Analysis of Elastic Scattering of ^{6,8}He and ^{11}Li on Protons and ^{6}He on ^{12}C Using Microscopic Optical Potentials

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Abstract
The data of elastic scattering of ^{6,8}He and ^{11}Li on protons and ^{6}He on ^{12}C at beam energies less than 100 MeV/nucleon (MeV/N) are analyzed utilizing microscopic optical potentials obtained by a single (double)-folding procedure and also by using those inherent in the high-energy approximation. The calculated real and imaginary parts of the optical potentials are based on the neutron and proton density distributions of He and Li isotopes obtained within the large-scale shell-model (LSSM) method. The depths of the real and imaginary parts of the microscopic optical potentials are considered as fitting parameters using as a constrain the behavior of the volume integrals as functions of the incident energy. The ^{11}Li breakup effect on ^{11}Li+p elastic scattering at energy of 62 MeV/N is analyzed within a cluster model for ^{11}Li with ^{9}Li and $2n$ fragments. Predictions for the longitudinal momentum distribution of ^{9}Li fragments produced in the breakup of ^{11}Li on a proton target are given. The role of the spin-orbit and "surface" terms of the optical potential is also studied and estimations of the total cross sections within the both LSSM and breakup reaction model are made.

1 Introduction

The availability of radioactive ion beams facilities made it possible to carry out many experiments and to get more information regarding the structure of these nuclei and the respective reaction mechanisms (see, e.g., the review [1]). Experimental studies of exotic light nuclei, such as ^{6,8}He, ^{11}Li, ^{12}Be and others, with a localized nuclear core and dilute few-neutron halo or skin have also been an important test for various theoretical models used in the description of the data on cross sections of processes with such nuclei. Among the latter we should mention the microscopic analysis using the coordinate-space g-matrix folding method (e.g., Ref. [2]), as well as works where the real part of the optical potential (ReOP) is microscopically calculated (e.g., Ref. [3]) using the folding approach (e.g., Refs. [4, 5]). Usually the imaginary part of the OP's (ImOP) and the spin-orbit (SO) terms have been determined phenomenologically, which has led to the usage of a number of fitting parameters.

In this work we present results of our works [6–9] on calculations of ^{6}He+p [6], ^{8}He+p [7], ^{6}He+^{12}C [8], as well as on ^{11}Li+p [9] elastic differential cross sections in which we used microscopic both ReOP and ImOP. The latter was taken from the OP derived in [10,11] in the frameworks of the high-energy approximation (HEA) [12] that is known as the Glauber theory. Our main aim is to describe the existing experimental data using these microscopic OP's with a minimal number of fitting parameters. In particular we study: i) the limits of applicability of the HEA OP for different regions of angles and incident energies; ii) the sensitivities of the cross sections to the nuclear densities of ^{6,8}He and ^{11}Li; iii) the role of the SO interaction and the non-linearity in the calculations of the OP's; iv) the nuclear surface effects; v) the role of the renormalization of the depths of the ReOP and ImOP; vi) the possibility to involve additional physical criteria for a better description of limited number of experimental data.

2 Theoretical scheme

The optical potential used in our calculations has the form

$$U_{opt} = V^F(r) + iW(r). \tag{1}$$

The real part of the nucleon-nucleus OP is assumed to be a result of a single folding of the nuclear density and of the effective NN potential and involves the direct and exchange parts (e.g. Refs. [4,5]):

$$V^F(r) = V^D(r) + V^{EX}(r). \tag{2}$$

The direct part $V^D(r)$ is composed by the isoscalar (IS) and isovector contributions and expressions for them can be found in Ref. [6]. In our consideration the energy and density dependence of the effective NN interaction (of CDM3Y6-type) are taken in usual forms [5, 6]. The isoscalar part of the exchange contribution to the ReOP has the form:

$$V_{IS}^{EX}(r) = g(E) \int \rho_2(\mathbf{r}_2, \mathbf{r}_2 - \mathbf{s}) F\left(\rho_2(\mathbf{r}_2 - \mathbf{s}/2)\right) \times v_{00}^{EX}(s) j_0(k(r)s) d\mathbf{r}_2, \tag{3}$$

where for the density matrix $\rho_2(\mathbf{r}_2, \mathbf{r}_2 - \mathbf{s})$ an approximation [13] is used. It is shown in Ref. [6] how the isovector part of the exchange ReOP can be obtained. The local momentum $k(r)$ of the incident nucleon in the field of the Coulomb $V_C(r)$ and nuclear potential (ReOP) is:

$$k^2(r) = \frac{2m}{\hbar^2} \left[E_{c.m.} - V_C(r) - V(r)\right] \left(\frac{1 + A_2}{A_2}\right). \tag{4}$$

One can see from Eq. (4) that nonlinearity effects appear as ingredient of the approach and they have to be taken into account.

In our work we use proton and neutron densities calculated microscopically within the LSSM method using the Woods-Saxon (WS) basis of single-particle wave functions with realistic exponential asymptotic behavior [14].

The complex HEA OP was derived in [10] on the basis of the eikonal phase inherent in the optical limit of the Glauber theory. In our procedure this OP or only its imaginary part together with the ReOP from the folding procedure is used to calculate the cross sections by means of the code DWUCK4 [15] for solving the Schrödinger equation. The HEA OP is obtained as a folding of the form factors of the nuclear density and the NN amplitude $f_{NN}(q)$ [10,11]:

$$U_{opt}^H = V^H + iW^H = -\frac{\hbar v}{(2\pi)^2} (\bar{\alpha}_{NN} + i) \bar{\sigma}_{NN} \times \int_0^\infty dq q^2 j_0(qr) \rho_2(q) f_{NN}(q). \tag{5}$$

In Eq. (12) $\bar{\sigma}_{NN}$ and $\bar{\alpha}_{NN}$ are, respectively, the NN total scattering cross section and the ratio of the real to imaginary part of the forward NN scattering amplitude, both averaged over the isospin of the nucleus (see, e.g., [16,17]).

The expression for the spin-orbit contribution to the OP used in our work is added to the right side of Eq. (1) and its form can be seen in e.g., Refs. [7,15,18].

In the case of the ^{11}Li+p elastic scattering we consider also the simplest ^{9}Li+2n model of ^{11}Li (see, e.g. [19]) in which two clusters are suggested, the ^{9}Li core (c) and the correlated pair of neutrons $h = 2n$ with the spin of the $2n$ cluster set to $s = 0$. In the framework of this model, the ^{11}Li+p OP can be estimated as folding of two OP's of interaction of the c- and h-clusters with protons and the density $\rho_0(s)$ corresponding to the wave function of the relative motion of two clusters:

$$U^{(b)}(r) = V^{(b)} + iW^{(b)} = \int d\mathbf{s} \rho_0(s) \left[U_c\left(\mathbf{r} + (2/11)\mathbf{s}\right) + U_h\left(\mathbf{r} - (9/11)\mathbf{s}\right)\right]. \tag{6}$$

The potentials U_c and U_h in Eq. (6) are calculated within the microscopic hybrid model of OP [10], in which a single-folding procedure is applied for the real part $V^{(b)}$, while the imaginary part $W^{(b)}$ is derived using the optical limit of the Glauber theory. For the n-p interaction we adopt the one introduced by Suzuki *et al*. [20] $v_{np} = v(r)(1 + i\gamma)$, where $v(r)$ is taken from the Minnesota potential [21].

The differential and total cross sections (for elastic scattering, as well as for diffractive breakup and absorption) all require calculations of the probability functions $d^3P(\mathbf{b}, \mathbf{k})/d\mathbf{k}$ that depend on the impact parameter $\mathbf{b}$. The general expression for the probability functions can be written as [22]

$$\frac{d^3P_\Omega(\mathbf{b}, \mathbf{k})}{d\mathbf{k}} = \frac{1}{(2\pi)^3}\left|\int d\mathbf{r}\phi_k^*(\mathbf{r})\Omega(\mathbf{b}, \mathbf{r}_\perp)\phi_0(\mathbf{r})\right|^2, \tag{7}$$

where $\Omega(\mathbf{b}, \mathbf{r}_\perp)$ is expressed by means of the two profile functions S_c and S_h of the core and the di-neutron clusters, respectively:

$$|S_i(b)|^2 = e^{-\frac{2}{\hbar v}\int_{-\infty}^{\infty} dz W_i(\sqrt{b^2+z^2})}, \quad i = c, h \tag{8}$$

where W is the imaginary part of the microscopic OP (6).

As shown in [22], the diffraction breakup elastic cross section (the longitudinal momentum distribution) has the form

$$\left(\frac{d\sigma}{dk_L}\right)_{diff} = \int_0^\infty b_h db_h \int_0^{2\pi} d\varphi_h \int_0^\infty dk_\perp \frac{d^2P(\mathbf{k}, \mathbf{b})}{dk_L dk_\perp}, \tag{9}$$

where $d^2P_\Omega(\mathbf{b}, \mathbf{k})/dk_L dk_\perp$ is obtained by integration of Eq. (7) over the transverse angle φ_k of the momenta.

3 Results and discussion

In the case of ^{6}He+p elastic cross sections ($E < 100$ MeV/N) the optical potential has the form

$$U_{opt}(r) = N_R V(r) + iN_I W(r), \tag{10}$$

where N_R and N_I are fitting parameters, the ReOP V is taken either from single-folding calculations (V^F) or from HEA (V^H), while ImOP has the form $W = W^H$ or $W = V^F$. In the case of ^{8}He+p process we introduce a surface component:

$$U'_{opt}(r) = U_{opt}(r) - i4aN_S \frac{dV^F(r)}{dr}. \tag{11}$$

For the ^{6}He+^{12}C cross sections the OP has the form

$$U_{opt}(r) = N_R V^{DF}(r) + iN_I W(r) + iN_I W^{SF}(r), \tag{12}$$

where the ReOP $V^{DF}(r)$ is a result of a double-folding procedure (using the charge density of ^{12}C obtained from electron-^{12}C scattering experiments) and $W^{SF}(r)$ has various forms related to the derivative $dW(r)/dr$ (e.g., $dW(r)/dr$, $rdW(r)/dr$, $r^2dW(r)/dr$, $dW(r-\delta)/dr$). It was shown in [6] that a good agreement in the case of ^{6}He+p is obtained when LSSM density is used (in comparison with the phenomenological densities) for $E = 41.6$ and 71 MeV/N with values of N_R and N_I close to unity. However, an agreement for the case of $E = 25.2$ MeV/N was obtained for rather smaller values of N_R (0.35) and N_I (0.03), thus showing the limitation of the approach for small energies ($E \leq 25$ MeV/N). In Fig. 1 we give the results for the ^{8}He+p elastic cross sections at energies $E = 15.7, 26, 32, 66,$ and 73 MeV/N. It is known that because the procedure of fitting belongs to the class of the ill-posed problems (e.g., [23]), it is necessary to impose some physical constraints on the choice of the set of parameters N.

One of them is the total cross section of scattering and reaction. However, the corresponding values are missing at $E < 100$ MeV/N. Another physical criterion that was imposed on the choice of N's is the behavior of the volume integrals [4]

$$J_V = \frac{4\pi}{A}\int drr^2[N_R V^F(r)], \qquad J_W = \frac{4\pi}{A}\int drr^2[N_I W^H(r)] \tag{13}$$

as functions of the energy. It has been pointed out (see, e.g., [24]) that the values of J_V decrease with the increase of the energy at $0 < E < 100$ MeV/N, while J_W is almost constant in the same interval. In Fig. 1 one can see the result of the fitting procedure with the values of the parameters given in Table 1.

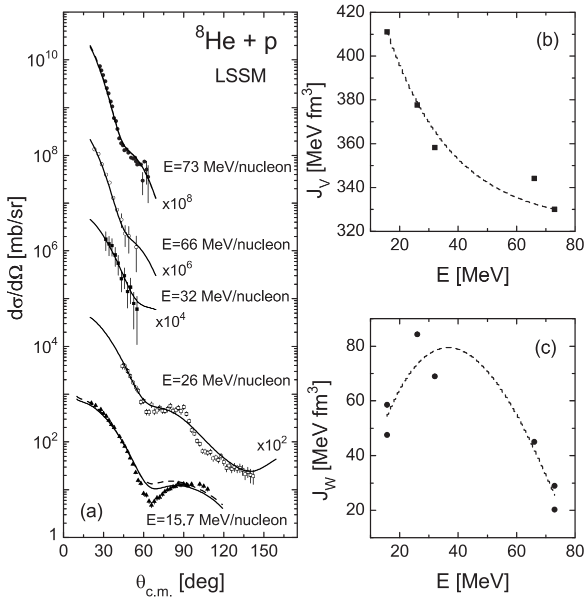

Fig. 1: The ^{8}He+p elastic scattering cross sections (a) at different energies using LSSM density of ^{8}He and parameters from Table 1. Experimental data are taken for 15.7 [25], 26 [26], 32 [27,28], 66 [27,28] and 73 MeV/N [27–29]. The obtained values of the volume integrals J_V (b) and J_W (c) (given by points) are shown as functions of the incident energy, while the dashed lines give the trend of this dependence.

It was shown in [6] that the inclusion of the surface term [see Eq. (11)] leads to a better agreement with the data for the lowest energy $E = 15.7$ MeV/N. Using the same physical constraint we obtained the best agreement of the calculations in the case of ^{6}He+^{12}C [by means of Eq. (11) and the surface term ($-iN_I^{SF} r^2 dW(r)/dr$)] that are presented in Fig. 2 for $E = 3$, 38.3, and 41.6 MeV/N. In Fig. 3 we show the results of our calculations of the ^{11}Li+p elastic cross sections for three energies $E = 62$, 68.4, and 75 MeV/N with and without accounting for the SO term.

Finally, in Fig. 4 we give as an example the calculated cross sections for the diffractive breakup elastic ^{11}Li+p reaction at $E = 62$ MeV/N. These results give predictions because there are not experimental data for such a process at ^{11}Li+p scattering at $E < 100$ MeV/N.

4 Conclusions

The results of the present work can be summarized:

1. The optical potentials and cross sections of ^{6}He+p ($E = 25.2$, 41.6 and 71 MeV/N), ^{8}He+p ($E =$

Table 1: The parameters N_R, N_I, N_R^{SO} and N_I^{SO}, the volume integrals J_V and J_W (in MeV.fm^3) as functions of the energy E (in MeV/N), and the total reaction cross sections σ_R (in mb) for the ^{8}He+p scattering in the case of LSSM density.

E	N_R	N_I	N_R^{SO}	N_I^{SO}	J_V	J_W	σ_R
15.7	0.630	0.064	0.139	0.070	411.1	58.6	722.0
15.7	0.630	0.052	0.166	0.057	411.1	47.6	701.2
26	0.644	0.128	0.035	0.026	377.7	84.35	381.2
32	0.648	0.120	0.062	0.022	358.3	69	302.7
66	0.852	0.131	0	0	344.2	45	95.2
73	0.869	0.090	0.004	0	330.0	29	60.9
73	0.869	0.063	0.010	0	330.0	20.25	43.9

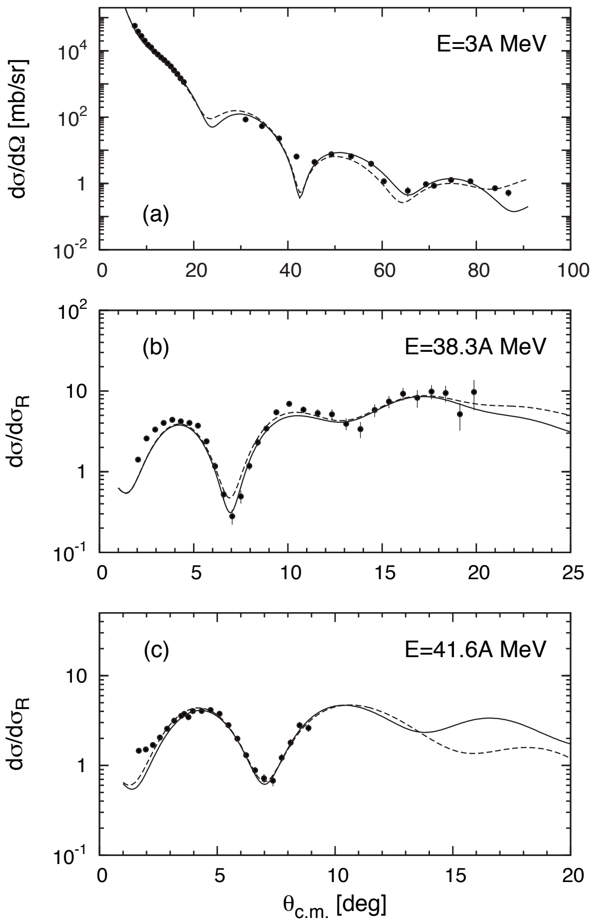

Fig. 2: Differential cross section of elastic ^{6}He+^{12}C scattering at $E = 3$ (a), 38.3 (b) and 41.6 MeV/N (c). Solid line: $W = W^H$, dashed line: $W = V^{DF}$. The experimental data are taken from Refs. [30–32].

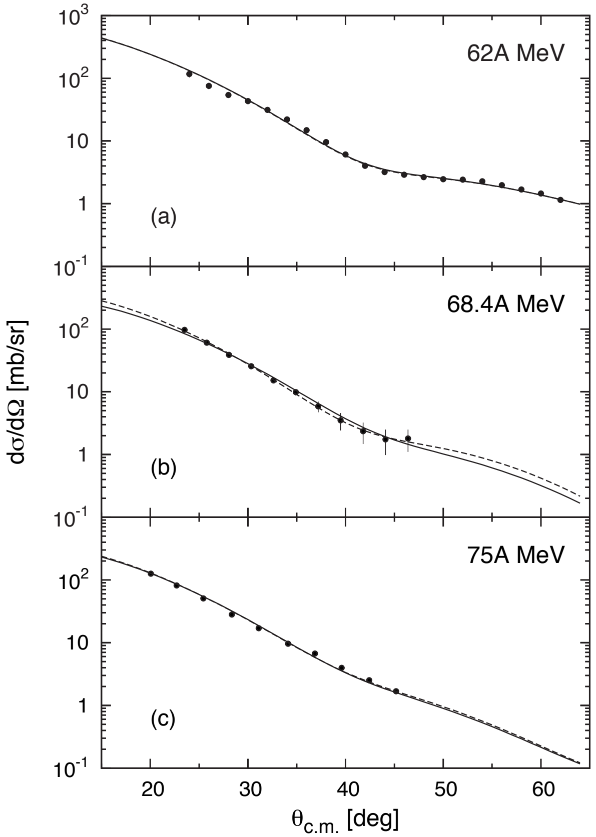

Fig. 3: The ^{11}Li+p elastic scattering cross section at $E = 62, 68.4,$ and 75 MeV/N. Solid line: without SO term; dashed line: with SO term. The experimental data are taken from [33] for 62 MeV/N, [34] for 68.4 MeV/N, and [35] for 75 MeV/N.

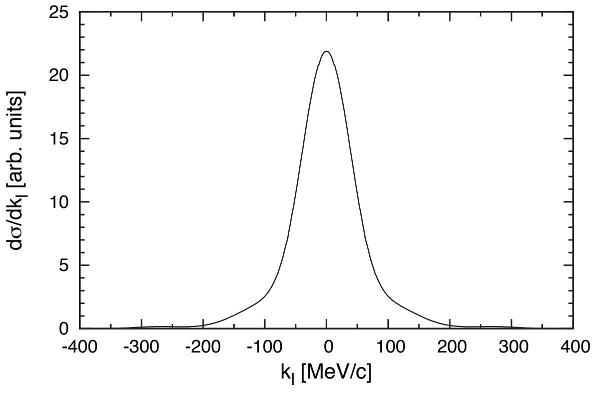

Fig. 4: Cross section of diffraction breakup in ^{11}Li+p scattering at $E = 62$ MeV/N.

15.7, 26, 32, 66 and 73 MeV/N), ^{11}Li+p ($E = 62$, 68.4 and 75 MeV/N), and ^{6}He+^{12}C ($E = 3$, 38.3 and 41.6 MeV/N) elastic scattering were calculated and comparison with the available experimental data was performed. The direct and exchange parts of ReOP (V^F) were calculated microscopically using the folding procedure and M3Y (CDM3Y6-type) effective interaction based on the Paris NN potential. The ImOP (W^H) was calculated within the high-energy approximation. Different model densities of protons and neutrons in ^{6}He, ^{8}He and ^{11}Li were used in the calculations: LSSM method, Jastrow correlation method (also Tanihata and COSMA). The SO contribution to the OP was included in the calculations. The cross sections were calculated by numerical integration of the Schrödinger equation by means of the DWUCK4 code using all interactions obtained (Coulomb plus nuclear optical potential).
2. The problem of the ambiguity of the values of the depths of OP's contributions: the parameters N_R, N_I, N_R^{SO}, and N_I^{SO} when the fitting procedure is applied to a limited number of experimental data is considered. A physical criteria imposed in our work on the choice of the values of the parameters N were the known behavior of the volume integrals J_V and J_W as functions of the incident energy in the interval $0 < E_{inc} < 100$ MeV/N, as well as the values of the total reaction cross section.
3. We considered also another folding approach that includes ^{11}Li breakup suggesting a ^{9}Li+2n cluster model, computing the potentials of the interactions of the two clusters with the proton. Predictions for the longitudinal momentum distributions of ^{9}Li fragments produced in the breaking of ^{11}Li at 62 MeV/N on a proton target are given and calculations of the diffraction and stripping reaction cross sections are performed. The necessity of experiments on these reactions of ^{11}Li+p at $E < 100$ MeV/N is emphasized.

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