REM - the Shape of Potentials for f(R) Theories in Cosmology and Tachyons

Dumitru N. Vulcanov [a, *], *Goran S. Djordjevic* [b, c, †] *and Ciprian A. Sporea* [a,‡]
[a] West University of Timisoara, Faculty of Physics, B-dul V. Parvan no. 4, 300223, Timisoara, Romania
[b] University of Nis, Faculty of Science and Mathematics, Visegradska 33, Nis, Serbia
[c] Theory Group, Physics Department, CERN, CH-1211, Geneva 23, Switzerland

Abstract
We investigated the reverse engineering method (REM) for constructing the potential of the scalar field in cosmological theories based on metric f(R) gravity and Friedman Robertson Walker (FRW) metric. Then transposing the new field and Friedman equations in an algebraic computing special library (in Maple + GrTennsorII platform) we graphically investigate the shape of the potentials in terms of the scalar field in at least two type of cosmology with exponential and linear scale factor expansion. Some perspectives and conclusions relating these results with tachyonic cosmology theories are noticed.

1 Introduction

The reverse engineering method (REM) was proposed [1] in order to reconstruct the shape of the scalar field potentials, including tachyon-like, in cosmology starting with a certain time behavior of the scale factor. Recently we studied REM in cosmologies with minimally and non-minimally coupled scalar field [2] for Friedmann-Robertson-Walker metrics. We extended our study for higher order metric theories of gravity (generically called $f(R)$ theories [3–6]).

This time we are mostly focused on a lagrangian of the theory having the form $L = R + \alpha R^2 + L_\phi$, where R is the Ricci scalar and L_ϕ represents the lagrangian of the scalar field (minimally coupled with gravity). The field equations (modified Friedmann equations in FRW metric) will contain second order derivatives of the scale factor. It is hard to find analytical solutions for this type of field equations and thus a numerical investigation is necessary. For this reason we developed a series of Maple+GrTensorII programs, organized in a special library, using the symbolic computation and graphical facilities for processing REM in several cases: exponential, linear or sinusoidal expansion of the Universe. The shape of the potential as a function of the scalar field is graphically investigated in order to establish a convenient theory producing the respective time evolution of the universe.

We also present the classical behavior of several tachyon-like scalar-field potentials [1] which are still good candidates for describing a realistic scale factor of the Universe, in particular for describing inflation, with accent on its early phase on archimedean and nonarchimedean spaces. This approach applied in [7] on classical and quantum dynamics for a tachyonic field with exponential potential, here is briefly extended applied on a model with $V(T) \sim \cosh^{-1}(T)$. Possibility for its generalization and quantization is pointed out in Section 4.

2 Scalar field cosmology in metric f(R) gravity

We are dealing with cosmologies based on Friedman-Robertson-Walker (FRW) metric

$$ds^2 = c^2 dt^2 - a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2 \right) \right] \tag{1}$$

*E-mail: vulcan@physics.uvt.ro
†E-mail: gorandj@junis.ni.ac.rs
‡E-mail: sporea_89@yahoo.com

where $a(t)$ is the scale factor and $k = -1, 0, 1$ for open, flat or closed cosmologies. The dynamics of the system with a scalar field minimally coupled with metric $f(R)$ gravity is described by the following action [4, 6]

$$\mathcal{S} = \int \mathrm{d}^4x\,\sqrt{-g}\left\{\frac{1}{2}\frac{M_P^2}{8\pi}f(R) + \frac{1}{2}\nabla_\mu\phi\,\nabla^\mu\phi - V(\phi)\right\} + \int \mathrm{d}^4x\,\mathcal{L}_M(g_{\mu\nu}, \psi_M) \tag{2}$$

where $f(R)$ is in principle an arbitrary function of the Ricci scalar R, $V(\phi)$ is the potential of the scalar field and $\mathcal{L}_M$ represents the Lagrange density for regular matter (dust, radiation, etc).

As in the Starobinsky model [5] (known to be an excellent model of chaotic inflation) we choose for $f(R)$ the following expression

$$f(R) = R + \alpha R^2 \tag{3}$$

where we consider α to be a real constant (inversely proportional with the rest mass of the scalar particle in the Starobinsky model).

Varying the action $\mathcal{S}$ with respect to the metric $g^{\mu\nu}$ we get the new field equations

$$\begin{aligned}(1 + 2\alpha R)\,R_{\alpha\beta} - \frac{1}{2}\left(R + \alpha R^2\right)g_{\alpha\beta} - 2\alpha g^{\mu\nu}\left(\nabla_\alpha\nabla_\beta - g_{\alpha\beta}g^{\sigma\tau}\nabla_\sigma\nabla_\tau\right)R_{\mu\nu} =\\ = -\phi_{,\alpha}\phi_{,\beta} + g_{\alpha\beta}\left(\frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - V(\phi)\right)\end{aligned} \tag{4}$$

3 Algebraic computing, "Cosmo" library and some graphical results

As reported previously [1] we composed a new library (called generically "Cosmo") for use in cosmology embedded in Maple+GrTensorII algebraic computing platform (see at http://grtensor.org). GrTensorII is a free library (developed by a community of users) which incorporates a Riemannian geometry in a Maple symbolic computation environment. The Cosmo library defines the main cosmological functions and operators (Hubble constant/function, deceleration, scale factor etc.) producing the main Friedman equations from Einstein eqs. in FRW metric. This time for our purposes we cannot use the predefined (in GrTensorII) Einstein equations, thus we have to define new tensor components for the field equations above mentioned by introducing a sequence of GrTensorII commands as

$> grdef(`Ec\{a\ b\} := (1+2*alpha*Ricciscalar)*R\{a\ b\}-(1/2)*g\{a\ b\}*(Ricciscalar + alpha*Ricciscalar^2) - s\{,a\}*s\{,b\} + (1/2)*g\{a\ b\}*((1/2)*g\{\wedge i\ \wedge j\}*s\{,i\}*s\{,j\} - V)`);$

where s represents the scalar field and Ec{a b} is a tensorial object having as components the main field equations here replacing the Einstein equations.

Fortunately we still can use the Riemannian environment defined by GrTensorII (Riemann and Ricci tensors, Christoffell symbols for covariant derivatives, the Ricci scalar - called Ricciscalar in the above command line, etc.). Finally after loading the FRW metric (using qload GrTensorII command) and a sequence of gralter, simplify and subs commands we obtain the new Friedmann equations as

$$V(\phi) = 3\dot{H}(t) + 6H(t)^2 + 3\frac{k}{a(t)^2} + h_1(k, \alpha, \dot{H}, \ddot{H}..) \tag{5}$$

$$\dot{\phi}^2 = -2\dot{H}(t)^2 + 2\frac{k}{a(t)^2} + h_2(k, \alpha, \dot{H}, \ddot{H}..) \tag{6}$$

where $H(t) = \dot{a}(t)/a(t)$ is the Hubble function. We arranged the equations to resemble the form of eqs. (12)-(13) in [1] and we collected in h_1 and h_2 the terms containing higher order time derivatives of $H(t)$.

Thus treating analytically these equations as we done in REM for previous cases mentioned above is almost impossible so we analyzed them graphically, using the numerical and graphic facilities of Maple. Actually the aim of REM is to recover the shape of the potential in term of the scalar field (eliminating the time from the above equations).

To illustrate our graphical results obtained after processing the Cosmo library, we concentrate on some examples (mainly the exponential and the linear expansion of the universe) taken from Table no. 1 of [1]. For the exponential case, when we have

$$a(t) = a_0 e^{\omega t}, \tag{7}$$

ω being a real constant, we obtained the results presented in Figure 1.

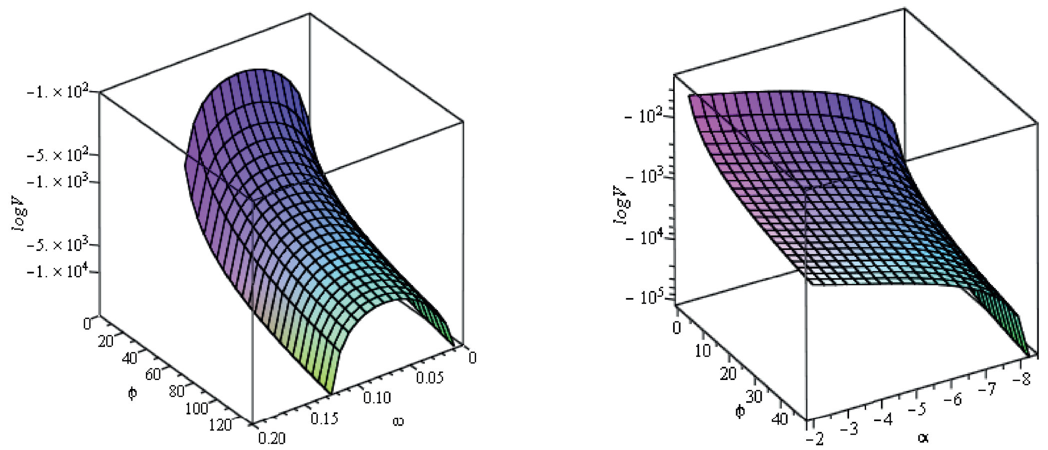

Fig. 1: $V(\phi)$ in terms of different ω at $k = 0$ (left panel) and of different α at $k = 1$ and $\omega = 0.1$ (right panel)

Similar results were obtained in the case of a linear expansion. For the sake of completeness we analyses also several cases for the time behavior of the potential which also can offer valuable information on the interaction theory governing the universe. Figure no. 2 below illustrates some of these results for the two cases we processed, the exponential and the linear one.

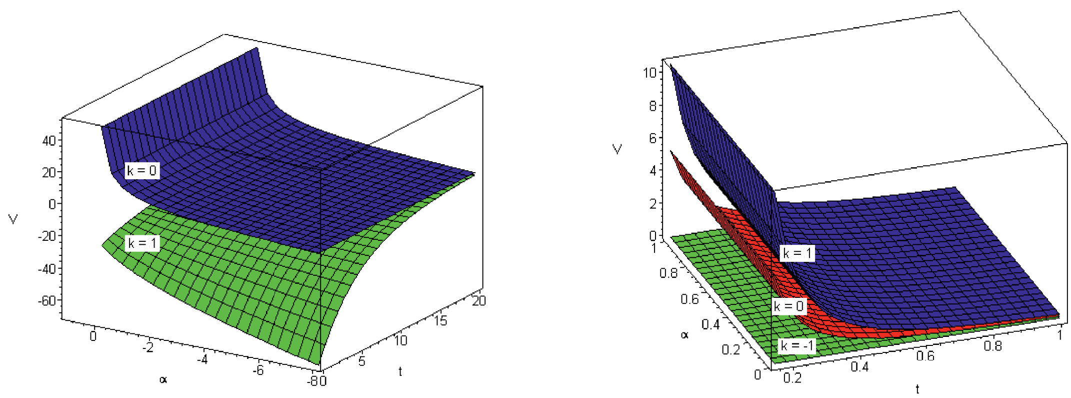

Fig. 2: Time behavior of V in terms of different α at $k = 0, 1, -1$ in the exponential case (left panel) and the linear case (right panel)

4 Concluding remarks and the connection with tachyonic cosmological theories

Our investigations proved that the REM is feasible even in non-Einsteinian gravity cosmologies when no analytical solution of the REM is possible to obtain. Actually the graphical solution we applied and processed can show the shape of the potential of the scalar field providing good information for building an interaction theory in non-standard cosmologies.

Besides several possible approaches to include tachyonic potentials in cosmology, first of all in its inflationary phase, we will notice here an approach triggered by Sen‘s conjecture on tachyonic matter in context of D-brane dynamics [8]. In short, he proposed the following lagrangian

$$L_{tach} = L_{tach}(V, \partial_\mu T) = -V(T)\sqrt{1 + g^{\mu\nu}\partial_\mu T \partial_\nu T} \tag{8}$$

For a spatially homogenous (tachyon) field $T(t)$, a simplified lagrangian and the corresponding equation of motion are respectively

$$L_{tach} = L_{tach}(T(t), \dot{T}(t)) = -V(T)\sqrt{1 - \dot{T}^2} \tag{9}$$

$$\ddot{T}(t) - \frac{1}{V(T)}\frac{dV}{dT}\dot{T}^2(t) + \frac{1}{V(T)}\frac{dV}{dT} = 0 \tag{10}$$

There are numerous interesting potentials that can be incorporated in L_ϕ, either motivated by string theory or by its solvability in context of inflation theory and “Friedmanology“. For instance

$$V_1(T) = e^{-\alpha T} \qquad V_2(T) = \frac{V_m}{\cosh(\beta T)} \tag{11}$$

One gets, up to some unimportant constants, equations of motion for these potentials

$$\ddot{T}(t) + \alpha \dot{T}^2(t) - \alpha = 0 \tag{12}$$

$$\ddot{T}(t) + \beta \tanh(\beta T)\dot{T}^2(t) = \beta \tanh(\beta T) \tag{13}$$

However, this form of lagrangian (action), as well equation of motion, is highly nonlinear and quite unsuitable for analytic solutions of Friedman equations, and in particular for quantization of any particular potential, i.e. model. However, as it was shown in [7] in case of exponential potential it is possible to find a locally equivalent lagrangian in the standard form producing the same equation of motion. In case of the later of of above mentioned potentials it can be shown [9] that

$$L_1(Y_1, \dot{Y}_1) = \frac{1}{2}\dot{Y}_1^2(t) + \frac{\alpha^2}{2}Y_1^2(t) \tag{14}$$

$$L_2 = \frac{1}{2}\cosh^2(\beta T)(1 + \dot{T}^2(t)) = \frac{1}{2}\dot{Y}_2^2(t) + \frac{\beta^2}{2}Y_2^2(t) + \frac{1}{2} \tag{15}$$

where

$$T \to Y_1 = \alpha^{-1}e^{\alpha T} \qquad T(t) \to Y_2(t) = \beta^{-1}\sinh(\beta T(t)) \tag{16}$$

A generalization of this approach for a wide class of tachyonic potentials and quantization of the corresponding models with quadratic lagrangians via path integral technique will be presented elsewhere [9]. These results, as well as our REM approach to $f(R)$ theories and (non)minimally coupled tachyonic fields with gravity [2] are a good point for testing string-theory motivated potentials and scale factor obtained from this approach, in least complicated solvable models and shape of potentials obtained from rather realistic models using REM methods [10].

Acknowledgements

This work was supported by ICTP - SEENET-MTP project PRJ-09 Cosmology and Strings. One of the authors (D.N.V.) expresses his gratitude to the organizers of the BW2013 Workshop, Vrnjackja Banja, Serbia, where a part of this work was considered. G.S.Dj would like to thank to CERN-TH, for kind hospitality and support, where the final part of the paper was worked out, and to acknowledge support of the Serbian Ministry for Education, Science and Technological Development under project No 176021.

References

[1] D.N. Vulcanov, Cent. Eur. J. Phys., 6, No.1, (2008), p.84

[2] D.N. Vulcanov, G.S. Djordjevic, Rom. Journ. of Phys., 57, No. 5-6 (2012), p. 1011

[3] M.B. Mijic, M.S. Morris, Wai-Mo Suen, Phys.Rev. D, 34, No. 10, (1986) , p. 2934

[4] A. De Felice, S.Tsujikawa, arXiv: 1002.4928v2 [gr-qc]

[5] A.A. Starobinsky, Phys. Lett. B, 91 (1), 99-102 (1980).

[6] T.P. Sotiriou, V. Faraoni, Rev. Mod. Phys., 82, 451-497 (2010); arXiv:0805.1726 [gr-qc]

[7] D. D. Dimitrijevic, G. S. Djordjevic and Lj. Nesic, Fort. der Phys., 56, No. 4-5, p. 412

[8] A. Sen, JHEP 0204, (2002) 048

[9] D.D. Dimitrijevic, G.S. Djordjevic and M. Miliosevic, *On classicalization and quantization of tachyon-like matter (non)archimedean spaces*, to appear in Rom. Rep. of Phys.

[10] D.D. Dimitrijevic, G.S. Djordjevic, M. Miliosevic, C.A. Sporea and D.N. Vulcanov, *On analytical and numerical DBI cosmologies*, work in progress.