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ATOM SIZE OF RARE GASES FROM THEIR MAGNETIC BIREFRINGENCE

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ABSTRACT

We present a relation between the size of a monoatomic gas atom and the value of its magnetic birefringence. The results, in spite of the simplicity of the model, are in good agreement with the experimental data.

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The light propagation is not isotropic in a gas region where a magnetic field B is present. The magnetically induced birefringence consists in a difference in the two indices of refraction of light propagating orthogonally to the $\vec{B}$ direction, linearly polarized along $\vec{B}$ ($n_{\parallel}$) and orthogonally to it ($n_{\perp}$), respectively.

The birefringence is measured by the difference

$$\Delta n = n_{\parallel} - n_{\perp} , \qquad (1)$$

and it turns out that (Cotton-Mouton effect)

$$\Delta n = C\, B^2\, \lambda , \qquad (2)$$

where λ is the light wavelength and C is the Cotton-Mouton constant of the gas under study.

The Cotton-Mouton constant of rare gases has recently been accurately measured [1].

Theoretical estimates [2] of such a constant disagree by a factor of three with the experimental values, as shown in Table 1. The disagreement is possibly due to the use of the pseudopotential method in the derivation of the Cotton-Mouton constant.

In this letter we relate the Cotton-Mouton constant of a monoatomic gas to its first excited state linear dimensions, using a simplified atom model.

We show that the rare gas atom excited state dimensions derived from the experimental values of the Cotton-Mouton constant agree rather well with those of the alkali atom which follows the rare gas under study in the Mendeleev table. Our simplified model of monoatomic gas consists of an S ground state together with only a P excited state ($m=0,\pm1$), separated by an energy difference $\Delta E = \hbar\omega_0$; moreover only the ground state is populated. Without taking into account the electron spin, in the presence of a constant homogeneous magnetic field B directed along the z axis, the energy levels are split and shifted according to Fig.1. Apart from the Zeeman splitting

$$\omega_L = \mu B \ ; \quad \mu = |e|/2mc \ , \tag{3}$$

of the excited states, the quadratic term in B present in the perturbed Hamiltonian

$$\mathcal{H} = (1/2m)\ [\vec{p} - (e/c)\vec{r} \times \vec{B}]^2 \ + V \ , \tag{4}$$

produces (see Fig.1):

a) a shift of the ground state of $\hbar UB^2$;

b) a shift of $\hbar WB^2$ of the $m = 0$ excited state;

c) a shift of $2\hbar WB^2$ of the $m = \pm1$ excited states;

where

$$U = (\alpha/12\ mc) \int dr\ r^4\ |R_0(r)|^2 = (\alpha/12mc)\ \langle r_0^2 \rangle\ , \tag{5}$$

$$W = (\alpha/20\ mc) \int dr\ r^4\ |R_1(r)|^2 = (\alpha/20mc)\ \langle r_1^2 \rangle\ , \tag{6}$$

α being the fine structure constant, R_1 and R_0 the radial functions of the excited and ground state, respectively.

In the hypothesis of no magnetic polarizability, the birefringence of a medium is described by the symmetric susceptivity tensor χ [3], defined by

$$D_i = (\delta_{ij} + 4\pi\ \chi_{ij})\ E_j\ , \tag{7}$$

which reads

$$\chi_{ij} = N\ \frac{2\ e^2}{3\hbar}\ \sum_t\ \frac{\omega_t}{\omega^2_t - \omega^2}\ \mathrm{Re}\ \Xi_{ij}^{\ t}\ , \tag{8}$$

where E, D, e, $\hbar$, ω have the usual meaning and

N is the number of atoms per cm^3,

ω_t is the angular frequency for the transition from the ground state to the t state,

Ξ_{ij}^t is defined as

$$\Xi_{ij}^t = \langle 0|x_i|t\rangle\langle t|x_j|0\rangle, \tag{9}$$

with x the usual position operator and $|0\rangle$ the ground state.

By assuming in the calculations of the tensor χ the energy-level situation shown in Fig. 1, it turns out that the only dielectric polarizability elements χ_{ij} which are not vanishing are

$$\chi_{11} = N\alpha c \ |\langle 0||x||1\rangle|^2 \left\{ \frac{\omega_1 + \omega_L}{(\omega_1+\omega_L)^2-\omega^2} + \frac{\omega_1 - \omega_L}{(\omega_1-\omega_L)^2-\omega^2} \right\}, \tag{10}$$

$$\chi_{22} = \chi_{11} \ , \tag{11}$$

$$\chi_{33} = 2 \ N\alpha c \ |\langle 0||x||1\rangle|^2 \ \frac{\omega_2}{\omega_2{}^2 - \omega^2} \ , \tag{12}$$

where $<0||x||1>$ is the Wigner-Eckart reduced matrix element [4] of the position operator between the ground state and the excited state, and we have put

$$\omega_1 = \omega_0 + (2W-U)B^2, \tag{13}$$

$$\omega_2 = \omega_0 + (W-U)B^2. \tag{14}$$

For propagation of light in the xy plane (i.e. orthogonal to the magnetic field direction), the two indices of refraction $n_\parallel$ and $n_\perp$ are such that

$$n_\parallel^2 - 1 = 4\pi \chi_{33} \cong$$
$$\cong (n^2 - 1)\{1 - B^2[(\omega_0^2 + \omega^2)(W-U)]/[\omega_0(\omega_0^2 - \omega^2)]\} , \tag{15}$$

$$n_\perp^2 - 1 = 4\pi \chi_{11} = 4\pi \chi_{22} \cong$$
$$\cong (n^2 - 1)\{1 - B^2[(\omega_0^2 + \omega^2)(2W-U)]/[\omega_0(\omega_0^2 - \omega^2)]$$
$$+ \mu^2 B^2(\omega_0^2 + 3\omega^2)/(\omega_0^2 - \omega^2)^2\} , \tag{16}$$

where n is the index of refraction of the gas, in the absence of external fields. On the same hypothesis, the refraction index n satisfies the equation

$$n^2 - 1 = 8\pi N\alpha c \ |<S||x||P>|^2 \ \omega_0/(\omega_0^2-\omega^2). \tag{17}$$

According to eqs. (15) and (16), the birefringence Δn induced by the magnetic field in the gas is given by

$$\Delta n = n_{\parallel} - n_{\perp} \cong$$
$$\cong (n - 1)\, B^2\{[W(\omega_0^2 - \omega^2)]/[\omega_0(\omega_0^2 - \omega^2)] - \mu^2(\omega_0^2 + \omega^2)/(\omega_0^2 - \omega^2)^2\} \,, \tag{18}$$

and therefore, from eq. (2), the Cotton-Mouton constant C reads

$$C = \frac{\omega(n-1)}{2\pi c}\left[\frac{W(\omega_0^2+\omega^2)}{\omega_0(\omega_0^2-\omega^2)} - \mu^2 \frac{\omega_0^2+3\omega^2}{(\omega_0^2-\omega^2)^2}\right] . \tag{19}$$

If we put in eq. (19) the expression of W as given by eq. (6), and we use for (n-1) the dispersion relation

$$n - 1 = A/(\omega_0^2 - \omega^2) \,, \tag{20}$$

we get an equation relating the Cotton-Mouton constant C with the value of $\sqrt{\langle r_1^2 \rangle}$. Table 2 shows the parameters A and ω_0 which fit the dispersion relation (20) for noble gases in the visible region (see ref. [5]).

In this way, by using the experimental values of the Cotton-Mouton constants, we have determined the dimension $\sqrt{\langle r_1^2 \rangle}$ of the excited state of the noble gas under study, as obtained in this simplified atom model. In Table 3 we compare the above dimensions with those of the corresponding alkali atom in the ground state given, for instance, in ref. [6]. The comparison shows that this very simplified model predicts values which are in rather good agreement with experimental results.

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Table 1: Cotton-Mouton constants of noble gases

gas	$C_{exp} \times 10^{18}$ (*) $(G^{-2}cm^{-1})$	$C_{th} \times 10^{18}$ (*) $(G^{-2}cm^{-1})$
Ar	1.32 ± 0.20	0.41
Kr	1.93 ± 0.21	0.76
Xe	4.45 ± 0.19	1.67

(*) C_{exp} is taken from ref. [1], whereas C_{th} is taken from ref. [2] .

Table 2: Dispersion in noble gases

gas	$A \times 10^{-28}$ (rad/s)^2	$\omega_0^2 \times 10^{-32}$ (rad/s)^2	$(n-1) \times 10^5$ λ=514.5 nm
Ar	5.92670	2.13740	29.6
Kr	6.7162	1.60446	45.7
Xe	7.6917	1.12819	77.4

Table 3: Atomic radii of noble gases in excited states

gas	$C \times 10^{18}$ (G^{-2} cm^{-1})	$\sqrt{\langle r_1^2 \rangle}$ (Angstrom)	Alkali atom	r_0 (*) (Angstrom)
Ar	1.32	2.63	K	2.38
Kr	1.93	2.75	Rb	2.51
Xe	4.45	3.07	Cs	2.70

(*) These values are obtained from ref.[6].

FIGURE CAPTIONS

Fig.1: Energy-level splittings due to the presence of the magnetic field B.

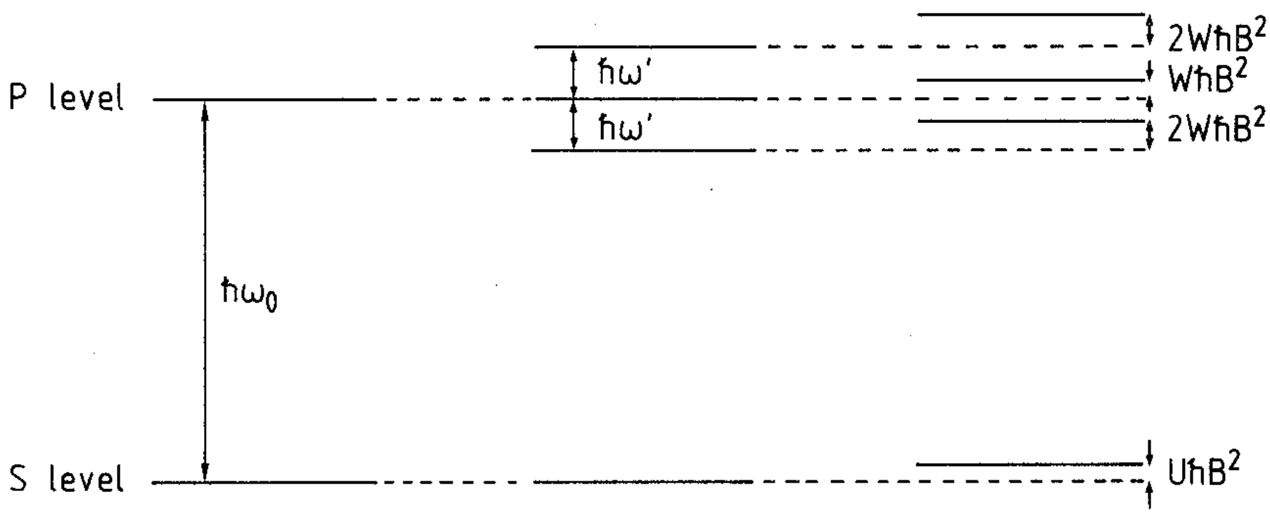
P level
S level
$\hbar\omega_0$
$\hbar\omega'$
$\hbar\omega'$
$2W\hbar B^2$
$W\hbar B^2$
$2W\hbar B^2$
$U\hbar B^2$

Fig. 1