

Further measurements of the impedance of LEP

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Summary

Measurements of the bunch length as function of RF voltage result in better estimates of the corrections which have to be applied to the "raw data", and hence of longitudinal impedance in LEP. For the short bunch length of about 5 mm at injection, one obtains an "effective impedance" of only 22 mOhms, which corresponds to a low-frequency value of Z/n of about 1/4 Ohm. Simultaneous measurements of the bunch length and the horizontal emittance as functions of the RF frequency yield the damping aperture and the variations of the damping partition numbers. These can be used to estimate the RF frequency for which the orbit passes through the centers of the quadrupoles, and which permits an absolute energy calibration of LEP.

Also the transverse broad-band impedance is recalculated with the improved estimates of the bunch length. The transverse impedance of the RF cavities is found to be slightly larger than the original estimate, while the impedance of the shielded bellows is somewhat lower. The threshold for transverse mode-coupling has not been measured yet, but is still expected to be near 0.75 mA/bunch if the bunches are lengthened to 4 cm.

1 Introduction

Preliminary measurements of the impedance of LEP have been reported in Commissioning Note 6 [1]. The conversion of the directly measured quantities to longitudinal or transverse impedances relies on the knowledge of the bunchlength, which therefore has to be measured first.

The "raw data" for the bunch length at the injection energy of 20 GeV (see Fig.1) were obtained with the signal from a pick-up button connected by a short cable to a sampling scope in the tunnel. The exact transfer function of the pickups is not known, but we approximate its response with a Gaussian of standard deviation σ_{corr} . The correction must be applied quadratically

$$\sigma_{true}^2 = \sigma_{meas}^2 - \sigma_{corr}^2 \quad (1)$$

Below the turbulent threshold, the measured bunchlength was found to be approximately constant at 32 ps for $V_{RF} = 84MV$, which is much larger than the expected zero current value of 18 ps. Without correction to the bunch length, this would lead to a very small

energy damping partition number $J_E = 2(\sigma_{t0}/\sigma_\tau)^2$ of only 0.6 (instead of the natural value of two).

Also the horizontal emittance of LEP was measured (with a wire scanner), and found to be almost twice the expected value[2]. The corresponding damping partition number J_x would then be only about 0.6, again smaller than the natural value (one). In a flat machine like LEP (for which $J_y = 1$), Robinson's sum rule for the damping partition numbers requires $J_E + J_x = 3$, while this sum is only 1.2 with the values found above. Corrections obtained by including the cable length and the finite rise-time of the oscilloscope gave only $\sigma_{corr} \approx 14ps$, which still leads to a strong violation of the sum rule.

If we assume that the measurements of the horizontal emittance can be trusted, the corrections to the measured bunch length must be much larger. A value of $\sigma_{corr} = 27 ps$ is required to bring the low-current bunch length to about 16 ps, where the energy damping partition number becomes 2.4 and the correct partition number sum is obtained. The increased correction could be explained by the physical size of the pick-up buttons (32 mm diameter, corresponding to 106.7 ps full width). In first order, the signal is expected to have the shape of the electrode width, i.e. a semi-circle. The equivalent Gaussian with the same area then has a standard deviation divided by $8/\sqrt{\pi} \sim 4.51$ or 23.6 ps. Quadratic addition of the two corrections then yields the same value of 27 ps. However, rather than relying on this ad-hoc explanation, we attempted to measure the correction in the machine directly as described below.

2 Bunch Length versus RF Voltage

In LEP, where bunches are short compared to the RF period, and which is operated at a synchronous phase angle near π at injection energy, a linear approximation to the RF voltage can be applied. Then the product of bunch length and synchrotron tune is proportional to the energy spread $\sigma_s Q_s = \alpha R \delta_E$. For bunch currents below the "turbulent threshold" ($I_b < I_{thr}$) the energy spread does not change with current. Assuming quadratic correction of the bunch length, we get

$$\sigma_{meas}^2 - \sigma_{corr}^2 = \left(\frac{\alpha \delta_E}{2\pi f_s} \right)^2 \quad (2)$$

Since the square of synchrotron frequency is proportional to the RF voltage $f_s^2 \propto V_{RF} \cos \phi_s$, we can make the RHS zero by extrapolating the measured values to $V_{RF} \rightarrow \infty$. In fig.2 we plot the squared bunch length (full width at half maximum), which intercepts the vertical axis near $5000 ps^2$, i.e. $(FWHM)_{corr} = 70.7ps$. Assuming a Gaussian distribution, this should be divided by $2\sqrt{2 \ln 2} = 2.355$ to obtain the standard deviation $\sigma_{corr} = 30ps$.

The 'measured' correction is somewhat larger than the 27 ps estimated earlier. This fact may be due to resonances of the pickup: for short bunches, oscillations at a frequency of about 6 GHz can be seen behind the signal proper (see Fig.4). The phase of the oscillations appears to be such as to shorten the signal, and this effect would now depend on the bunch length itself. The exact determination of the bunch length has thus to be postponed until instruments with a better resolution (e.g. a streak camera) become available.

2.1 Longitudinal Impedance

Calculation of the longitudinal impedance from the measured threshold current and bunch length can be obtained using the stability criterion for bunched beams[1]

$$\left| \frac{Z}{n} \right| = \frac{FhV_{RF} \cos \phi_s}{\sqrt{2\pi} I_{thr} (\omega_o \sigma_{thr})^3} \quad (3)$$

where the form factor $F = 1$ for a capacitive impedance (expected for short bunches), and about 1.4 for resistive impedances. Substituting the parameters of LEP yields the extremely low impedance of about $22m\Omega$ (for $F=1$). However, one has to specify that this is the "effective impedance" acting on a very short bunch, and *not* the usually quoted low frequency limit Z/n . Assuming a resonator impedance with resonant frequency ω_r , the effective impedance of short bunches $\sigma < 1/\omega_r$ is strongly reduced due to the overlap of the bunch spectrum with both the positive low-frequency inductance, and the negative high-frequency capacitance. The result is given approximately by

$$\left(\frac{Z}{n} \right)_{eff} = 2 (\omega_r \sigma_r)^2 \left| \frac{Z}{n} \right| \quad (4)$$

for short bunches ($\omega_r \sigma_t < 1$), whereas for longer ones ($\omega_r \sigma_t \geq 1$)

$$\left(\frac{Z}{n} \right)_{eff} = \left| \frac{Z}{n} \right| \quad (5)$$

applies.

In LEP, the major part of the longitudinal impedance is expected to come from the RF cavities, for which a broad-band resonator frequency of about $2GHz$ has been estimated [3]. For a bunch length $\sigma_r = 16ps$ we find $\omega_r \sigma_t \sim 0.21$, and we have to use the short bunch expression to get the correction factor for the effective impedance of $2(\omega_r \sigma_t)^2 \sim .09$. Therefore, the low-frequency limit of the impedance then becomes

$$\left(\frac{Z}{n} \right) = 0.25\Omega \quad (6)$$

Since this result depends both on the form factor and on the assumed resonator frequency, it may be even larger (since $F > 1$ and ω_r possibly smaller than the 2 GHz assumed).

We have also computed the bunch length as function of current with the program BBI, which has the Hofmann-Maidment model[4] as one of its options. The result for an impedance of 0.25 Ohm is shown in figure 1 in addition to measured values.

3 Damping aperture and central frequency

a) Variation of the RF frequency will shift the particle orbit. This changes the magnetic field seen by the particles in the quadrupoles (and higher multipoles), and hence the amount of synchrotron radiation emitted. This fact is usually expressed as a change of the "damping partition numbers" J_E and J_x . In particular, J_x will become zero at some

frequency below the nominal one - where the beam passes through the center of the lenses - and J_E vanishes at a frequency above it. The machine is stable over the (frequency) range where both J_x and J_E are positive, which is usually called the "damping aperture".

b) Fig. 3 shows the damping partition numbers at injection energy as function of the (last 4 digits of the) RF frequency (in Hz). They have been computed from the measured (and corrected) values of the bunch length and emittance with the expressions

$$J_E = 2 \left(\frac{\sigma_{t0}}{\sigma_t} \right)^2 \quad J_x = \frac{E_{x0}}{E_x} \quad (7)$$

The theoretical values are $\sigma_{t0} = 18ps$ for $V_{RF} = 84MV$, and $E_{x0} = 6.7nm$. The damping aperture is thus found to be about 560 Hz, in good agreement with computations. However, a number of inconsistencies appear in this plot:

i) for the previously determined bunch length correction of 30 ps, the points for J_E lie on a straight line, but this line exceeds the expected maximum of 3 (and even 4): This is in contradiction to Robinson's sum rule.

ii) the correction can be adjusted so that J_E remains below 3 by choosing $\sigma_{corr} = 27ps$. While this is quite acceptable in itself, it makes the line strongly curved - in contradiction to analytical estimates that its slope should not change by more than about 1 %.

iii) the horizontal damping partition number appears to have a slope too small by a factor of about 2, and J_x reaches only 1.5 at the upper limit of the damping aperture, consistent with previous measurements[2].

c) The "central frequency", i.e. where the orbits go through the centers of the quadrupoles (as well as sextupoles and higher multipoles), can be determined by intersecting the curve for the damping partition number J_E with the theoretical value. This is normally 2 in a separate function machine, but only about 1.95 in LEP due to a small combined function quadrupole component caused by a thin layer of magnetic nickel on the vacuum chamber in the dipoles (used for binding the lead shielding to the aluminum chambers). We see from fig.2 that this value is reached at about 145 Hz if we use the straight line connecting $J_E = 3$ on the lower edge of the damping aperture with $J_E = 0$ at the upper one, in quite good agreement with other measurements of this value (about 160 Hz) [6].

d) A possible explanation of the difficulties with the damping partition number J_x may be due to a blow-up of the horizontal beam size by synchro-betatron resonances. However, one cannot exclude measurement errors with the wire scanner as well as with the synchrotron light monitor. Recent measurement with a beam scraper at 45 GeV gave a much smaller emittance than the one obtained with synchrotron light, which had agreed reasonably well with the wire scanner in the past[7].

The difficulties with the energy damping partition number could be resolved if the natural bunch length was shorter than calculated, e.g. due to bunch shortening at low currents. In some models of bunch lengthening this is expected to occur for short bunches seeing a capacitive impedance[4], but no clear observation of shortening has been made in LEP. A more likely explanation is the inaccuracy of the measurement with a button pickup which is much larger than the bunch length. This problem should be solved when the streak camera will become operational in the near future.

4 Transverse Impedance

In ref.(1), the transverse impedance has been calculated from measurements of the betatron tune shifts with current. However, the impedance is also proportional to the bunch length, and we expect to obtain a smaller (effective) impedance when the bunch length is corrected for instrumental errors. The tune shift caused by N impedances can be written

$$\frac{\Delta Q}{\Delta I} = \frac{R}{2\pi\sigma E/e} \sum_i i = 1N \langle \beta \rangle_i Z_{T_i}^{eff} \quad (8)$$

where $\langle \beta \rangle_i$ is the average beta function at the i -th transverse impedance Z_{T_i} . The effective transverse impedance for a Gaussian in a resonator impedance can be expressed as

$$Z_T^{eff} = Z_T F(\omega_{res} \sigma_t) \quad (9)$$

where the form factor can be approximated by $F(x) \approx 2x^2$ for $x < 1$ (bunches short compared to the resonant wavelength), and by $F(x) = 1$ for $x \geq 1$ (long bunches or high resonant frequency).

Since the RF cavities have circular beam holes, their contributions to the horizontal and the vertical shift must be equal, i.e. somewhat less than the smaller of the two shifts. Since the resonant frequency of the RF cavities is estimated at about 2 GHz, the "short bunch" expression applies when the bunches have only $\sigma_t = 18ps$. Thus a strong reduction of the effective impedance by a factor of about 9 will occur. The average beta function in the RF cavity region has been kept to less than 40 m in the design of LEP [5] in order to maximize the threshold of the transverse mode-coupling instability.

Much effort has been spent on shielding the vacuum chamber bellows, of which there are almost 3000 in LEP. They have a chamber height slightly above half their width. Since the transverse impedance is varies with the second to third power of the radius, we expect their effect on the vertical tune shift to be about 5 times larger than on the horizontal one. The broad band resonant frequency of the bellows has been estimated to be larger than 8 GHz[3], and hence their effective impedance will be equal to the actual one even for the rather short 18 ps bunches. Furthermore, the average beta function at the bellows is about 75 m in both planes, almost twice the value at the RF cavities.

Substituting these conditions into the above expressions, we find two equations with two unknowns for the (vertical) transverse impedances of the RF cavities and the bellows, which we expect to be the two main contributors in LEP:

$$\begin{aligned} \frac{-\Delta Q_H}{\Delta I} &= \frac{R}{2\pi\sigma E/e} \left[2(\omega_r \sigma_t)^2 Z_T^{cav} \langle \beta \rangle_{cav} + 0.2 Z_T^{bel} \langle \beta \rangle_{bel} \right] \\ \frac{-\Delta Q_V}{\Delta I} &= \frac{R}{2\pi\sigma E/e} \left[2(\omega_r \sigma_t)^2 Z_T^{cav} \langle \beta \rangle_{cav} + Z_T^{bel} \langle \beta \rangle_{bel} \right] \end{aligned} \quad (10)$$

The measured frequency shifts per mA at LEP were[1]

$$\frac{\Delta Q_H}{\Delta I} = -0.070, \quad \frac{\Delta Q_V}{\Delta I} = -0.125, \quad (11)$$

The constant in front of the brackets is approximately $(1.78 \cdot 10^5)^{-1}$ for the bunch length of 18 ps at injection energy. Then the solution of these equations yields

$$\begin{aligned} Z_T^{cav} &\approx 2.23 M\Omega/m \\ Z_T^{bell} &\approx 0.16 M\Omega/m \end{aligned} \quad (12)$$

In spite of the seemingly much smaller value for the bellows, their contribution to the vertical tune shift for the natural bunch length at injection is just as large as that of the RF cavities. The total transverse loss factor is only slightly higher than original estimate of about $2 M\Omega/m$, hence we expect that the threshold for transverse mode-coupling instability will be near the originally estimated values of 3/4 mA/bunch, or 3 mA per beam, if the bunches are lengthened to $\sigma = 4cm$, e.g. with the wigglers which are already installed in LEP.

5 Conclusions

The bunch length in LEP can be measured only with limited accuracy since it is usually well below the diameter of the button pickups used presently. The corrections which have to be applied to the "raw data" are often larger than the measurements themselves. This situation should improve when the streak camera will become operational.

The longitudinal impedance depends on the third power of the bunch length, and is thus particularly sensitive to measurement errors. Nevertheless, we can obtain a reasonable estimate of the broad band impedance by using the machine itself for calibration. We thus find the value of approximately 1/4 Ohm for the low frequency limit of Z/n , while the effective impedance seen by a bunch of 16 ps RMS is only about 20 mOhm.

In the transverse plane, the total impedance is found by separating the contributions of the RF cavities and of the bellows. For the assumptions made, the transverse impedance of the cavities appears to be slightly higher than computed, while the bellows impedance comes out slightly lower. Nevertheless, the contribution of the bellows to the vertical tune-shift is actually larger than that of the RF cavities for the short bunches at injection. The total transverse impedance found thus is only slightly larger than estimated, and hence the threshold for the transverse mode-coupling instability is still expected near 3 mA per beam (4 bunches, lengthened to $\sigma \approx 4cm$).

References

- [1] D. Brandt et al, LEP Commissioning Note 6 (Oct.1989)
- [2] J.P. Koutchouk, Commissioning Note 10 (1989)
- [3] B. Zotter, CERN LEP Divisional report TH/87-34 (1987)
- [4] A. Hofmann, J. Maidment, LEP Note 168 (1979)
- [5] B. Zotter, LEP Note 448 (1983)
- [6] A. Hofmann et al, Commissioning Note 12 (1989)
- [7] J.P. Koutchouk, private communication

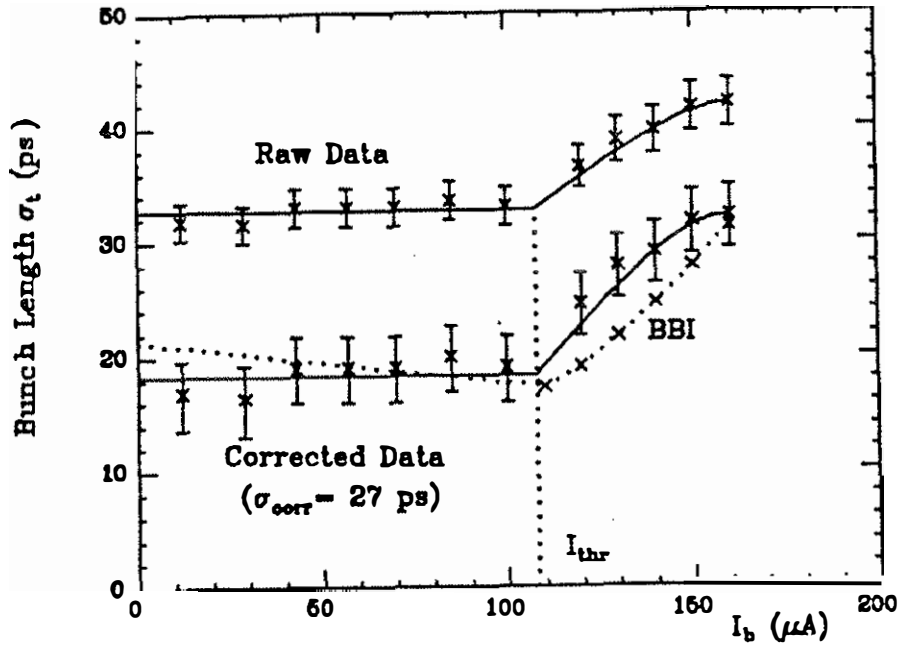


Figure 1: Variation of bunch length with current in a single bunch: raw data, corrected values, and results from BBI.

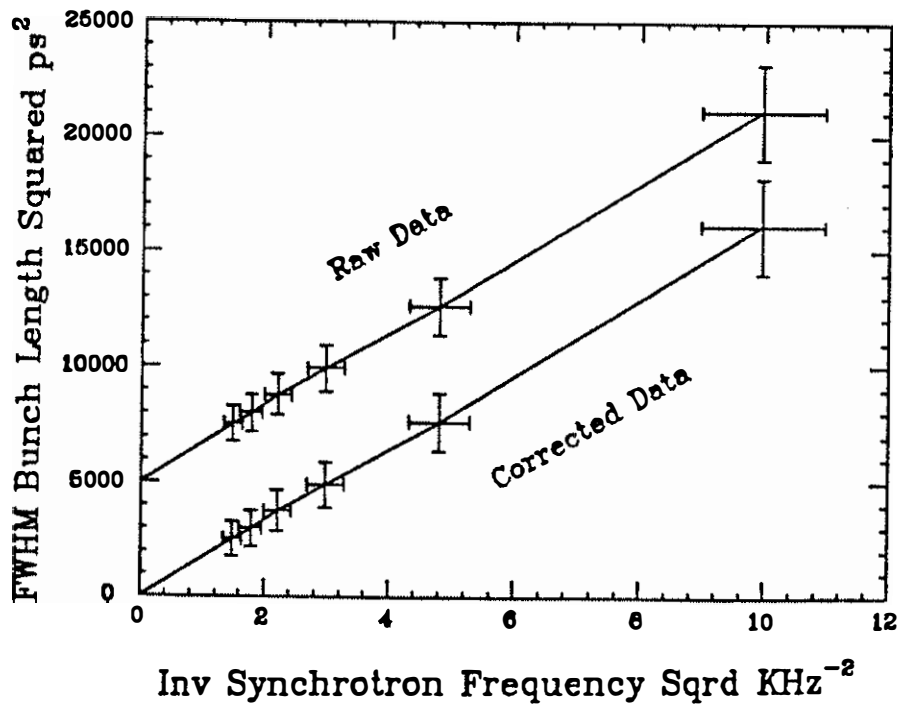


Figure 2: Variation of (squared) bunch length with RF voltage: the upper curve shows "raw data", the lower one corrected values.

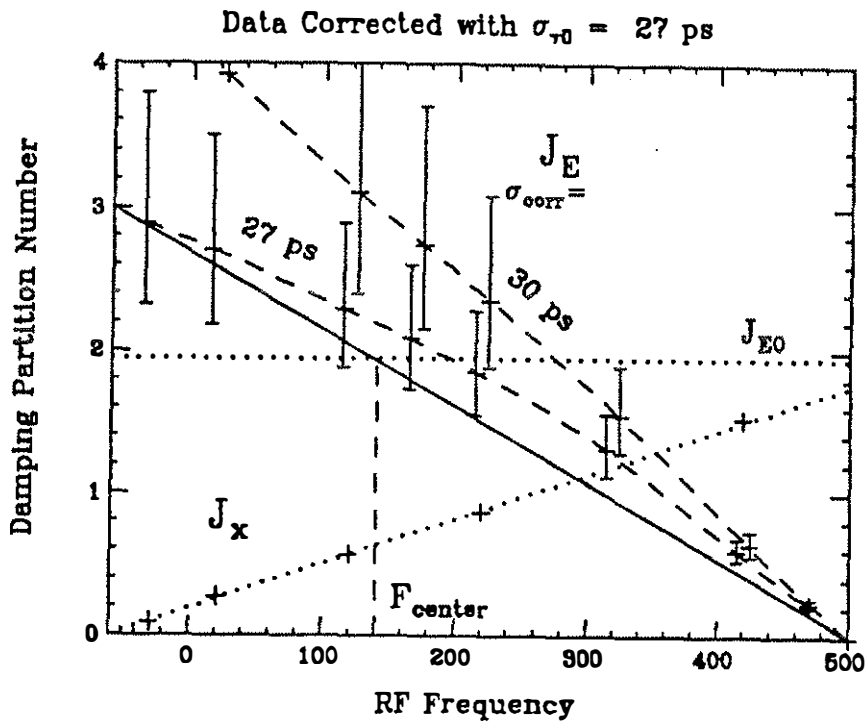


Figure 3: Variation of damping partition number J_E with RF frequency at 20 GeV for two different corrections of raw data: upper line - 30 ps, lower - 27 ps, solid - theoretical. For comparison we also show the measured damping partition number J_x (dotted line).

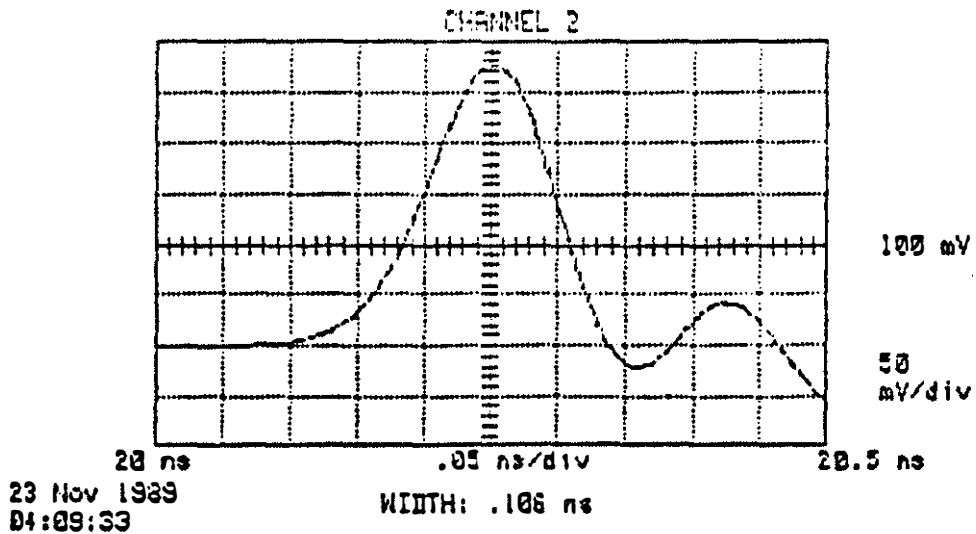


Figure 4: Signal from sampling oscilloscope connected to pickup for short bunch length: the horizontal scale is 50 ps/div.