Determination of ⁸**B(**p**,**γ**)** ⁹**C reaction rate from** ⁹**C breakup**

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Abstract

The astrophysical factor of the ${}^{8}B(p,\gamma){}^{9}C$ at zero energy, $S_{18}(0)$, is determined from three-body model analysis of ⁹C breakup processes. The elastic breakup $^{208}Pb(^{9}C, p^{8}B)^{208}Pb$ at 65 MeV/nucleon and the one-proton removal reaction of ${}^{9}C$ at 285 MeV/nucleon on C and Al targets are calculated with the continuum-discretized coupled-channels method (CDCC) and the eikonal reaction theory (ERT), respectively. As a result of the present analysis, $S_{18}(0)$ extracted from the two reactions show good consistency, in contrast to in the previous studies.

1 Introduction

In low-metallicity supermassive stars, the proton capture reaction of ${}^{8}B, {}^{8}B(p,\gamma){}^{9}C$ ignites the explosive hydrogen burning [1]:

$$
{}^{8}B(p,\gamma) {}^{9}C(\alpha,p) {}^{12}N(p,\gamma) {}^{13}O(\beta^{+}\nu) {}^{13}N(p,\gamma) {}^{14}O.
$$

This process called hot pp chain is expected to be a possible alternative path to the synthesis of the CNO elements. Because of the difficulties in measuring the ${}^{8}B(p,\gamma){}^{9}C$ cross section $\sigma_{p\gamma}$ at very low energies, several alternative reactions have been proposed [2–4] to indirectly determine the astrophysical factor $S_{18}(\varepsilon)$

$$
S_{18}(\varepsilon) = \sigma_{p\gamma}\varepsilon \exp[2\pi\eta].\tag{1}
$$

Here, ε is the relative energy between p and ⁸B in the center-of-mass (c.m.) frame and η is the Sommerfeld parameter. Because an astrophysical factor has quite weak energy dependence, several previous studies have paid special attention to the evaluation of $S_{18}(\varepsilon)$ at zero energy, $S_{18}(0)$ [1–5]. Experimental results seem to support the $S_{18}(0)$ obtained by a cluster model calculation [5]. There is, however, still a significant discrepancy of about 50% between the $S_{18}(0)$ obtained by Coulomb dissociation method [4] and the ANC method [2, 3].

In this paper, we reinvestigate the Coulomb dissociation [4] (elastic breakup) and the proton removal process [3] of ⁹C by means of coupled-channel calculation with a three-body ($p + {}^{8}B + \text{target}$) model. We adopt the continuum-discretized coupled-channels method (CDCC) [6–8] for the former and the eikonal reaction theory (ERT) [9, 10] for the latter; we use the ANC method [11] for both reactions. The main purpose of the present study is to show the consistency between the two values of $S_{18}(0)$ extracted from these two types of breakup, and thereby determine $S_{18}(0)$ with high reliability.

2 Theoretical framework

In Fig. 1 we show schematic illustration of the three-body $(p + {}^{8}B + \text{target})$ system. The scattering between ${}^{9}C$ and a target nucleus A is described by the Schrödinger equation

$$
\left[-\frac{\hbar^2}{2\mu}\nabla_{\mathbf{R}}^2 + h + U(r_p, r_B) - E\right]\Psi(\mathbf{r}, \mathbf{R}) = 0, \tag{2}
$$

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Fig. 1: Illustration of the three-body system.

where $\Psi(r, R)$ is the tree-body wave function and $r(R)$ is the coordinate of ⁸B (⁹C) relative to p (A). The reduced mass between ${}^{9}C$ and A is denoted by μ and E is the total energy of the three-body system in the c.m. frame. The internal Hamiltonian of ⁹C is shown by h. The interaction $U(r_p, r_B)$ is given by

$$
U(r_p, r_B) = V_p^{(N)}(r_p) + V_p^{(C)}(r_p) + V_B^{(N)}(r_B) + V_B^{(C)}(r_B),
$$
\n(3)

where $V_{\text{X}}^{(N)}$ and $V_{\text{X}}^{(C)}$ are the nuclear and Coulomb interactions, respectively, between X and A; X represents a fragment particle of the projectile, i.e., p or ${}^{8}B$. Similarly, r_X denotes the relative distance between X and A.

In the present analysis of the elastic breakup of ${}^{9}C$, we solve Eq. (2) with eikonal-CDCC (E-CDCC) [12, 13]. E-CDCC assumes eikonal approximation to the scattering wave between ${}^{9}C$ and A. As a result, the total wave function $\Psi(r, R)$ is expressed by

$$
\Psi(\mathbf{r}, \mathbf{R}) = \sum_{c} \Phi_c(\mathbf{r}) e^{-i(m-m_0)\phi_R} \psi_c(b, z) \phi_{\mathbf{K}_c}^C(b, z), \tag{4}
$$

where $\Phi_c(\mathbf{r})$ is the internal wave function of ⁹C with c the channel indices {i, ℓ , S, I, m}; i > 0 $(i = 0)$ stands for the *i*th discretized-continuum (ground) state, and ℓ , S, and I are, respectively, the orbital angular momentum, the channel spin, and the total angular momentum of the p and ${}^{8}B$ system. m is the projection of I on the z-axis taken to be parallel to the incident beam; m_0 is the value of m in the incident channel. b is the impact parameter defined by $b = \sqrt{x^2 + y^2}$ with $\mathbf{R} = (x, y, z)$ in the Cartesian representation. The use of the Coulomb incident wave $\phi_{\mathbf{K}_c}^{\text{C}}(b, z)$ instead of the plane wave $\exp(K_c \cdot R)$ in the eikonal approximation is one of the most important features of E-CDCC; K_c is the asymptotic wave-number vector of ${}^{9}C$ in channel c from A. In the actual calculation, we use an approximate asymptotic form of $\phi_{\mathbf{K}_c}^{\mathbf{C}}(b, z)$. E-CDCC is shown to work very well for describing both the nuclear and Coulomb breakup processes with high accuracy and computational speed [12, 13].

The one-proton removal reaction, its stripping component in fact (see below), is analyzed by means of the eikonal reaction theory (ERT) [9, 10], which can calculate an inclusive cross section, such as a nucleon removal cross section, in the CDCC framework. ERT uses a formal solution (the scattering matrix S) to the coupled-channel equations of E-CDCC, and makes adiabatic approximation to only the nuclear part of S. Then one can obtain the most important result of ERT, i.e., the product form of S [9]

$$
S = S_{\rm b} S_{\rm c},\tag{5}
$$

where S_b and S_c show the contributions from the constituents b and c of the projectile, respectively. At this stage, however, this result can be derived only when b or c is chargeless, which is not the case for the ${}^{9}C$ projectile consisting of p and ${}^{8}B$. Therefore, in the present study, we neglect the Coulomb breakup process in the one-proton removal process and replace the Coulomb interaction $V_p^{(\text{C})}(\mathbf{r}_p)$ with

$$
V_p^{(C)}(r_p) \to V_p^{(C)}(R). \tag{6}
$$

Then we can calculate the one-proton removal cross section σ_{-p} with

$$
\sigma_{-p} = \sigma_{\text{bu}} + \sigma_{\text{str}},\tag{7}
$$

as in Refs. [9, 10]. In Eq. (7), σ_{bu} and σ_{str} denote the elastic breakup cross section and the stripping cross section, respectively; ERT is used to evaluate σ_{str} . The accuracy of the replacement of Eq. (6) can be examined by calculating σ_{-p} with and without the Coulomb breakup. It is confirmed that the Coulomb breakup contributes to σ_{-p} for C and Al targets by about 5%. Thus, we conclude that the Coulomb breakup by these two targets can be neglected with 5% errors. Below we include this amount in uncertainties of $S_{18}(0)$ extracted from σ_{-p} . The detail of our numerical setups are shown in the Ref. [14].

3 Results and discussion

Fig. 2: Breakup spectrum of the ²⁰⁸Pb(${}^{9}C, p{}^{8}B$)²⁰⁸Pb at 65 MeV/nucleon as a function of the relative energy ε between p and ⁸B. The dashed line shows the result of calculation with a normalized p -⁸B wave function, whereas the solid line is the result multiplied by 1.1 to fit the experimental data $[4]$.

First, we analyze the elastic breakup $^{208}Pb(^{9}C,p^{8}B)^{208}Pb$ at 65 MeV/nucleon. In Fig. 2, we show the breakup cross section as a function of the relative energy ε between p and ⁸B. We have included the experimental efficiency $e(\varepsilon)$ [15] and resolution Γ in the calculation. We adopt $\Gamma = 0.23$ MeV extracted from the experimental breakup spectrum of ${}^{12}C({}^{9}C,p{}^{8}B){}^{12}C$ at 65 MeV/nucleon [15]. In order to determine $S_{18}(0)$ we fit the theoretical result (dashed line) to the experimental data [4], and the solid line is obtained. The renormalization factor is 1.10, which results in $S_{18}(0) = 67.3$ eVb.

In Fig. 2, our calculation describes well the breakup spectrum below $\varepsilon \sim 1.0$ MeV, i.e., both the transition to the $1/2^-$ resonant state and breakup to low-energy nonresonant states of ⁹C. It should be noted that we treat the resonant and nonresonant breakup continua on the same footing in the CDCC calculation. In the higher ε region than the resonance energy, however, the calculation significantly underestimates the experimental data. It is expected that this is due to incompleteness of our present framework. The back-coupling effects of three-body breakup states of ⁹C to $p + p + 7$ Be on the $p + {}^{8}B$ state observed will become important as ε increases. In addition, more accurate description of the $p+{}^{8}B$ continua for higher partial waves with a proper p -⁸B interaction $V_{pB}^{(N)}$ will be needed. At low ε , these possible problems will not exist, because only the tail of the overlap between ${}^{9}C$ and p - ${}^{8}B$ contributes to the breakup process.

Table 1: Results of the one-proton removal reactions with ¹²C and ²⁷Al targets. The experimental data of σ_{-p} are taken from Ref. [17].

Target				
	calc.	expt.	calc.	expt.
σ_{-p} [mb]	44.9	48(8)	53.9	55(11)
$S_{18}(0)$ [eVb]	65.2		62.2	

Second, we analyze the one-proton removal reaction of ⁹C at 285 MeV/nucleon on ¹²C and ²⁷Al targets. We neglect the Coulomb breakup of ⁹C in this case. We calculate σ_{bu} by CDCC and the stripping cross section σ_{str} by ERT, and obtain the one-proton removal cross section σ_{-p} , as the sum of the two. Then we renormalize the calculated σ_{-p} to fit the experimental value taken from Ref. [17], which determines $S_{18}(0)$. These values are summarized in Table 1. One sees that the two results of $S_{18}(0)$, corresponding to ¹²C and ²⁷Al targets, agree well with each other. By taking an average of the two values, we obtain $S_{18}(0) = 63.7$ eVb.

We here remark that in our three-body coupled-channel analysis, the values of $S_{18}(0)$ extracted from two different breakup reactions, 67.3 eVb (elastic breakup) and 63.7 eVb (proton removal), show very good agreement. This indicates reliability of the present analysis and the result of $S_{18}(0)$. As a principal result of the present study, we obtain

$$
S_{18}(0) = 66 \pm 10 \text{ eVb.}
$$
 (8)

In Fig. 3, the $S_{18}(0)$ extracted by the present work is compared with previous values. Previous results mentioned above can be categorized into two, i.e., one is around 80 eVb (Ref. [4, 5]) and the other is around 45 eVb (Ref. [2, 3]). Our result exists in between them, slightly favoring the former.

In Ref. [4], the E1 contribution to the elastic breakup of ${}^{9}C$ by ${}^{208}Pb$ at 65 MeV/nucleon was extracted by subtracting the contributions of the nuclear and E2 breakup processes ($\sim 10\%$) from the measured cross section, with a help of the ${}^{9}C$ breakup data by ${}^{12}C$ at the same energy. The rather good consistency between the present and previous results of $S_{18}(0)$ will indicate that the procedure for extracting the E1 contribution worked quite well. It was reported in Ref. [4], however, that about 80% of the peak in the ²⁰⁸Pb(${}^{9}C_{v}P^{8}B$)²⁰⁸Pb breakup spectrum around $\varepsilon = 0.9$ MeV was explained by nonresonant E1 breakup processes. On the other hand, in the present analysis, the peak is found to be mainly generated by the nuclear and E2 transition to the 1/2[−] resonance state. Reason for this large discrepancy in the resonant part between the present and previous studies needs further investigation; this is our important future work. If we adopt a one-step calculation including nuclear and Coulomb breakup with all multipolarities, $S_{18}(0) = 54$ eVb is obtained, i.e., 20% difference appears. This behavior is the same as in the study of $S_{17}(0)$ for the ⁷Be(p,γ)⁸B reaction [12].

Our result is quite larger than the result of Ref. [3], in which the one-proton removal reactions $({}^{9}C, {}^{8}B)$ at 285 MeV/nucleon were analyzed by the extended Glauber model, with carefully evaluating the uncertainty regarding the nucleon-nucleon effective interactions (profile functions). By a detailed analysis, it is found that the difference between the $S_{18}(0)$ obtained in the present work and Ref. [3] is mainly due to the proton optical potential. In Fig. 4 of Ref. [3], the reaction cross section σ_R of the p^{-12} C (solid line) is compared with experimental data. As shown in the figure, the data have quite

Fig. 3: $S_{18}(0)$ extracted by this work (circle) is compared with the results of the Coulomb dissociation method (cross) [4] and the analysis of σ_{-p} with the extended Glauber model (triangle) [3]. Theoretical results with a cluster model calculation (squares) [5] and the value extracted from the $d(^{8}B, ^{9}C)n$ reaction (diamond) [2] are also shown.

large uncertainty; there seem to be two data groups between 250 MeV and 600 MeV. Our microscopic calculation based on the Melbourne q matrix gives $\sigma_R = 198$ mb at 285 MeV, which is smaller than the value used in the previous study by about 10%. It should be noted that both the theoretical values of σ_R are consistent with the experimental data, within their uncertainty mentioned above. This 10% difference is indeed crucial for the evaluation of σ_{-p} , which eventually gives the difference in $S_{18}(0)$ by about 35%. Thus, more accurate and reliable data of σ_R is highly desirable to judge the microscopic theoretical calculations of σ_R , although we have shown in this study a very good agreement between the two $S_{18}(0)$ extracted from different breakup reactions.

4 Summary

We have analyzed the elastic breakup of ${}^{9}C$ by ${}^{208}Pb$ at 65 MeV/nucleon and the one-proton removal reaction of ⁹C at 285 MeV/nucleon on C and Al targets by a three-body coupled-channel framework, i.e., CDCC for the elastic breakup process and ERT for the stripping process. We determined the astrophysical factor at zero energy, $S_{18}(0)$, for the ${}^{8}B(p,\gamma){}^{9}C$ reaction. Our principal result is $S_{18}(0) = 66 \pm 10$ eVb. We have confirmed that the results of $S_{18}(0)$ extracted from the two independent experimental data agree very well with each other, and thus resolved a significant discrepancy of $S_{18}(0)$ in the previous studies. Although the $S_{18}(0)$ is determined well in the present analysis, description of the breakup spectrum at higher p ⁸B relative energies is not sufficient. Extension of the present reaction model to incorporate the $p + p + 7$ Be configuration will be very important for deeper understanding of the breakup of ⁹C. Investigation on the $d(^{8}B, {}^{9}C)n$ transfer reaction, which gives a quite smaller $S_{18}(0)$ than in the present study, will also be important.

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