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The production of large mass lepton pairs and weak bosons in hadronic collisions can provide important tests of perturbative QCD. In this talk I will review a recent study [1] of the production of vector bosons V ($=\gamma^*$,W,Z) of mass M at small transverse momentum q_T ,in which the predictions of a complete two loop analysis are confronted with experimental data.

Following the analyses of [2] and [3] the cross section at small transverse momentum, $q_T \ll M, \sqrt{s}$,can be written

$$\frac{d^2\sigma^{H_1+H_2\to V+\cdots}}{dq_T^2\,dy} = \frac{2\pi^2\alpha}{3s}\int_0^\infty db\, b\, J_0(bq_T)\, e^{-S(b,M)}\, F\left(x_1,x_2;\frac{b_0}{b}\right)$$

$$x_1 = \frac{M}{\sqrt{s}}e^{y} \qquad x_2 = \frac{M}{\sqrt{s}}e^{-y} \qquad b_0 = 2e^{-\gamma_E} \tag{1}$$

up to corrections of order q_T/M . In this result

1. exp(-S) is a 'Sudakov form factor':

$$S(b,Q) = \int_{\left(\frac{b_0}{b}\right)^2}^{Q^2}\frac{dq^2}{q^2}\left[A(\alpha_s(q))\ln\frac{Q^2}{q^2} + B(\alpha_s(q))\right]$$

$$A(\alpha_s) = \sum_{i=1}^{\infty}\left(\frac{\alpha_s}{2\pi}\right)^i A^{(i)} \qquad B(\alpha_s) = \sum_{i=1}^{\infty}\left(\frac{\alpha_s}{2\pi}\right)^i B^{(i)} \tag{2}$$

2. F is a product of structure functions:

$$F(x_1,x_2;Q) = \sum_{ij}\sigma_{ij}\left\{D_{q_i/H_1}(x_1,Q)\,D_{\bar{q}_j/H_2}(x_2,Q) + q\leftrightarrow\bar{q}\right\} \tag{3}$$

where the σ_{ij} are the appropriate electroweak charges.

The Q evolution of the moments of the structure functions is, as usual, controlled by (a) universal anomalous dimensions $\gamma_N^{(i)}$ and (b) process dependent coefficient functions $C_N^{(i)}$.

In [1] the above cross section was compared to an exact order α_s^2 parton-level calculation. It was found that all large logarithms of M/q_T and M/μ (μ is the renormalisation scale) were correctly accounted for, at least to second order in perturbation theory. In addition the two loop coefficients $A^{(2)}$ and $B^{(2)}$ were determined, and the universality of the two loop anomalous dimension was verified. The results of the calculation are ($\overline{\text{MS}}$ convention):

$$A^{(1)} = 2C_F \qquad B^{(1)} = -3C_F$$

$$A^{(2)} = 2C_F\left[\left(\frac{67}{18}-\frac{\pi^2}{6}\right)C_A - \frac{10}{9}T_R\right] \qquad B^{(2)} = C_F^2\left[\pi^2-\tfrac{3}{4}-12\zeta(3)\right] + C_FC_A\left[\frac{11}{9}\pi^2 - \frac{193}{12} + 6\zeta(3)\right] + C_FT_R\left[-\frac{4}{9}\pi^2 + \frac{17}{3}\right]$$

$$\gamma_N^{(i)} = \int_0^1 dx\, x^N\left[P^{(i)}(x)\right]_+ \qquad P^{(1)}(x) = \frac{1+x^2}{1-x}\cdot C_F$$

$$\begin{aligned}P^{(2)}(x) = {} & C_F^2\left[-2\frac{1+x^2}{1-x}\ln x\ln(1-x) - \left(2+4x+\frac{3}{1-x}\right)\ln x\right. \\ & \left. -\frac{1}{2}(1+x)\ln^2 x - 9(1-x) + 2\frac{1+x^2}{1+x}S(x)\right] \\ & + C_FC_A\left[\left(\frac{1+x^2}{1-x}\right)\left(\frac{1}{2}\ln^2 x + \frac{11}{6}\ln x + \frac{67}{18} - \frac{\pi^2}{6}\right)\right. \\ & \left. + 2(1+x)\ln x + \frac{26}{3}(1-x) - \frac{1+x^2}{1+x}S(x)\right] \\ & + C_FT_R\left[\frac{1+x^2}{1-x}\left(-\frac{2}{3}\ln x - \frac{10}{9}\right) - \frac{4}{3}(1-x)\right]\end{aligned}$$

$$S(x) = -\frac{1}{2}\ln^2 x + 2\ln x\ln(1+x) + 2\,\mathrm{Li}_2(-x) + \pi^2/6$$

$$C_N^{(1)} = C_F\left[\pi^2/2 - 4 + \frac{1}{(N+1)(N+2)}\right] \tag{4}$$

Note that

1. the anomalous dimensions extracted from the small q_T cross section agree with those obtained by the usual methods. [4]

2. the $A^{(i)}$ coefficients can be related to the $x\to 1$ behaviour of the corresponding $P^{(i)}(x)$ functions. Thus if

$$P^{(i)}(x) = \frac{k^{(i)}}{1-x} + r^{(i)}(x) \tag{5}$$

then $A^{(i)}=k^{(i)}$ (i=1,2). This was in fact how $A^{(2)}$ was first determined [3]. It is also interesting to note that

$$B^{(1)}(x) = 2\int_0^1 dx\, r^{(1)}(x)$$

$$B^{(2)}(x) = 2\int_0^1 dx\, r^{(2)}(x) + \beta_0 C_F\left(8-\frac{4\pi^2}{3}\right) \tag{6}$$

3. in [2] the Sudakov form factor has the more general representation

$$S(b,Q) = \int_{c_1^2b^{-2}}^{c_2^2Q^2}\frac{dq^2}{q^2}\left[\hat{A}\ln\frac{c_2^2Q^2}{q^2} + \hat{B}\right] \tag{7}$$

Different values of c_1, c_2 correspond to different factorization and renormalization prescriptions. The invariance of S under such scheme changes can be used to relate the general coefficients $\hat{A}^{(i)}(c_1,c_2)$, $\hat{B}^{(i)}(c_1,c_2)$ to those of eqn.4, which correspond to $c_1=b_0=2\exp(-\gamma_E)$, $c_2=1$. Thus

$$\hat{A}^{(1)} = A^{(1)}, \quad \hat{B}^{(1)} = B^{(1)} - 2A^{(1)}L_2$$

$$\hat{A}^{(2)} = A^{(2)} - 2\beta_0 A^{(1)}L_1$$

$$\hat{B}^{(2)} = B^{(2)} - 2\beta_0 B^{(1)}L_1 + 8\beta_0 A^{(1)}L_1L_2 - 2A^{(2)}L_2$$

$$L_1 = \ln\left(\frac{2}{c_1}\right) - \gamma_E \qquad L_2 = L_1 + \ln c_2 \tag{8}$$

4. the above results are valid only for the non-singlet part of the cross section. The extension to the singlet case is discussed in [1]

As it stands, the cross section given in eqn.1 cannot be used for numerical predictions. This is because the functions S and F which appear inside the b integral can only be calculated perturbatively up to $b=O(\Lambda^{-1})$. The large b behaviour must be parametrized and determined from data. There are of course arbitrarily many ways of doing this. For example [2] the variable

$$b^* = b\left[1 + b^2 Q_0^2\right]^{-\frac{1}{2}} \tag{9}$$

is such that $b^* \to Q_0^{-1}$ as $b \to \infty$. A value for Q_0 can be chosen such that when b is replaced by b^* in S and F these functions are always evaluated at scales where perturbation theory should apply. The non-perturbative region is then represented by

$$\exp\left[-g_1(b)\ln\frac{Q^2}{Q_0^2} - g_2(b;x_1,x_2)\right] \tag{10}$$

In [1] the functions g_1 and g_2 were fixed by fitting FNAL and ISR lepton pair data. Fig.1 shows the resulting prediction for W production at the CERN $p\bar{p}$ collider, compared to data from the UA1 collaboration. The two curves correspond to the two sets of structure functions given in [5]. The difference in shape is due mainly to the different values of Λ (.2 GeV and .4 GeV) used in the two sets. Within the limited statistics available, the data is clearly quite well described. Note that the effects of non-perturbative smearing become negligible for q_T values >5 GeV. It is interesting to study the contribution to the cross section of the various higher order coefficients in the Sudakov form factor. Fig.2 shows the effect on the W cross section at $\sqrt{s}$=630 GeV of "switching off" the coefficients $B^{(2)}$, $A^{(2)}$ and $B^{(1)}$ in turn. The size of the the contribution to the cross section evidently decreases in the order $A^{(1)} > B^{(1)} > A^{(2)} > B^{(2)}$ indicating that the perturbative series is apparently well-behaved.

The conclusion therefore is that W and Z production in the region $5 < q_T < 15$ GeV is insensitive to non-perturbative effects and should be firmly predicted by QCD. The main uncertainty in this region is the value of Λ.

The work reported in this talk was the result of a most stimulating and enjoyable collaboration with Christine Davies and Bryan Webber.

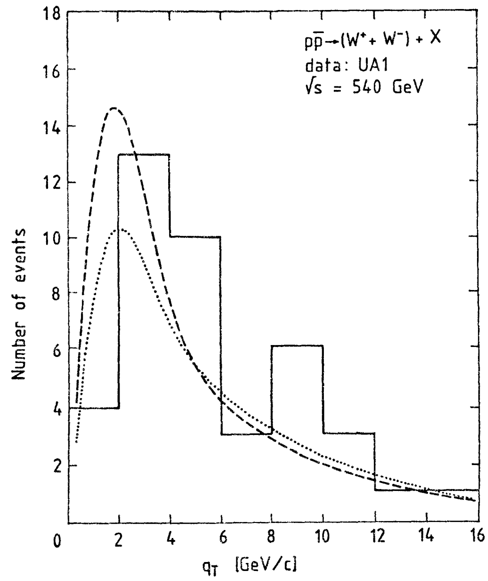

Fig.1 : The cross section for W production at $\sqrt{s}$ = 540 GeV in $p\bar{p}$ collisions. The dashed, dotted lines use set 1,2 structure functions from [5] respectively. The data are from the UA1 $W \to e\nu$ events.

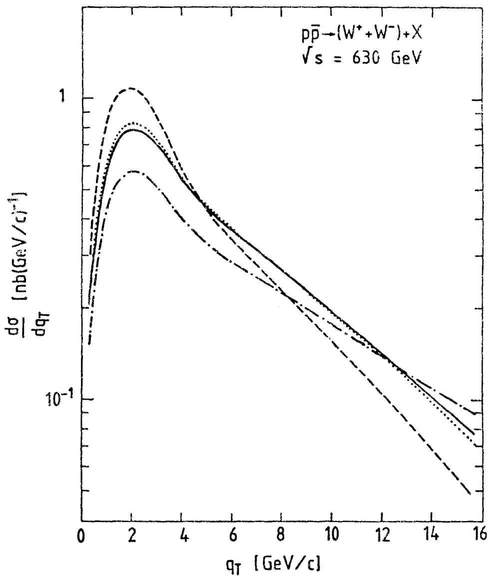

Fig.2 : The effect on the W cross section of successively adding the calculated coefficients to S(b,Q). $A^{(1)}$ only (dashed-dotted line); add $B^{(1)}$ (dashed line); add $A^{(2)}$ (dotted line); add $B^{(2)}$ (solid line).

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