7) we find a tensor structure of OP and then the discrete part H_D of the stationary subgroup H . The knowledge of H makes possible to establish the degeneration space. The results of the calculations are presented in table 1. The homotopy groups of the degeneration space π_1 and π_2 are also described there. It s nontriviality means the possibility of existence of stable vortices (π_1) and (π_2) - singular points in the bulk of the liquid.

We note, that in all ($J \neq 0$) phases the nuclear superfluid is anisotropic. The axis of pair's angular momentum quantization is physically picked out. So in nuclei with triplet pairing the spherical symmetry is spontaneously broken. (In (J , M = 0) - phases only the axial symmetry remains.)

2. Wave functions of nuclei with triplet pairing

The wave functions (WF) $|J\,M\rangle$ of a nuclei in different phases (J , M) must be "strongly orthogonal" because of necessity of global changes in whole condensate's state. It means that matrix elements

$$\langle J'\,M'|\,a_1^+ \cdots a_n^+\, a_{n+1} \cdots a_{2n}\,|\,J\,M\rangle \tag{2.1}$$

of product of any finite number of quasiparticle operator is small exponentially in particle number N. In macroscopic systems values (2.1) are negligibly small. But in nuclei $N \sim 10^2$, and we must estimate the level of hindering of interphase γ - transitions.

We construct the triplet pairing wave functions by the quasispin method. I.E. we perform the hamiltonian $\hat{H}$ of the system as a sum of some Lee group generators $\vec{I}_k$

$$\hat{H} = \sum_k \vec{\alpha}_k \vec{I}_k \tag{2.2}$$

In (2.2) $\vec{k}$ is the quasiparticle momentum, α_k are numerical functions of $\vec{k}$. (We suggest that pairs consist of quasiparticles with opposite momenta).

The hamiltonian $\hat{H}$ we diagonalize with transformation $\mathcal{D}(\omega_k)$ from quasispin group.

$$\mathcal{D}(\omega_k)\,\vec{I}\vec{\alpha}\,\mathcal{D}^+(\omega_k) = (\,\ldots\,\lambda_n(k)\,\ldots\,) \tag{2.3}$$

The eigenfunction $|J\,M\rangle$ of the hamiltonian are obtained from physical vacuum state $|0\rangle$ with the "rotation"

$$|J\,M\rangle = \prod_{k} \mathcal{D}(\omega_k)\,|0\rangle \tag{2.4}$$

One could use the group SO(5) for this aim, as has been done in ref./6/. But we shall demonstrate that the general case could be described by simpler group SO(3) $\otimes$ SO(3).

Since nucleus size is sufficiently larger than $1/k_F$ (the inverse Fermi momentum) we can classify the quasiparticle states with momentum quantum number. On the other hand, the correlation length is larger than the nucleu's radius and we are interested in homogeneous superfluid states only ($B(r) = \text{const}$). We introduce the hamiltonian corresponding to such p-pairing states in the form

$$\hat{H} = \hat{H}_0 - G_{lml'm'} \sum_{kk'} \hat{k}_l (i\sigma_2\sigma_m)_{\alpha\beta}\, a^+_{k\alpha} a^+_{-k\beta} \otimes$$

$$\otimes\, \hat{k}'_{l'} (-i\sigma_{m'}\sigma_2)_{\alpha'\beta'}\, a_{-k'\alpha'}\, a_{k'\beta'} \tag{2.5}$$

$$\hat{H}_0 = \sum_{k} \epsilon_k\, a^+_{k\alpha}\, a_{k\alpha} \tag{2.6}$$

(We reckon the quasiparticle energy ϵ_k from the Fermi-level's energy.) In (2.5) the tensor G contains effective constants g_J of short range interactions in channels with different angular momenta $J = 0, 1, 2$ of pairs of interacting particles:

$$G_{lml'm'} = g_0 \delta_{lm}\delta_{l'm'} + g_1 e_{lmn} e_{l'm'n} + g_2 \left(\delta_{ll'}\delta_{mm'} + \delta_{lm'}\delta_{ml'} - \frac{2}{3}\delta_{lm}\delta_{l'm'} \right) \tag{2.7}$$

The sum on $\vec{k}$ in (2.5) includes $k_3, k_3' > 0$ only.

We suppose, that the system prevents some J - pairing. Since we shall work with approximate hamiltonian

$$\hat{H}' = \hat{H}_0 - g_J \sum_k \{[i\sigma_2 \vec{\sigma}\vec{b}(k)]_{\alpha\beta} a^+_{k\alpha} a^+_{-k\beta} + e.c.\} + g_J \, Sp \, B^+ B \tag{2.8}$$

For it s eigenstates

$$\hat{H}' | f \rangle = E_f | f \rangle \tag{2.9}$$

we shall look among coherent states

$$\hat{\Psi}_\alpha(\vec{r}_1) \, \hat{\Psi}_\beta(\vec{r}_2) | f \rangle = f_{\alpha\beta}(\vec{\rho}, \vec{r}) \, | f \rangle \tag{2.10}$$

In expression (2.8) we use OP B_{ij} normalized as

$$B_{ij} = \frac{\partial b_j(\vec{\rho})}{\partial \rho_i} \Big|_{\rho \to 0} \tag{2.11}$$

Vector $\vec{b}(k)$ is connected with OP with an equation analogous to (1.4)

$$b_j(k) = k_i B_{ij} \tag{2.12}$$

To distinguish a convenient quasispin group structure for the hamiltonian (2.8), we must know commutation relations between it s terms. In the triplet case the spin matrix construction takes place in (2.8) and the commutator

$$[\vec{\sigma}\vec{b}, \vec{\sigma}\vec{b}^*] = 2i \vec{\sigma} (\vec{b} \times \vec{b}^*) \tag{2.13}$$

plays an important role.

The transformation of the tensor B^{JM} under rotations around axis $\hat{z}$ follows the law (1.8). From expressions (1.8) and (2.12) we ascertain that $\vec{b}$ is collinear to $\vec{b}^*$ in (M = 0) - case.

Hence $\vec{b}\times\vec{b}^* = 0$. Therefore $(M = 0)$-case is almost identical to singlet pairing, because the presence of spin matrix does not influence on dinamical group. In $(M \neq 0)$-case $\vec{b}$ and $\vec{b}^*$ change under rotations in different way, and vector $\vec{b}\times\vec{b}^*$ differs from zero. As a result, the quasispin group in $(M\neq 0)$-case differs from $M = 0$ one, and we shall consider these cases separately.

Now we introduce operators

$$I_+(\vec{k}) = \frac{1}{2} a^+_{k\alpha} a^+_{-k\beta} (i\sigma_2 \vec{\sigma})_{\alpha\beta} (\hat{z} + i\hat{y}) \quad ; \quad I_- = I_+^+ \tag{2.14}$$

$$I_3(\vec{k}) = \frac{1}{4} \sum_{q=\pm k} (a^+_{q\alpha} a_{q\alpha} + \vec{\sigma}_{\alpha\beta}\, \hat{x}\, a^+_{q\alpha} a_{q\beta}) - \frac{1}{2} \tag{2.15}$$

with $\hat{x}\perp\hat{y}\perp\hat{z}$ (these vectors are defined in Table 1). The three operators satisfy the commutation relations of the Lee algebra of the group SO(3). In addition,

$$I_+^2 = I_-^2 = 0$$

and the weights of it s irreducible representations could be 0 or 1/2.

The hamiltonian (2.8) we rewright in the form (2.2) as

$$\hat{H}' = 2\sum_k \epsilon_k + g_J \, Sp\, B^+ B + \hat{H}(I) \tag{2.16}$$

with

$$\hat{H}(I) = \sum_k \left(2\epsilon_k I_3(\vec{k}) - \sqrt{2}\, g_J \,|b(k)|\,[I_+(\vec{k}) + I_-(\vec{k})]\right)$$

Rotation

$$\mathcal{D}(\omega_k) = \prod_\alpha \left[\cos\frac{\omega_k}{2} - 2\alpha a^+_{k\alpha} a^+_{-k\alpha} \sin\frac{\omega_k}{2} \right] \tag{2.17}$$

with

$$\cos\omega_k = \epsilon_k \Big/ \sqrt{\epsilon_k^2 + g_J^2\, |b(k)|^2} \tag{2.18}$$

diagonalizes the hamiltonian (2.16). ($\alpha = \pm 1/2$ is $\hat{z}$ - axis spin projection.)

The vacuum state $|0\rangle$ is the eigenstate of $I_3(\vec{k})$ with eigenvalue $-1/2$. We obtain the ground state of superfluid condensate acting with transformation $\mathcal{D}$ (2.17) on the vacuum. The energy of this state is directly calculated

$$E_{J0} = \langle J0 \,|\, \hat{H}' \,|\, J0 \rangle =$$

$$= g_J \,\mathrm{Sp}\, B^+ B + \sum_k 2 \left\{ \epsilon_k - \sqrt{g_J^2 |b(k)|^2 + \epsilon_k^2} \right\}$$

These results are identical to formulas for singlet pairing. (We must replace the 2α coefficient with 1 and vector $\vec{b}$ with scalar function a .)

For $M \neq 0$ case, $\vec{b} \times \vec{b}^* \neq 0$. Using Table 1 we can see, for example, that in (2,2)-phase $i \cdot \vec{b} \times \vec{b}^* = \hat{z} \cdot |\vec{b} \times \vec{b}^*|$, and $\vec{b} \perp \hat{z}$.

The convinient quasispin group for (J , $M \neq 0$)-phases are defined with operators

$$I_+(\vec{k},\mu) = \frac{1}{2} a^+_{k\alpha} a^+_{-k\beta} (i\sigma_2 \vec{\sigma})_{\alpha\beta} \frac{\vec{b} - 2i\mu \vec{b} \times \hat{z}}{\sqrt{|b|^2 - 2\mu |\vec{b} \times \vec{b}^*|}} \tag{2.20}$$

and

$$I_3(\vec{k},\mu) = \frac{1}{4} \sum_{q=\pm k} (a^+_{q\alpha} a_{q\alpha} + 2\, a^+_{q\alpha} a_{q\beta} (\vec{\sigma}\hat{z})_{\alpha\beta} \quad) - \frac{1}{2} \tag{2.21}$$

$\mu = \pm 1/2$ is quasiparticle's spin projection to the axis $\hat{z}$.

For fixed μ and $\vec{k}$ three operators I_+, $I_- = I_+^+$ and I_3 are the SO(3)-group generators, and operators $\vec{I}(\vec{k},\mu)$ with different μ (and $\vec{k}$) commute. Therefore we obtain the Lee algebra of the group SO(3) $\otimes$ SO(3). It s irreducible representations are of 0 and 1/2 weights. Hamiltonian (2.8) we rewright in form (2.16) with

$$\hat{H}(I) = \sum_{k\mu} \{ 2\epsilon_k I_3(k,\mu) - - \sqrt{2}\, g_J [|b|^2 - 2\mu|\vec{b}\times\vec{b}^*|]^{1/2} [I_+ + I_-]$$

To obtain the operators of quasispin rotation (which diagonalize hamiltonian $\hat{H}(I)$) we use the expression (2.17) with some changes. Rotational parameters ω depend not on momenta $\vec{k}$ only but also on spin variables α :

$$\cos\omega_{k\alpha} = \epsilon_k \Big/ \sqrt{\epsilon_k^2 + g_J^2 [|\vec{b}|^2 - 2\alpha|\vec{b}\times\vec{b}^*|]} \tag{2.22}$$

The WF $|J\,M\rangle$ in $M \neq 0$ phases we obtain by acting with transformation (2.17) on $|0\rangle$ state. The wavefunctions $|J\,M\rangle$ obtained by this way are of positive P-parity, bacause all operators $\vec{I}$ are invariant under space reflections.

The ground state energy is

$$E_{JM} = g_J\, Sp\, B^+B + \sum_{\mu k} \{\epsilon_k - \sqrt{\epsilon_k^2 + g_J^2(|b|^2 - 2\mu|\vec{b}\times\vec{b}^*|)}\} \tag{2.23}$$

One can see from (2.23) that quasiparticle energy in ($M\neq0$)-phases depends on spin projection to the axis $i\vec{b}\times\vec{b}^*$, and the quasiparticle spectrum evinces two-gap structure. This will be discussed in details in Section 3.

Now we find the level of interphase orthogonality. We calculate for this aim the scalar product $\langle {}^1S_0 | {}^3P_0 \rangle$ of WFs of condensates with singlet S-pairing and (J = 0) triplet pairing. The quasispin groups in these cases have SO(3)-structure. To obtain the simplest estimation we consider the case of equal pairing gaps ($\Delta^2_{triplet} = g^2|\vec{b}|^2 = g^2|a|^2 = \Delta^2_{singlet}$) and parameters ω_k in these two states.

$$\langle {}^1S_0 | {}^3P_0 \rangle = \prod_k \cos\omega_k = \exp\left\{ \sum_k \ln \frac{\epsilon_k}{\sqrt{\epsilon_k^2 + \Delta_k^2}} \right\}$$

After using an ordinary approximations (we put $\Delta(k) = \Delta(k_F)$ and cut off the integration at k_F) the matrix element takes form

$$\langle {}^1S_0 | {}^3P_0 \rangle = \exp\left\{ -\frac{3\pi}{8} \cdot \frac{\Delta}{\epsilon_F} \cdot N \right\} \tag{2.24}$$

For nuclei with $N \sim 100$, $\epsilon_F \sim 40$ MeV and $\Delta \sim 1$ MeV

$$\langle {}^1S_0 | {}^3P_0 \rangle \sim 5 \cdot 10^{-2}$$

and the factor of hinding of interphase transitions is of 10^{-3}.

So we had shown that the existence of different types of pairing in nuclei results in new qualitative features in nuclear spectra: the existence of bands of nonintercombining collective levels.

3. Quasiparticle excitations

Accordingly to scheme proposed in Section 2, we obtain the n - quasiparticle wave functions after quasispin rotation (2.17) of vector

$$\left(\prod_i^n a^+_{k_i \alpha_i} \right) |0\rangle$$

The quasiparticle energy spectrum

$$\epsilon_{JM}(k,\mu) = \sqrt{\epsilon_k^2 + g_J^2(|b|^2 - 2\mu|\vec{b}\times\vec{b}^*|)} \quad (3.1)$$

is found to be of two-gap character in ($M \neq 0$)-phases. (We remind that μ is the quasiparticle spin projection onthe axis $i\vec{b}\times\vec{b}^*$ This splitting also exists/7/ in the so called α and β- phases*) of superfluid He^3. The existence of phases with two-gap spectra is a qualitative peculiarity of triplet pairing, contrary to always spin-independent spectrum of singlet pairing. Two phases (singlet (L, M) and triplet (J = L, M)) with similar symmetry properties differ, first of all, in structure of quasiparticle spectra described above. The gap equations are also different in th se cases. In ref./1/ authors didn't take this fact into account and therefore their results need an additional consideration.

The two-gap character of some triplet phases could result in interesting features in spectra of even-even nuclei. The states

$$\mathcal{D}\, a^+_{k\mu_1} a^+_{-k\mu_2}|0\rangle \qquad , E^* = \epsilon(k,\mu_1) + \epsilon(k,\mu_2)$$

with two quasiparticles with opositely directed momenta are splitted in common spin projection on some axis (for example, $\hat{z}$ - axis in (2,2)-phase). This splitting

$$\Delta E^* = |\epsilon(k,+1/2) - \epsilon(k,-1/2)|$$

is of the gap order. The γ - transitions between ($\mu_1 = \mu_2$) and ($\mu_1 = -\mu_2$) levels are of M1 character, because they are connected with $\Delta\mu = \pm 1$ spin reorientation. The $\Delta\mu = \pm 2$ E2-transition could be the result of relativistic corrections only and is hindered by factor 10^{-2}.

*) In ref./8/ the existence of β - phase in vortex core was proposed.

So the existence of successive M1-transitions among two-quasiparticles states could be the test for band with triplet pairing in nuclei. Each cascade consists of two γ-rays with close energies. Because of gap dependence on $\vec{k}$ direction (i.e. on orbital quasiparticle properties) the transition energy could vary in wide range (o.1÷1 MeV).

4. The rotational spectra

The combination of anisotropy of superfluid in ($J \neq 0$)-phases with finite sizes of nucleus makes possible the collective rotation of condensate. In this section we are interested in the dependence of nuclear moment of inertia on angular momentum. We also define here relative directions of rotation and an axis of cooper pair spin quantization.

The basic point of our approach is the OP.

We use the Ginzbourg-Landau approximation (we consider the energy of the system to be a polynomial function of OP). On this way we can obtain the simple qualitative description of properties of nuclear rotation.

We rewright the energy $E_{JM}(\vec{I})$ of rotating nucleus in form

$$E_{JM}(\vec{I}) = E_{JM}(\vec{\Omega}) + \vec{I}\vec{\Omega} \tag{4.1}$$

Rotational frequency is denoted by $\vec{\Omega}$, and $E_{JM}(\vec{\Omega})$ is the energy in rotating frame. We express $E_{JM}(\vec{\Omega})$ as a sum of scalars of the lowest orders constructed of OP and $\vec{\Omega}$.

In the case of (J , $M \neq 0$)-phases the expression for $E_{JM}(\vec{\Omega})$ begins with the first order term $\Omega_i e_{ijl} B_{jn} B^*_{ln}$.
Using the Table 1 we calculate

$$\Omega_i e_{ijl} B_{jn} B^*_{ln} = c\,\vec{\Omega}\hat{z}A^2 \qquad ; \quad c = \begin{cases} 1\ , & J = M \\ 1/2, & J = 2 \\ & M = 1 \end{cases} \tag{4.2}$$

with $A^2 = Sp(BB^*)$. We assume, that the terms of higher orders in $\vec{\Omega}$ are neglegibly small. Then

$$E_{JM}(\vec{\Omega}) = (-\alpha + \beta\vec{\Omega}\hat{z})A^2 + \gamma A^4 \tag{4.3}$$

Effective parameter γ differs for different phases, because there are three independent invariants of the fourth order of OP. We suppose that α is positive and some equilibrium OP exists. β could be taken positive too. Energy (4.3) is minimized by condition $\vec{\Omega}\hat{z}/\Omega = -1$. So the nuclear matter rotates around the axis of the quantyzation of the condensate's spin. This unusial rotational axis orientation is connected with the axial anisotropy of OP (in $(M\neq 0)$-phases OP is built on $\vec{V} = \hat{x} + i\hat{y}$ vectors, $\vec{V}\perp\hat{z}$). At the same time the effective potential might be axially symmetric, bacause it is connected with matrix element $\langle\hat{\psi}^+\hat{\psi}\rangle$, i.e. some BB^* combination. So our approach differs from the standard cranking model.

After minimizing energy $E_{JM}(\vec{\Omega})$ we obtain

$$A^2(\Omega) = \frac{\alpha + \beta\Omega}{2\gamma}\ , \quad \gamma > 0 \tag{4.4}$$

Using the definition

$$\vec{I} = -\frac{\partial E_{JM}(\Omega)}{\partial\vec{\Omega}} \tag{4.5}$$

and eq.(4.3)-(4.4), we obtain

$$E_{JM}(I) - E_{JM}(0) = \frac{1}{2\mathcal{J}_0}\left(\sqrt{I^2} - I_0\right)^2 \tag{4.6}$$

with

$$\mathcal{J}_0 = \beta^2/2\gamma \;;\; I_0 = \frac{\alpha\beta}{2\gamma} \;;\; E_{J0}(0) = -\frac{\alpha}{4\gamma} \qquad (4.7)$$

Note that I_0 is the nuclear spin value in (J , M ≠ 0) phase without any rotation.

Effective moment of inertia we define with expression

$$\frac{1}{2\mathcal{J}(I)} = \frac{\partial E_{JM}(I)}{\partial(\vec{I}^2)} = \frac{1}{2\mathcal{J}_0}\left(1 - \frac{I_0}{\sqrt{\vec{I}^2}}\right) \qquad (4.8)$$

Formula (4.8) qualitatively differs from formula

$$\frac{1}{2\mathcal{J}(I)} = C + D\vec{I}^2 \qquad (4.9)$$

used in nuclear rotation spectra. We see that in high ordinary states spin the (J , M ≠ 0)-phase has almost constant effective moment of inertia

$$\mathcal{J}(I \gg I_0) \approx \mathcal{J}_0 \qquad (4.10)$$

The (J , 0)-phase shows quite another behavior. The vector $e_{ijl}B_{in}B^*_{ln}$ equals zero, and nucleus could rotate in ortogonal to $\hat{z}$ - axis direction only. This situation is analogous to the standard cranking model.

For small $\vec{\Omega}$ we define

$$E_{J0}(\Omega) = (-\alpha - \eta\Omega^2)A^2 + \gamma A^4 \qquad (4.11)$$

with positive α, η, γ. The $\eta > 0$ requirement is connected with negativity of the second order correction to the ground state energy in perturbation theory. Now we find the difference between expressions (4.3) and (4.11). In the first the sign of Ω - correction to energy is not connected with perturbation theory laws. Therefore we shall call (J , M ≠ 0)-phases as rotatively-

-stable phases.

Minimizing $E_{JO}(\Omega)$ we obtain

$$A^2(\Omega) = \frac{\alpha + \eta\Omega^2}{2\gamma} \tag{4.12}$$

Using formulas (4.1), (4.5), (4.11) and (4.12) we find the rotational spectrum structure

$$E_{JM}(I) - E_{JM}(0) = \tag{4.13}$$

$$= \frac{E_{JO}(0)}{3} \left\{ \left[\sum_{\lambda=\pm 1} \left(I^2/I_c^2 + 1 + \lambda\sqrt{(I^2/I_c^2 + 1)^2 - 1} \right)^{1/3} - 1 \right]^2 - 1 \right\}$$

with $E_{JO}(0)$ defined with expressions (4.7) and

$$I_c^2 = \frac{2}{27} \frac{\alpha^3\eta}{\gamma^2}$$

In low spin region $I \ll I_c$ we can use polynomial approximation (4.9) for effective moment of inertia with the coefficients

$$C = \frac{\gamma}{2\alpha\eta} \qquad \text{and} \qquad D = \frac{19}{4} \frac{\gamma^3}{\alpha^4\eta^2}$$

In high spin region $I \gg I_c$ eq.(4.13) shows asymptotic behavior of rotational energy

$$E_{JO}(I) - E_{JO}(0) \longrightarrow \frac{3}{4}(\gamma/\eta^2)^{1/3} \cdot (\vec{I}^2)^{2/3} \tag{4.14}$$

and effective moment of inertia

$$\frac{1}{2\mathcal{J}}(I) = (\eta^2/\gamma)^{1/3} \cdot I^{2/3} \tag{4.16}$$

Similar asymptotic was obtained in ref. /10/ for some variant of IBM and in ref. /11/

The results presented above are also valid for nontrivial singlet pairing (for example, quadrupole pairing).

So we see that multiphase superfluidity must suggest the existence of rotational bands with qualitatively different spin-dependence of moment of inertia. This circumstance makes probable the band crossing. Moreover, the $I^{4/3}$ energy dependence (4.14) could result the two-fold band crossing.

Let us turn now to comparison of formulas (4.6) and (4.19) with experimental data. We studied 34 isotopes in rare-earth and actinide regions (Xe^{130}, $Ce^{128,130}$, Ba^{128}, Gd^{156}, $Dy^{156,158}$, $Er^{158,164,166}$, $Yb^{164,166,174}$, $Hf^{166,168,170,174}$, $W^{166-176}$, $Hg^{184,186}$, $Th^{222,228,230}$, $U^{232-236}$, $Pu^{242,244}$). The parameters varied were α, β, γ, η, which stay constant along each described band. In all cases the theoretical and experimental data coinsidence was in the range of 1-5%.

An expressive example of Dy^{156} spectra is represented in Fig.2. The data for two rotational bands were taken from ref./12/. The ground state band we interprete as (J , 0)-phase (4.13) and side-band as (J , M $\neq$ 0)-phase (4.6). We want to turn reader's attention to the possible inverse band crossing at high spin, mentioned above.

The spin-dependence of effective moment of inertia in nuclei Hf^{168} and Er^{164} are represented in Fig.3 (experimental data from ref./13/).

5. The boundary conditions

Near the boundary of nucleus (on the $1/k_F$ scale) the OP is nonhomogenueos in $\vec{n}$ direction ($\vec{n}$ is normal to the nuclear surface). The requirement of continuity of normal part of the OP and it s derivative in $\vec{n}$ direction define the boundary conditions.

$$n_i B_{ij}(R) = 0 \tag{5.1}$$

For axially symmetric phases (J , 0) eq.(5.1) nullifies the OP. For phases (J , J) eq.(5.1) could be satisfied without destruction of OP. In ref./14/ for analogous situation in He^3 it was proposed, that the surface phase could differ from volume phase. For finite system condition (5.1) means, that (J , J) phases are prefered. The possibility of superfluidity conservation on the nuclear surface could change the surface energy of the drop and influence on equilibrium shape of a nucleus.

Simultaniously, the tangential vector field $V_i B_{ij}^{JM}$ (with vector $\vec{V}$ from Table 1) exists on the surface. According to the Poincaré theorem the singularity of that field always exists. The theorem or nullifies OP, either demands the nonhomogeneous OP's structure. In the second case the problem of the scale of such nonhomogeneity exists, because the size of nucleus is smaller than correlation length. Nevertheless, if we suggest the existence of topological singularities, we should consider the new type of excitations connected with the motion of the surface singularity.

Conclusions

In present paper we distinguished the possible consequences of hypothetical triplet pairing in nuclei. The most interesting of them are the existence of nonintercombining collective bands and the two-gap spectrum of quasiparticle excitations with successive M1-transitions between multiplet components. If our interpretation of experimental data has some relation with reality, we should expect the two-gap multiplets in rotatively-stable band region(Section 4).

Note, that some indications to existence of group of levels nonintercombining with the low energy part of spectrum was obtained in ref./15/.

In this previos investigation we did not consider the effects of the space quantization on quasiparticles properties. In our oppinion the main qualitative features of triplet pairing don't depend on the concrete initial quasiparticle level structure.

The whole microscopic analysis of the possibility of different types of pairing needs the self consistent consideration not of effective potential only but also of some superfluid collective modes. The usual potential scheme hardly can answer the question, does the triplet pairing in nuclei exist.

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FIGURE CAPTION

Fig. 1 The two-gap multiplet of two-quasiparticle excitations in even-even nuclei. The successive M1-transitions between states with different quasiparticle spin projections μ_1, and μ_2 are shown.

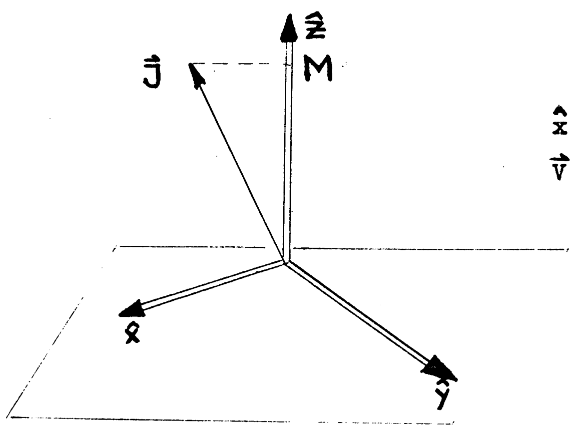
Fig. 2 Rotational spectrum of Dy^{156}. Experimental data are taken from ref. /12/. The theoretical energies are presented in brekkets. We describe the ground state band with formula (4.13) and side-band with formula (4.6) for rotatively-stable band.

Fig. 3 The moment of inertia $\mathfrak{J}$ dependence on angular momentum. The experimental data ($\times$) for Hf^{168}(a) and Er^{164} (b) are taken from ref. /12/. The theoretical results are given by curves I and II.

Table 1

The tensor structure of OP for different phases of triplet pairing with strong spin-orbital coupling.

J,M	B_{ij}	H_D	G/H	π_1	π_2
0, 0	$\delta_{ij}e^{i\phi}$	-	S^1	Z	0
1, 0	$e_{ijk}z_k e^{i\phi}$	$z \to -z$ $\phi \to \phi+\pi$	$S^2 \otimes R^1/Z$	Z	Z
1, 1	$e_{ijk}V_k$	-	SO(3)	Z_2	0
2, 0	$e^{i\phi}(z_i z_j - \frac{1}{3}\delta_{ij})$	$z \to -z$	$S^2 \otimes U(1)/Z_2$	$Z_2 \otimes Z$	Z
2, 1	$V_i z_j + z_i V_j$	-	SO(3)	Z_2	0
2, 2	$V_i V_j$	-	$SO(3)/Z_2$	Z_4	0



$\hat{x} \perp \hat{y} \perp \hat{z}$

$\vec{V} = \hat{x} + i\hat{y}$

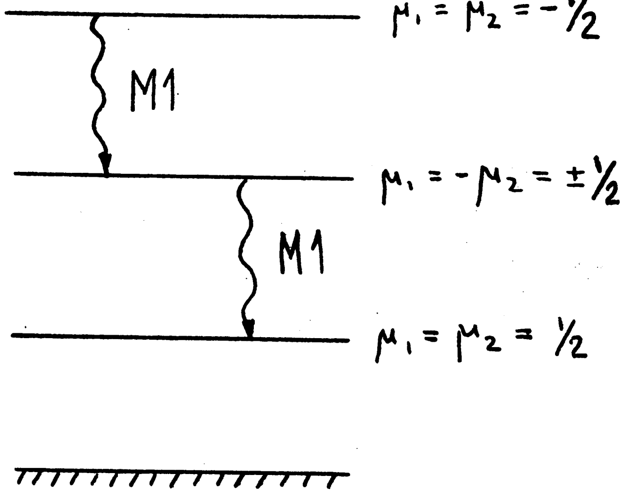
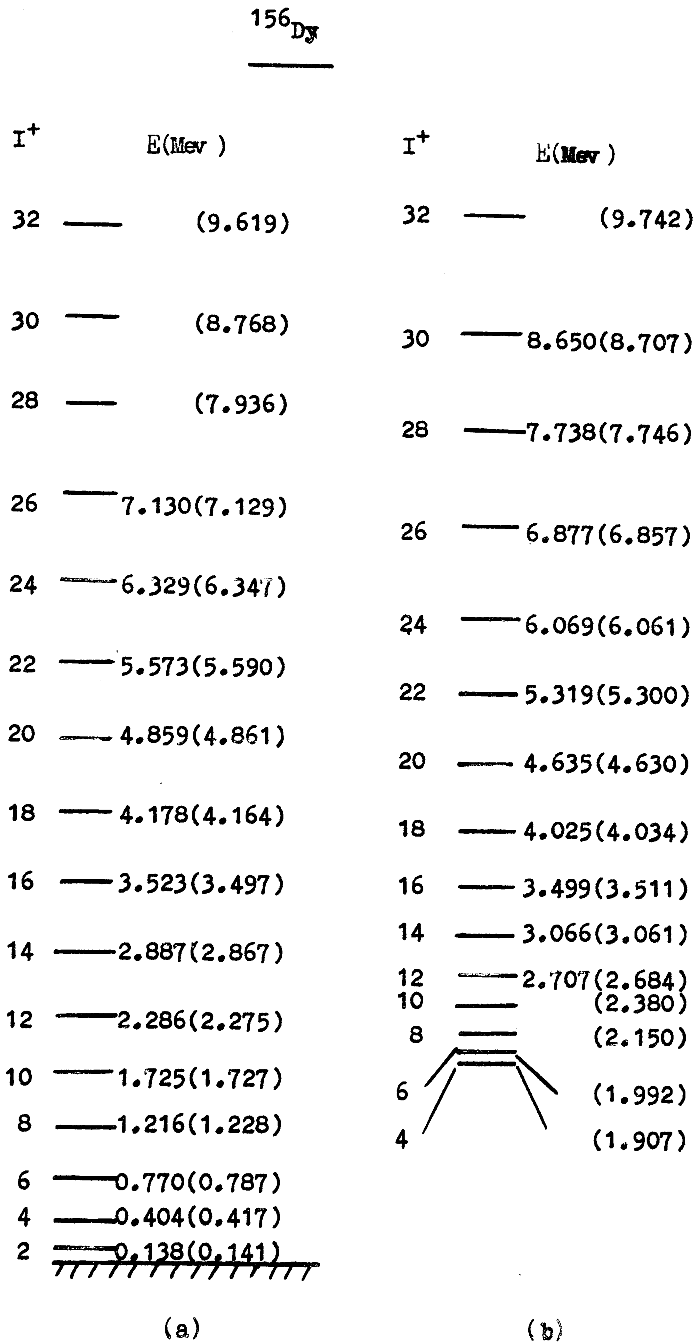
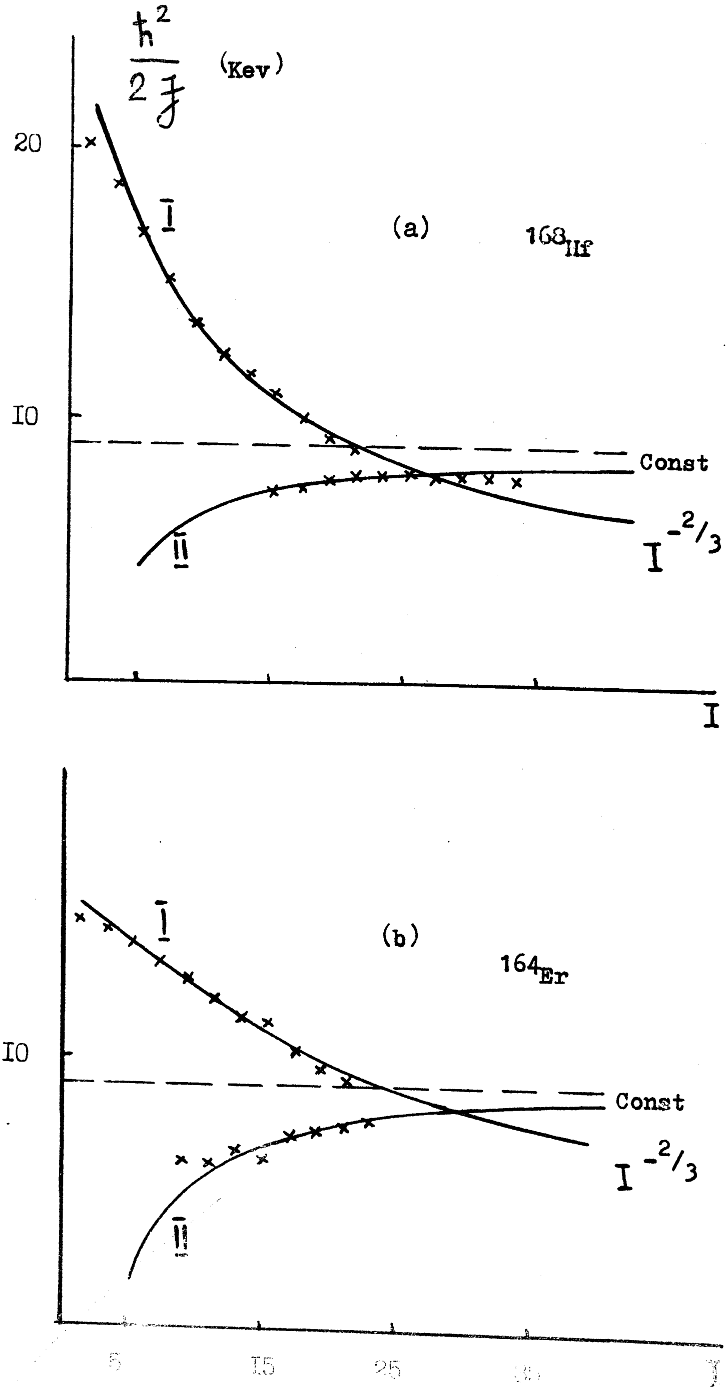


Fig. 1



^{156}Dy
I$^+$
E(Mev)
32 (9.619)
30 (8.768)
28 (7.936)
26 7.130(7.129)
24 6.329(6.347)
22 5.573(5.590)
20 4.859(4.861)
18 4.178(4.164)
16 3.523(3.497)
14 2.887(2.867)
12 2.286(2.275)
10 1.725(1.727)
8 1.216(1.228)
6 0.770(0.787)
4 0.404(0.417)
2 0.138(0.141)
(a)
I$^+$
E(Mev)
32 (9.742)
30 8.650(8.707)
28 7.738(7.746)
26 6.877(6.857)
24 6.069(6.061)
22 5.319(5.300)
20 4.635(4.630)
18 4.025(4.034)
16 3.499(3.511)
14 3.066(3.061)
12 2.707(2.684)
10 (2.380)
8 (2.150)
6 (1.992)
4 (1.907)
(b)

Fig. 2



$\frac{\hbar^2}{2\mathcal{J}}$ (Kev)
20
I0
(a)
168Hf
I
II
Const
$I^{-2/3}$
I
I0
(b)
164Er
I
II
Const
$I^{-2/3}$
5
I5
25
I

Fig. 3

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