# Reaction theory for studying rare isotopes

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**Abstract**
Rare isotopes are often studied through nuclear reactions. Reliable reaction theory is needed to be able to extract the desired information from data. In these proceedings we briefly summarize some of the advances in reactions theory, including tests on exisiting models to better understand their range of validity and accuracy. We focus on deuteron induced reactions for which a number of important benchmarks have been recently performed.

## 1 Motivation

Low energy nuclear physics has seen for the last couple of decades important investments toward more powerful accelerators. Laboratories such as RIKEN (Japan), GSI (Germany) and NSCL/FRIB (USA) have either completed or are in the process of building new facilities with increased capabilities. This means that one can explore new isotopes, but also it represents increased particle production rates which then allows for more detailed reaction studies.

Nuclear reactions are the most diverse tool for studying the properties of nuclei. Through the wide range of kinematic choices, one can probe the surface, the asymptotic behavior, excitations, deformations, etc, etc. It is thus of paramount importance for the low energy programs to have reliable reaction theory that enables a meaningful analysis of the data.

The study of reactions with rare isotopes can provide critical information for structure many-body theories, which in turn will test assumptions made on the effective nucleon-nucleon interaction. This is ultimately the big goal of our field, to understand the force that binds nucleons together to make the matter that surrounds us. Studying rare isotopes provides a level of sensitivity to parts of the nuclear force which is otherwise hidden, for example the isospin dependence, the density dependence and role of many-body forces. To have a many-body structure theory with predictable power, one needs to include reactions with rare isotopes in the study.

One of the most important physics questions in our time, is to know how and where the heavy elements were formed. Although we do not fully have an answer to this question, we certainly know that part of the elements were formed in hot explosive environments and involved neutron rich isotopes, many nucleons away from stability (the so-called r-process). Neutron capture reactions relevant for the r-process may never be measured in the laboratory because they involve species that decay quickly, but there are indirect methods involving nuclear reactions from which one can extract the information of astrophysical interest [1]. Here again, reactions with rare isotopes play an important role.

With reactions playing such an important role, both for understanding the nuclear force, and the origin of the elements, it is critical to have reliable reaction theory. Here we briefly summarize some advances made in benchmarking existing theories, and present some ongoing efforts to advance the methods used today. We focus on deuteron induced reactions, and mostly on one nucleon transfer A(d,p)B. For closed shell targets, this problem can be addressed within a three-body model $n + p + A$ and the ingredients of the theory are effective interactions which can be constrained by elastic scattering thus reducing ambiguities. In Section II we discuss some of the existing theories and the underlying assumptions, in Section III we summarize some of the results obtained in the comparisons, and in Section IV an overview of current and future work is presented.

## 2 General aspects of theories for A(d,p)B reactions

Most of us learned about the distorted wave Born approximation (DWBA) in our graduate studies [1]. This method, in its common implementation, assumes the transfer proceeds through a single step, and takes an effective d-A interaction to represent the incoming wave and a p-B interaction for the outgoing wave. DWBA continues to be the most popular method to analyse (d,p) transfer angular distributions [2]. DWBA neglects deuteron breakup, beyond what might be included in the d-A potential, an assumption that has been long questioned given the loosely bound nature of the deuteron (2.2 MeV).

Specifically to address this drawback of DWBA, the adiabatic wave method (ADWA) was developed. In this method the deuteron continuum is explicitly included, although in a simplified form [3–5]. The approximations in ADWA work best if $np$ are close together and thus should be adequate to compute transfer cross sections in post-form.

To include deuteron breakup in a more detailed manner, the continuum discretized coupled channel (CDCC) method [6, 7] was constructed. In CDCC, the three-body wavefunction is expanded in the full set of eigenstates of the $n - p$ subsystem. Given the necessary truncations of the basis, this method is not exact, particularly when rearrangment channels are involved.

In order to solve the three-body problem $n + p + A$ exactly, it is best to include explicitly in the basis the rearrangement channels. The Faddeev method [8] starts then from this overcomplete basis, includes all Jacobi components providing additional flexibility to describe what happens not only when the neutron and proton are close and correlated, but also when the $n - p$ subsystem has absorbed a large part of the energy of the system and are no longer spatially correlated. The Faddeev-AGS method [9, 10] transcribes the problem into a T-matrix integral equation in momentum space and together with techniques specifically developed to handle Coulomb, can provide exact solutions to $d + A \rightarrow d + A$, $d + A \rightarrow p + B$, $d + A \rightarrow n + B'$ and $d + A \rightarrow n + p + A$.

## 3 Comparing methods for A(d,p)B reactions

### 3.1 DWBA versus ADWA

Recent work [11] analysed angular distributions resulting from the reaction $^{10}$Be(d,p)$^{11}$Be for a wide range of energies. Spectroscopic factors for the ground state and the first excited state of $^{11}$Be were extracted using DWBA and subsequently finite range ADWA. Three important conclusions can be drawn from the study: i) the spectroscopic factors extracted show a large dependence on beam energy for DWBA while virtually no dependence is observed for ADWA, ii) the spectrosopic factors depend strongly on the choice of the optical potentials in DWBA but a much weaker dependence is found in ADWA and iii) the overall magnitude of the spectroscopic factors obtained with ADWA is systematically smaller than with DWBA.

The first point can be understood based on the fact that DWBA leaves out important physics in the dynamics, namely deuteron breakup. By including it, ADWA is able to capture the dependence on beam energy reliably. The second point is related to this fact that ADWA is constructed solely on nucleon-optical potentials which are better understood. We will come back to the dependence on optical potentials in 4.1. Finally, the last aspect is criticial for the interpretation of the results obtained through (d,p) experiments. When reduction factors are discussed (e.g. [12]) and interpreted as missing correlations in the many-body structure theory, one needs to consider deuteron breakup in the analysis.

In Fig.1 we compare DWBA and ADWA for another reaction of interest $^{132}$Sn(d,p)$^{133}$Sn(g.s.) at $E_{\rm d}$ = 9.5 MeV [13]. This reaction was recently measured and offers an excellent example of the sort of reactions one needs to describe. Shown are the angular distribution as a function of the center of mass angle and the interactions included in the DWBA and ADWA calculations are the same as those in [13]. ADWA predicts a cross section around 20% larger than DWBA and consequently the extracted spectroscopic factor is 20% smaller.

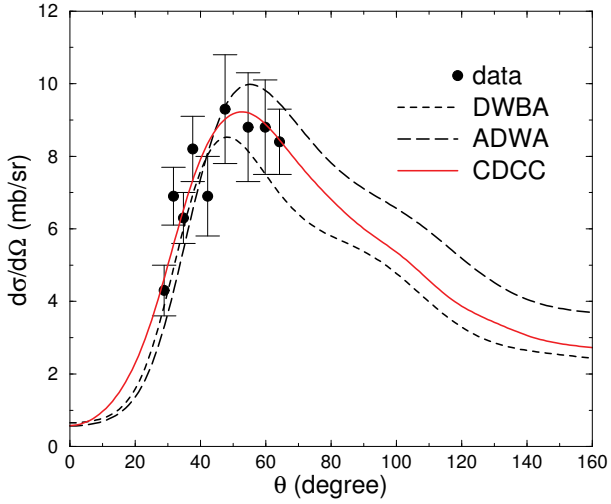


**Fig. 1:** Comparing methods for $^{132}$Sn(d,p)$^{133}$Sn(g.s.) at $E_{\rm d}$ = 9.5 MeV. Angular distributions as a function of the center of mass angle for DWBA (black dashed), ADWA (blue long-dashed), CDCC (red solid) and experiment (black circles) [13].

### 3.2 Zero range versus finite range ADWA

The study of Nguyen et al. [14] compares, in a systematic manner, the zero-range ADWA [3] and the finite-range ADWA [4]. A large number of cases were chosen with the condition that data exist and ADWA provides a good description of the angular distribution. Results show that the zero-range approximation introduces an error of the order of 10% for low energy (d,p), and this value increases to 50% or more for higher energies. Although the work in [14] concentrates on the ground state, recent work developed by Liu *et al.* for (d,p) reactions populating excited states [15] show similar trends. In [15] a comparison with DWBA was also made but for many cases the standard optical potential parameterizations did not provide an adequate description of the angular distribution, confirming the strong sensitivity to the details of the optical potential found in [11]. Since it is not computationally demanding, results in [14, 15] provide a clear preference for the use of the finite-range ADWA in the analysis of (d,p), over its zero-range counterpart or the traditional DWBA. From now on we will use ADWA to refer to the finite-range version only.

### 3.3 ADWA versus Faddeev

Since ADWA provides a practical way in which to analyse (d,p) data, if one is to replace DWBA by ADWA as the working tool for experimentalists, it is necessary to better understand the validity of the approximations and the type of precision obtained in ADWA. After all, it does assume a Sturmian expansion truncated to one term [4]. From its formalism one expects ADWA to do better for low energy, but it was unclear at what beam energy would it breakup down.[1] With this motivation in mind, a comparison of ADWA with Faddeev was performed in [16]. Results for a number of reactions, at a wide range of energies, demonstrate that ADWA compares very well with the exact Faddeev-AGS transfer cross sections for reactions at around 10 MeV/u. Indeed for s-wave final states the agreement was close

[1] Unfortunately the name ADWA is misleading because adiabatic usually refers to a high energy approximation. This is not the case for the method introduced in [4].

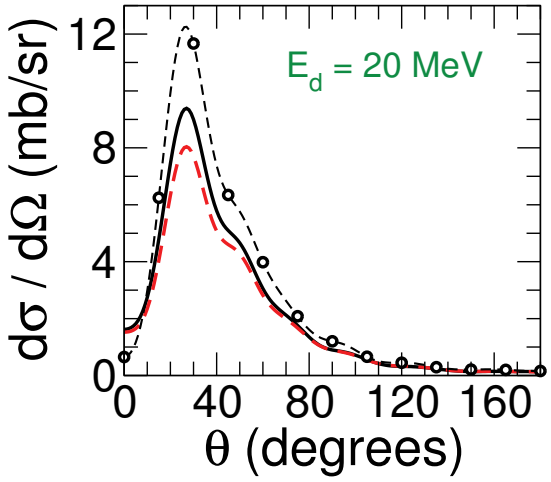


**Fig. 2:** Optical potential dependence of the angular distribution of $^{208}$Pb(d,p)$^{209}$Pb at $E_{\rm d}$ = 20 MeV. We compare CH89 [18] with KD [19] and BG [20]. All calculations performed within CDCC.

to perfect. However the agreement deteriorates with beam energy and with increasing angular momentum transfer. Both these points can be well understood given that in ADWA one assumes that the full three-body deuteron wave has the same radial behaviour as the first term of the Sturmian expansion and that would naturally apply to low energy and zero angular momentum. We also found in [16] a larger sensitivity to the energy at which the optical potential parameters were obtained for reactions at higher beam energies. Note that global parameterizations of optical potentials are always energy dependent and the Faddeev-AGS formulation in [16] uses a fixed energy independent Hamiltonian.

### 3.4 CDCC versus Faddeev and ADWA

Most recently, a detailed comparison between CDCC and Faddeev has been completed [17]. In this work, all relevant channels (elastic, breakup and transfer) are studied. Given the difficulties in obtaining convergence for breakup channels, the comparison was performed neglecting the spin of the nucleons. Here we focus on the results for transfer. As for ADWA, very good agreement is found for reactions around 10 MeV/u, where most experiments in ISOL facilties take place. However that agreement is somewhat lost at the higher energies, where a larger dependence on the interactions is present. One can therefore unambiguously validate CDCC and ADWA against the exact calculations only for the lower energies.

ADWA can be thought of as a simplified CDCC calculation. One would then expect that CDCC remains valid in regions where ADWA is not. Particularly, since CDCC takes the expansion in $n-p$ partial waves explicitly up to convergence, we expected it would do much better than ADWA in cases where the angular momentum transfer is not zero. Results for (d,p) reactions on $^{12}$C or $^{48}$Ca do not show this trend. The examples studied in [17] suggest a similar level of agreement of ADWA and CDCC with the exact Faddeev calculations.

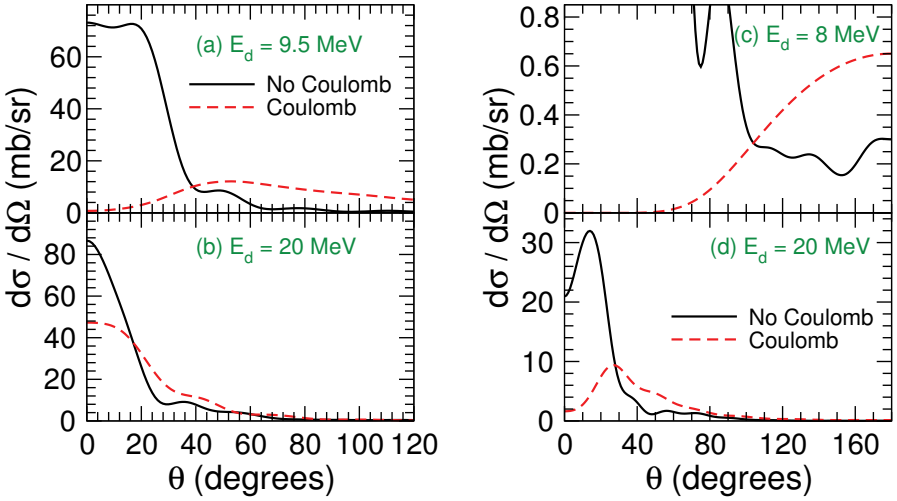


**Fig. 3:** Coulomb effects on the breakup cross sections for $^{132}$Sn(d,p)$^{133}$Sn(g.s.) at (a) $E_{\rm d}$ = 9.5 MeV and (b) $E_{\rm d}$ = 50 MeV and for $^{208}$Pb(d,p)$^{209}$Pb(g.s.) at (c) $E_{\rm d}$ = 8 MeV and (d) $E_{\rm d}$ = 20 MeV.

## 4 Outlook

### 4.1 Better constraints on the optical potential

Given the interest in neutron-rich heavy isotopes in connection to the r-process, it is important to understand the dependence on the nucleon optical potential. We performed CDCC calculations for a number of cases, exploring different nucleon-nucleus parameterizations [18–20]. In Fig.2 we present results for the angular distributions following $^{208}$Pb(d,p)$^{209}$Pb at $E_{\rm d}$ = 20 MeV. While all these potentials provide identical $p - Pb$ elastic angular distributions at 10 MeV, the corresponding (d,p) transfer distributions differ considerably. Even constraining the optical potential with elastic is not sufficient to obtain the desired accuracy. This result suggests that one does need to combine data with microscopic theory to reduce the ambiguities in the optical potential.

One aspect of the optical potentials that arises naturally from microscopic theories and is most often neglected in reaction calculations is non-locality. The connection of the optical potentials with many-body structure models, including non-locality, requires additional efforts in the future.

### 4.2 The Coulomb problem

One interesting outcomes from these comparative studies [16, 17] was a better understanding of the limitations of the present implementation of Faddeev-AGS [10]. Of particular relevance is the Coulomb force which is well known to create problems in three-body reactions models. The development of Coulomb screening and renormalization techniques were essential to enable the application of the method to nuclear reactions [10]. However only light to medium nuclei can be tackled, while much of the interest in the future lies in nuclei with $Z > 20$. $^{132}$Sn is a cornerstone example since it is one of the only bound doubly magic nuclei, far from stability, and accessible to experimental studies. In Fig.1 we show the angular distribution for the transfer $^{132}$Sn(d,p)$^{133}$Sn(g.s.) at $E_{\rm d}$ = 9.5 MeV computed in DWBA, ADWA and CDCC (multiplied by the appropriate statistical factor to correct for neglecting spin). No Faddeev solution could be obtained for this case.

Since Coulomb represents large technical difficulties, it is fair to ask to what extent is it important

in transfer reactions, and in the analysis of (d,p) angular distributions. We performed CDCC calculations in the energy range where it was demonstrated to be valid, for a number of cases, to explore this aspect. For light systems $A \approx 12$ Coulomb has a small effect in the transfer cross section and could be safely neglected or approximated. Fig.3 shows the effect on heavy systems. At sub-Coulomb energies, the reaction dynamics is dictated by the Coulomb force and therefore one cannot expect any sensible answer when ignoring the Coulomb interactions (Fig3(c)). The other panels in Fig. 3 show clearly that the Coulomb effect in (d,p) reactions above the Coulomb barrier is still very important for these heavier systems. Thus, it is critical to treat the Coulomb force in the three-body problem accurately.

This problem is being addressed by the TORUS collaboration [21]. A new method that does not rely on Coulomb screening, has recently been suggested to enable the treatment of Coulomb for any case [22]. Work along these lines is in progress.

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