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SIMULATION OF INSTABILITIES IN THE PRESENCE OF BEAM FEEDBACK

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Abstract

The effect of longitudinal and transverse instabilities in electron storage rings is simulated by tracking many superparticles for many turns through a model of a machine lattice. This lattice model is defined by a series of machine elements such as RF stations (including longitudinal and transverse wake fields), beam pick-ups, feedback kicker magnets etc. The machine elements may be interconnected in any specified way so as to produce for example feedback on the longitudinal or transverse beam motion. Each superparticle is treated in six-dimensional phase space and the effects of quantum excitation and radiation damping are included. Insofar as possible the program has been structured to allow study of all known single-beam effects (such as synchro-betatron resonances, transverse mode coupling etc.) in the presence or the absence of some form of beam feedback. The primary goal of the program was to study the effect of a reactive beam feedback system on the threshold for transverse mode coupling.

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1. Introduction

The computer simulation technique has been widely and successfully employed for the study of the beam-beam effect[1,2,3)] in colliding beam accelerators. More recently simulations[4,5)] have been used to study phenomena produced by longitudinal and transverse wakefields. However, as far as the authors are aware there have been no simulations written to deal with the response of single-beam effects to external feedback.

In nearly all electron accelerators the single-beam intensity limitation is caused by either longitudinal or transverse instabilities. For the more recent larger electron storage rings the single-beam intensity limitation is caused by a transverse instability thought to be provoked by transverse mode coupling[6,7,8)]. The theory of transverse mode coupling has been applied[4)] to the LEP machine and a threshold current of around 3 mA per beam of four bunches has been found. A reactive feedback system has been proposed[9)] to increase the threshold current for this instability. A simplified analytical model[10)] has shown that such a feedback system should significantly increase the limit for transverse stability. Subsequent improvements and extensions to this model[11,12,13)] have removed some of the worrying simplifications. However even in its most extended form, the model neglects many possibly important effects. In view of the importance of the subject to the LEP performance, a realistic computer simulation has been developed to study this problem. From the outset the philosophy adopted was that the simulation should be structured in a realistic way so as to include <u>all</u> known single-beam effects and to allow easy future inclusion of effects not yet known.

The main aims however were to allow a realistic study of the effect of feedback systems on beam stability and to study synchro-betatron resonances under quasi-realistic conditions.

2. Description of Program

A simplified flow chart of the program FEEDBAK is given in Fig. 1. This flow chart shows only the basic program without any of the possible options and should not be considered as fixed since the (user-specified) options may considerably change the program flow chart.

$$\begin{pmatrix} x \\ x' \end{pmatrix}_2 = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_1 \qquad (1)$$

where the matrix elements are given by

$$\begin{aligned} t_{11} &= \sqrt{\frac{\beta_2}{\beta_1}} (\cos \Delta\mu + \alpha_1 \sin \Delta\mu) \\ t_{12} &= \sqrt{\beta_1 \beta_2} \sin \Delta\mu \\ t_{21} &= \frac{\cos \Delta\mu[\alpha_1 - \alpha_2] - \sin \Delta\mu[1 + \alpha_1 \alpha_2]}{\sqrt{\beta_1 \beta_2}} \\ t_{22} &= \sqrt{\frac{\beta_1}{\beta_2}} (\cos \Delta\mu - \alpha_2 \sin \Delta\mu) \end{aligned} \qquad (2)$$

and $\Delta\mu$ is the betatron phase advance along the lattice

α is $-\beta'/2$

If the 'chromaticity' option is requested then the $\Delta\mu$ is evaluated for each particle using the calculated energy deviation ($\Delta E/E$) of that particle.

(ii) Longitudinal motion

Longitudinally the traversal between elements is accompanied by an energy loss and an RF phase change given by

$$\Delta E_{rad} = a_0 + a_1 \Delta E + \alpha_2 \Delta E^2 + a_3 \Delta E^3 \qquad (3)$$

$$\Delta\phi_{RF} = \frac{h\Delta\ell}{\gamma_t^2 R} \; \frac{\Delta E}{E_s} \qquad (4)$$

where a_0 is the average energy loss per turn

ΔE is the energy deviation

2.2 RF stations

Each superparticle which crosses an RF station 'experiences' energy changes due to

i) the externally applied electric field in the gap of the cavity

and

ii) the summation of the longitudinal wake fields set up by all the superparticles in advance of itself.

The superparticle motion in betatron phase space is also changed due to

i) non-zero momentum dispersion at the cavity location

and

ii) the summation of the transverse wake fields set up by all particles in advance of the particle under scrutiny.

At each RF station the cavity voltage, stable phase angle and frequency (to allow for higher harmonic systems) may be different. In addition a closed orbit error can be user-specified so as to simulate the situation where synchro-betatron resonances can be excited by the beam traversing the cavity off axis$^{16)}$.

The longitudinal and transverse delta function wake fields have been evaluated$^{14)}$ for the geometry of the LEP cavities. These wake fields are read from files and used for the simulation.

i) Longitudinal motion

As previously stated each superparticle of the simulated beam receives energy changes in the RF stations (ΔE_{RF}) due to the externally-applied electric fields and the induced longitudinal wake fields, i.e.

$$\Delta E_{RF} = eV(\phi,\phi_s) - e\Delta V_\ell \qquad (8)$$

where $V(\phi,\phi_s)$ is the voltage at the RF station

ϕ is the RF phase angle of the particle

ϕ_s is the stable phase angle

$$\Delta u' = \frac{e\Delta V_t}{E_s} \tag{12}$$

where D_{uRF} is the momentum dispersion at the RF
and ΔV_t is the transverse voltage caused by the transverse wakefield

$$V_t = \frac{i_b N_{CELL}}{N_s f_{rev}} \sum_j W_t(\Delta t_j) \frac{d_j}{r_{tube}} \tag{13}$$

where $W_t(t)$ is the delta function transverse wake field dependence (per cell) on time evaluated at the tube radius of the cavity (r_{tube})

d_j is the position of the particle which sets up the wake field

It should be noted here that d_j may be the superposition of the betatron motion (u_j), the momentum motion ($D_{uRF} \cdot \Delta E_j/E_s$) as well as any steady state closed-orbit errors at the cavity location. Consequently synchro-betatron resonances due to dispersion[17)] and/or orbit distortion at the RF[16)] may be simulated.

2.3 Beam position detectors (pick-ups)

For the purposes of the simulation the sole function of the beam position detectors is to provide a signal proportional to the position of the centre of gravity of the beam at the location of the detector. This signal is then used to determine the magnitude and sign of the kick to be applied at the beam deflector (kicker). In the simulation three types of pick-up are implemented. They all deliver a signal given by

$$u_{pu} = \left\langle u_i + D_{un} \frac{\Delta E_i}{E_s} \right\rangle_i + \text{noise}_n \tag{14}$$

control systems. These transfer characteristics are input as 'breakpoints' specifying the abscissa (i.e. the measured displacement at the pick-up) and the ordinate (the kick) at each point where the slope changes. This piecewise linear approach was preferred to a polynomial approach as the latter needs more terms for comparable accuracy over a wide range of pick-up values, and is much less suited to approximating common non-linear effects such as saturation. In order to generalise the approach each type of kicker may be 'connected' to any number of pick-ups of any type. In addition a vertical (horizontal) kicker may also produce a horizontal (vertical) kick, thereby simulating the effect of tilted kickers. The effect of noise is included in the pick-ups.

2.4.1 Injection damping kickers

A pulsed quadrupole may be used (in principle) to damp the centre of gravity motion of the injected beam. The simulation may be used to investigate the effect of such a system on the existing beam.

Briefly, the slope of the pulsed quadrupole is controlled by the signal from a pick-up, consequently the kick given to each particle is (for $\pi/2$ phase advance between the pick-up and the kicker)

$$\Delta u' = q(\bar{u}_{pu}) \cdot |u_k| \qquad (16)$$

where the function $q\ (u_{pu})$ is determined again by a linear combination of piecewise linear transfer characteristics and u_k is the particle position at the kicker. The effect of equation (16) is that the damping of the centre of gravity of the injected beam is dependent on its betatron amplitude. The use of a quadrupole ensures that the centre of gravity of the existing beam is unaffected.

2.5 Injection

The injection option allows simulation of a beam injected in betatron or synchrotron space with a beam already existing in the machine. The main purpose of this option was to study the behaviour of the injected and existing beam under the influence of beam feedback and wake fields. The parameters to be specified by the user are the number of turns between two injections, the number of superparticles per injection the program. However in order to see the effect of feedback against beam instabilities this option is very useful in providing the instabilities which are to be fought with the feedback option. This option has also been found useful for comparison of the simple step function wake field (used in two-particle models) with the realistic wake field$^{13)}$.

3.4 Quantum excitation and synchrotron damping

The simulation of quantum excitation and synchrotron damping are described in the section on the lattice. This has been installed in the program as an option so as to allow identification of phenomena which are enhanced by quantum noise or alleviated by synchrotron damping.

3.5 Noise

The addition of noise to the pick-up signal has already been described in the pick-up section. This option allows the study of varying amounts of noise on the effect of the feedback loop.

3.6 Coupling

The coupling of transverse energy from the horizontal plane to the vertical one can cause an apparent blow-up in the vertical beam size with a reduction in the luminosity. This option provides a coupling from the horizontal to the vertical plane in order to study the combined effect of feedback in one or two planes with coupling between the planes.

3.7 Chromaticity

Some head-tail instabilities are influenced by the dependence of the tune on energy deviation (chromaticity). This option causes the tune of each superparticle to have a component which depends on its energy deviation, i.e.

$$Q_i = Q_o + Q'\left(\frac{\Delta E}{E_s}\right)_i \tag{17}$$

This of course means that the lattice transfer matrices become dependent on the energy deviation of each superparticle. Consequently the transfer matrices must be calculated for each particle on each sector and cannot be calculated in advance as in the case for zero chromaticity. For this reason this option slows down considerably the running of the program.

it is therefore usually impossible to check quantitatively the results of any simulation until the machine is actually built. However a simulation can be extremely useful for obtaining some intuitive understanding of the physics behind any given phenomenon. This physical understanding can only be obtained if sufficient post-processing and analysis of the results are performed inside the program itself.

The usual outputs of a simulation program are the beam sizes (rms value) as a function of the number of turns. In addition to these parameters it is also very instructive to evaluate the following parameters.

4.1 Approximate invariants of motion

The Courant-Schneider invariant of motion is calculated for every particle. The mean and the rms values are also determined so as to be able to differentiate between coherent motion, incoherent growth and mismatch.

4.2 Tune values and distributions

The evaluation of the phase advance per turn allows the calculation of the non-integer part of the tune value. This value averaged over many turns (typically 40 to 100 depending on the tune) gives an accurate indication of the particle tune. Suitable binning of the results provides the distribution of the particle tune. When the same procedure is applied to the centre of gravity of the beam then the coherent tune value can be accurately obtained. This is particularly informative in cases where the tune value changes as a function of some other beam parameters such as feedback or wake fields. In this simulation program the particles' tunes (centres of gravity and distribution) are evaluated in all three phase planes.

4.3 Frequency spectra of centre-of-gravity motion

The position of the centre of gravity of the beam is measured on each turn and over many turns a fast Fourier transform of the motion is performed. In this way, for example, the shift in frequency of the head-tail mode $m = 0$ can be seen in relation to the higher modes which also have some centre-of-gravity motion.

functions. Note that these modes also take on negative values of number of particles. From equation (18) it is clear, however, that these modes have to be interpreted as the number of particles relative to the mean number. This mean number of particles is actually represented by the mode $H_0(x) \cdot H_0(y) \cdot H_0(\phi)$. Note also that these modes cannot provide any information about the coherent motion of the bunch since the three-dimensional window in which the modes are defined moves along with the bunch. This is not a drawback specific to the Hadamard functions however and it is remedied by the calculation of the motion of the centre of gravity of the bunch.

In performing a modal analysis, the user selects up to four combinations of three one-dimensional Hadamard functions for the x, y and ϕ dependence. The program then calculates the coefficients $a_i P_i(n\Delta t_s)$ of each selected mode, sampled at a user-specified sampling interval Δt_s. The resulting time series is Fourier transformed via the Fast Fourier Transform (FFT), after suitable precautions against artefacts like leakage$^{20)}$. This yields the frequency spectrum of the modes with the chosen spatial dependence. For the details of the computational procedure we refer to reference 20). The user can also specify the desired number of frequency spectra and how frequently and when he wants them to be calculated during the simulation. This allows valuable CPU time to be reserved for the actual simulation. This is important because the computational overhead inherent to the FFT and the calculation of the coefficients $a_i P_i(t)$ is still considerable, notwithstanding the computational advantages of the chosen modal expressions.

5. Example of the Use of the Simulation Program

The simulation program has been used$^{13)}$ to investigate the transverse mode coupling instability in the presence of reactive feedback for LEP. To this end the longitudinal and transverse delta function wake fields were computed and input to the program.

The results of a simulation of LEP at 51.5 GeV with two RF stations per turn are shown in Fig. 3. It can be seen that without feedback the measured coherent tune decreases fairly linearly with intensity until the tune shift is around 0.75 times the synchrotron tune. Further increase in intensity causes the beam to become transversely unstable (depicted by the sudden increase in the beam size). When the reactive feedback is cavities from bunches in one beam to bunches in the same beam or even in the other beam. There is also the possibility of injection for both beams and feedback from any bunch to any other bunch.

Further work is under way to implement an optimization routine which could be used to determine the optimum feedback parameters for a given realistic machine configuration. In addition it is intended to include the time response of the feedback system, including filtering of signals etc., as well as analysing the coherent multi-bunch instabilities resulting from long-term wake fields. Progress in these directions will be reported in due course.

7. Conclusions

The effect of longitudinal and transverse wake fields may be examined in the presence of longitudinal and transverse feedback by using a fairly complete simulation program. This simulation has the possibility (by using various options) of including a very wide range of known single-beam effects such as various driving mechanisms for synchro-betatron resonances. The program was written with the initial objective of studying the effect of reactive feedback on the threshold for the transverse mode coupling instability. This objective has been attained and the program may now be used to study practically any other single-beam effect.

Acknowledgements

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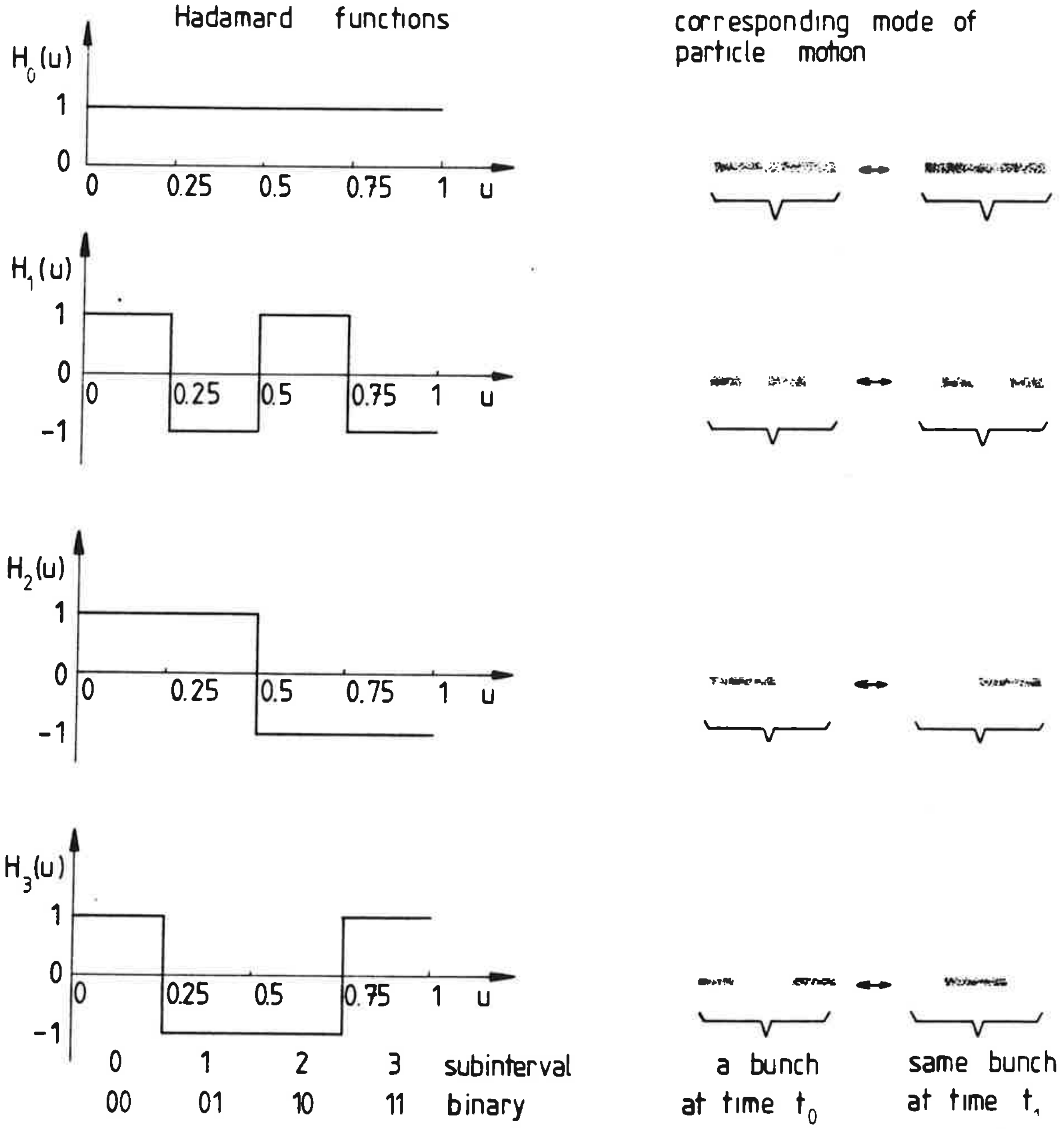

a)

b)

Fig. 2 a) the Hadamard functions on four subintervals
b) their interpretation in terms of one-dimensional particle motion

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LIST OF LEP REPORTS PUBLISHED

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