Gravitino properties in a conformal supergravity model

Nick E. Mavromatos

Theoretical Particle Physics and Cosmology Group, Department of Physics, King's College London, Strand, London WC2R 2LS, United Kingdom and Theory Division, Physics Department, CERN, CH-1211 Geneva 23, Switzerland

Vassilis C. Spanos

Institute of Nuclear Physics, NCSR "Demokritos," GR-15310 Athens, Greece

(Received 28 December 2012; published 21 February 2013)

In the context of a conformal supergravity model in the Einstein frame, in which the (next to) minimal supersymmetric standard model can embedded naturally to produce chaotic inflation scenarios, we study properties of gravitino in the cases where it is stable or unstable. In the latter case, we demonstrate that for large dilaton scale factors there is an enhanced magnitude of the gravitino width, when it decays to neutralino dark matter, as compared with the standard supergravity case. In this context, we discuss the associated consequences as far as big bang nucleosynthesis constraints and avoidance of gravitino overproduction are concerned.

DOI: 10.1103/PhysRevD.87.035025 PACS numbers: 12.60.Jv, 04.65.+e, 95.35.+d

I. INTRODUCTION

In Refs. [1,2] simple classes of supergravity (SUGRA), models describing superconformal coupling of matter to supergravity have been considered. The models contain nonminimal scalar/space-time-curvature couplings of the form ΦR, where Φ is a frame function, depending in general on matter supermultiplets, including dilatons. Such couplings have been argued to lead naturally to Higgs inflation in both nonsupersymmetric [3–6] and supersymmetric theories [7–9]. Scale-free globally supersymmetric theories, such as the next to minimal supersymmetric standard model (NMSSM) [10] with a scale-invariant superpotential, can be naturally embedded [2,7] into this class of theories, leading to new classes of chaotic inflationary scenarios [1]. Moreover, such models have been considered in Ref. [11] in connection with the possibility of dynamical breaking of supergravity theories, exploring further the conformal couplings of the gravitino four-fermion interactions.

It is the point of this article to discuss the properties of gravitino fields in such models, in particular in the context of the NMSSM. Specifically, we shall analyze decay processes involving gravitinos and calculate the corresponding life time. Depending on the strength of the conformal couplings, the width can be suppressed or enhanced significantly. In the case of enhancement the rapid decay of the gravitino implies a resolution of the gravitino overproduction, avoiding the big bang nucleosynthesis (BBN) constraints.

The structure of the article is as follows: in the next section, we describe the basic Lagrangian formalism and properties underlying the conformal supergravity models of Refs. [1,2]. In Sec. III we analyze the main decay processes involving gravitinos and calculate the associated widths (and lifetimes) and discuss how the latter are constrained by BBN. Conclusions and outlook are presented in Sec. IV.

II. LAGRANGIAN FORMALISM OF CONFORMAL SUPERGRAVITY MODELS

The action of the conformal supergravity models of Refs. [1,2], which we shall employ in our analysis below, in the Jordan frame, reads

$$e^{-1}\mathcal{L} = -\frac{1}{6}\Phi[R(e) - \bar{\psi}_\mu R^\mu] - \frac{1}{6}(\partial_\mu \Phi)(\bar{\psi} \cdot \gamma \psi^\mu) + \mathcal{L}_0 + \mathcal{L}_{1/2} + \mathcal{L}_1 - V + \mathcal{L}_m + \mathcal{L}_{\text{mix}} + \mathcal{L}_d + \mathcal{L}_{4\text{f}}, \tag{1}$$

where the curvature $R(e)$ uses the torsionless connection $\omega_\mu{}^{ab}(e)$, with e_a^μ the vielbeins, and e the vielbein determinant, and the gravitino kinetic term is defined using

$$R^\mu \equiv \gamma^{\mu\rho\sigma}\left(\partial_\rho + \frac{1}{4}\omega_\rho{}^{ab}(e)\gamma_{ab} - \frac{3}{2}\,\mathrm{i}\mathcal{A}_\rho\gamma_5\right)\psi_\sigma. \tag{2}$$

Here $\mathcal{A}_\mu$ is the part of the auxiliary vector field containing only bosons, namely,

$$\mathcal{A}_\mu = \frac{1}{6}\mathrm{i}(\partial_\mu z^\alpha \partial_\alpha \mathcal{K} - \partial_\mu \bar{z}^{\bar{\alpha}} \partial_{\bar{\alpha}} \mathcal{K}) - \frac{1}{3}A_\mu{}^A P_A, \tag{3}$$

where $A_\mu{}^A$ is the Yang-Mills gauge field, z^α are (complex) scalar fields, $\mathcal{K}(z, \bar{z})$ is the Kähler potential and P_A is a momentum map or Killing potential, which encodes the non-Abelian gauge transformations on the scalars and may also include Fayet-Iliopoulos terms.

The notation $\mathcal{L}_0$, $\mathcal{L}_{1/2}$ and $\mathcal{L}_1$ denote, respectively, the kinetic terms of spin 0, $\frac{1}{2}$, 1 fields in (1) [1]:

$$\mathcal{L}_0 = -\frac{1}{4\Phi}(\partial_\mu \Phi)(\partial^\mu \Phi) + \frac{1}{3} g_{\alpha\bar{\beta}} \Phi(\hat{\partial}_\mu z^\alpha)(\hat{\partial}^\mu \bar{z}^{\bar{\beta}}),$$
$$\mathcal{L}_{1/2} = -\frac{1}{2}\tilde{g}_{\alpha\bar{\beta}}\bar{\chi}^{\bar{\beta}}\not{D}\chi^\alpha + \frac{1}{2}\Phi\bar{\chi}^\alpha\gamma^\mu\chi^{\bar{\beta}}\hat{\partial}_\mu z^\gamma \times\left[-\frac{1}{3}g_{\gamma\bar{\beta}}L_\alpha + \frac{1}{4}L_{\alpha\gamma}L_{\bar{\beta}} - \frac{1}{4}L_\alpha L_{\gamma\bar{\beta}}\right] + \text{H.c.},$$
$$\mathcal{L}_1 = (\text{Re} f_{AB})\left[-\frac{1}{4}F^A_{\mu\nu}F^{\mu\nu B} - \frac{1}{2}\bar{\lambda}^A\not{D}\lambda^B\right] + \frac{1}{4}\,\text{i}[(\text{Im} f_{AB})F^A_{\mu\nu}\tilde{F}^{\mu\nu B} + (\hat{\partial}_\mu \text{Im} f_{AB})\bar{\lambda}^A\gamma_5\gamma^\mu\lambda^B], \tag{4}$$

where $f_{AB}(z)$ is a holomorphic kinetic gauge matrix, $L_\alpha \equiv \partial_\alpha \ln(-\Phi)$, $L_{\bar{\alpha}} \equiv \bar{L}_\alpha$, $L_{\alpha\beta} = \partial_\alpha L_\beta - \Gamma^\gamma_{\alpha\beta}L_\gamma$ and $g_{\gamma\bar{\beta}} = -\frac{1}{3}\Phi g_{\alpha\bar{\beta}} + \frac{1}{4}\Phi L_\alpha L_{\bar{\beta}}$, with $g_{\alpha\bar{\beta}} = \partial_\alpha\partial_{\bar{\beta}}\mathcal{K}$ as the Kähler metric, with the notation $\partial_\alpha \equiv \frac{\partial}{\partial z^\alpha}$, $\partial_{\bar{\alpha}} \equiv \frac{\partial}{\partial \bar{z}^{\bar{\alpha}}}$.

In the notation of Ref. [1], the covariant derivatives of the gauginos λ^A are defined as

$$D_\mu\lambda^A \equiv \left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{ab}(e)\gamma_{ab} - \frac{3}{2}\,\text{i}\mathcal{A}_\mu\gamma_5\right)\lambda^A - A^C_\mu\lambda^B f_{BC}{}^A, \tag{5}$$

with $f_{AB}{}^C$ the structure constants of the non-Abelian gauge group.

The fermion mass terms, $\mathcal{L}_m$, including gravitino bare mass terms (if any) and the mixed terms $\mathcal{L}_{\text{mix}}$ containing scalars and fermions, including factors of the frame function, are given explicitly in Ref. [1], and again will not be of interest to us in this work. We shall be explicitly interested in the penultimate of the terms on the right-hand side of Eq. (1), namely,

$$\mathcal{L}_d = \frac{1}{8}(\text{Re} f_{AB})\bar{\psi}_\mu\gamma^{ab}(F^A_{ab} + \hat{F}^A_{ab})\gamma^\mu\lambda^B + \frac{1}{\sqrt{2}}\left\{\bar{\psi}_\mu\gamma^\nu\gamma^\mu\chi^\alpha\left[\left(-\frac{1}{3}\Phi\right)g_{\alpha\bar{\beta}}\hat{\partial}_\nu\bar{z}^{\bar{\beta}} + \frac{1}{4}L_\alpha\partial_\nu\Phi\right] - \frac{1}{4}f_{AB\alpha}\bar{\chi}^\alpha\gamma^{ab}\hat{F}^{-A}_{ab}\lambda^B - \frac{1}{3}\Phi L_\alpha\bar{\chi}^\alpha\gamma^{\mu\nu}D_\mu\psi_\nu + \text{H.c.}\right\}, \tag{6}$$

where

$$\hat{F}_{ab}{}^A \equiv e_a{}^\mu e_b{}^\nu(2\partial_{[\mu}A^A_{\nu]} + g f_{BC}{}^A A^B_\mu A^C_\nu + \bar{\psi}_{[\mu}\gamma_{\nu]}\lambda^A). \tag{7}$$

The explicit expression for the four-fermion terms $\mathcal{L}_{4\text{f}}$, which also contain a significant dependence on the frame function Φ and its derivatives, will be presented below. Such four-fermion terms, in particular four-gravitino ones, have been argued in Ref. [11] to play an important role in some variants of the above class of conformal supergravity models, which can characterize certain low-energy limits of superstring theories, in which the frame function Φ may be identified with the dilaton-axion complex superfield, $\frac{\Phi}{3} \equiv \frac{1}{\kappa^2}e^{-2\varphi}$. Such models can serve as prototypes in which the Deser-Zumino [12] mechanism for *dynamical* breaking of the local supersymmetry (supergravity) scenario is realized explicitly. The important point to notice in this class of theories is the presence of the frame function Φ in front of the four-gravitino terms. This implies that, depending on the value of Φ, assumed to be stabilized appropriately by rolling to the minimum of an appropriate dilaton potential (generated by e.g., string loops, in case one embeds such conformal SUGRA models to string theory, or other ways of breaking the scale symmetry), the effective coupling of the four-gravitino interactions can be much larger than the gravitational coupling. Indeed, in the Einstein frame (denoted by E), the graviton field (in the original Jordan frame, denoted J) is redefined by means of

$$g_J^{\mu\nu} = e^{-2\varphi}g_E^{\mu\nu} = \left(\frac{\kappa^2}{3}\Phi\right)_{\text{bosonic}} g_E^{\mu\nu}, \qquad (\Phi|_{\text{bosonic}} > 0), \tag{8}$$

so that the curvature term in the target-space supergravity action has the canonical form, with coefficient the gravitational coupling $\kappa^2 = 8\pi G_N$, with G_N Newton's (four-dimensional) gravitational constant.

In the present work we shall work in the Einstein frame, which we consider as the *physical frame* in which cosmological observations are made. As already mentioned, in this frame, the Einstein-Hilbert part of the effective action, proportional to the scalar curvature of space-time, is normalized to its canonical form, with the coefficient in front proportional to the inverse of the Newton gravitational constant G_N. It is in this normalization that the observations of a Robertson-Walker cosmological observer are made, and the gravitational constant is constant in space-time, as seems to be indicated by observations. On the other hand, in the Jordan frame the effective gravitational "constant" would depend on the dilaton field, which in general could be space-time dependent. This was the case of Brans-Dicke theories which have been ruled out, at present, by the lack of experimental evidence for time variations in the gravitational constant.

The difference between the two frames, and the physical significance for cosmology of the Einstein frame, can be seen clearly, for instance, in the string-inspired cosmology of Ref. [13]. There, it is the Einstein-frame target-space metric that leads to a (linearly with the target time) expanding Universe, while the Jordan-frame metric is that of a static Minkowski space-time. The latter corresponds to a string observer who performs measurements using string rods and is distinct from the cosmological comoving observer who resides in the Einstein frame.

Therefore, it seems appropriate to evaluate the decay width of the gravitino and discuss its cosmological consequences in the Einstein frame, and this is what we do in this article. In the Einstein frame, the gravitational part of the effective conformal supergravity action, including the fermionic torsion induced four-gravitino terms, reads

$$\begin{aligned}\mathcal{L}^E(e^E)^{-1} &= -\frac{1}{2\kappa^2}R^E(e^E) + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}'_\mu\gamma_5\gamma_\nu D^E{}_\rho\psi'_\sigma - e^{2\varphi}V^E + \frac{11\kappa^2}{16}e^{-2\varphi}[(\bar{\psi}'_\mu\psi'^\mu)^2 - (\bar{\psi}'_\mu\gamma_5\psi'^\mu)^2] \\ &\quad + \frac{33}{64}\kappa^2 e^{-2\varphi}(\bar{\psi}'^\rho\gamma_5\gamma_\mu\psi'_\rho)^2 + \cdots \\ &= -\frac{1}{2\kappa^2}R^E(e^E) + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}'_\mu\gamma_5\gamma_\nu D^E_\rho\psi'_\sigma - e^{2\varphi}V^E + \rho^2(x) + \frac{\sqrt{11}}{2}\kappa\rho(x)e^{-\varphi}(\bar{\psi}'_\mu\psi'^\mu) + \pi^2(x) \\ &\quad + \frac{\sqrt{11}}{2}e^{-\varphi}\kappa i\pi(x)(\bar{\psi}'_\mu\gamma_5\psi'^\mu) + \frac{\sqrt{33}}{2}\kappa e^{-\varphi}i\lambda^\nu(\bar{\psi}'^\rho\gamma_5\gamma_\nu\psi'_\rho) + \cdots, \end{aligned} \tag{9}$$

where $R(e)$ denotes the curvature term with respect to the torsion-free spin connection, ψ'_μ denotes the canonically normalized gravitino with standard kinetic term as in $N = 1$ supergravity,

$$\psi'_\mu = e^\varphi\psi_\mu \tag{10}$$

while the ... denote structures, including auxiliary fields, that are not of direct interest to us here. In writing (9) we have expanded the four-gravitino terms into detailed structures to exhibit explicitly the terms that generate masses, and we linearize the four-gravitino terms. The condensate of interest to us is the vacuum expectation value (v.e.v.) of the linearizing field $\rho(x)$.

The reader should notice that the coefficients of the gravitino-ρ interaction terms in (9) contain dilaton-dependent factors $\sim e^{-\varphi}$, and are thus proportional, not to κ, but to

$$\tilde{\kappa} \equiv \kappa e^{-\varphi}. \tag{11}$$

A one-loop analysis shows that the effective potential for the condensate ρ field acquires a minimum at [11]

$$\rho_{\rm min} = \langle\rho\rangle = \pm 0.726 \tag{12}$$

at which it vanishes. The gravitino mass term, then, in (9) takes the following form:

$$-m_{3/2}\bar{\psi}'_\mu\Gamma^{\mu\nu}\psi'_\nu = -\frac{1}{2}m_{3/2}\bar{\psi}'_\mu\psi'^\mu, \tag{13}$$

with the dynamically generated gravitino mass of order

$$m_{3/2} = \sqrt{11}\tilde{\kappa}^{-1}\rho_{\rm min} = 2.408\kappa^{-1}e^\varphi = \frac{2.408}{\sqrt{8\pi}}e^\varphi M_P. \tag{14}$$

For large negative values of the v.e.v. $\langle\varphi\rangle < 0$, the resulting gravitino mass is much smaller than the Planck scale, and thus the effective coupling $\tilde{\kappa}$ (11) is much larger than the gravitational coupling G_N.

This implies that quantum gravitational corrections to the Minkowski space-time background, on which the above minimization of the effective potential has been considered, are not strong enough to destabilize (at least quickly) the gravitino condensate, unlike the case of standard $N = 1$ supergravity [14]. This prompted the authors of Ref. [11] to consider large positive values of

$$\langle e^{-2\varphi}\rangle \equiv \frac{1}{3}\kappa^2\langle\Phi|_{\rm bosonic}\rangle \gg 1, \tag{15}$$

and discuss their relevance to the above-mentioned scenario of dynamical *metastable* breaking of local supersymmetry (SUSY) and generation of gravitino mass.

It is the point of this article to examine the constraints implied by such an assumption on the dark sector of the Universe in the case of the NMSSM embedded in this conformal supergravity framework. However, we shall not only restrict ourselves to the case of negative expectation values of the dilaton, but we shall be more general and also consider the dark matter phenomenology/cosmology of the case of positive dilaton v.e.v. In this latter case, dynamical breaking of local SUSY is not possible in view of the destabilizing effects of the graviton fluctuations; nevertheless one assumes conventional breaking of SUSY, e.g., through gluino condensation [15], which is then communicating to the gravity sector to result in a nontrivial gravitino mass term $m_{3/2}$.

III. DECAY PROCESSES INVOLVING GRAVITINOS AND ASSOCIATED CONSTRAINTS

We first notice that, in the Einstein frame, one has to first normalize the kinetic terms of the scalars z^α and χ and λ fermions, by appropriate redefinitions involving the frame function Φ. In particular the gauginos should be renormalized in the Einstein frame as

$$\lambda' = e^{2\varphi}\lambda \tag{16}$$

in order to acquire canonical kinetic terms. On the other hand the gauge terms of the spin-1 part $\mathcal{L}_1$ (4) are already in canonical form, in view of the conformal nature of the (Maxwell-like) kinetic terms for the Yang-Mills fields. Here we consider the case $f_{AB} = {\rm const} = 1$.

Taking (10) and (16) into account, as well as the fact that the Yang-Mills gauge fields are not renormalized in the

Einstein frame by the frame function $e^{-2\varphi}$, we may write the first term of the right-hand side of (6) in the Einstein frame as

$$(L_d)_E = \frac{1}{8}(\mathrm{Re} f_{AB})\bar{\psi}'_\mu \gamma^{ab}(F'^A{}_{ab} + \hat{F}'^A{}_{ab})\gamma^\mu \lambda'^B. \quad (17)$$

This term is responsible for the gravitino–gaugino–gauge-boson interaction and it is not transformed going to the new frame.

Trying to recap the transformations we have

$$\psi'_\mu = e^{\varphi}\psi_\mu, \qquad \lambda' = e^{2\varphi}\lambda. \quad (18)$$

The scalar and vector fields don't change. Using these we can calculate how the interactions that are relevant to the gravitino decays change due to the dilation presence. In particular we are interested in the $\psi\chi Z(\gamma)$ decays, where with χ we denote the neutralino that is the lightest supersymmetric particle in our model. Doing so, we may then consider the terms $\mathcal{L}_d$ (6) in order to compute the decay rate of the massive gravitino field $\tilde{G}$ into, say, a neutralino χ in the NMSSM and a Z gauge boson:

$$\tilde{G} \to \chi + Z.$$

The so affected gravitino decay rate will in turn affect the dark matter relic density (assumed to be dominated by neutralinos in NMSSM) and this may imply stronger BBN constraints. It is therefore important that detailed cosmological studies of such dilaton extended minimal SUGRA models are performed.

In the usual minimal SUGRA, the gravitino satisfies the Rarita-Schwinger (RS) equations

$$\gamma^\mu \psi_\mu(x) = 0, \qquad (i\not{\partial} - m)\psi_\mu(x) = 0, \quad (19)$$

which result from the RS action [16]

$$\mathcal{L} = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu \gamma_5 \gamma_\nu \partial_\rho \psi_\sigma - \frac{1}{4}m_{3/2}\bar{\psi}_\mu[\gamma^\mu, \gamma^\nu]\psi_\nu. \quad (20)$$

The RS action including the dilaton effects can be written as

$$\mathcal{L}' = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}'_\mu \gamma_5 \gamma_\nu \partial_\rho \psi'_\sigma - \frac{1}{4}m'_{3/2}\bar{\psi}'_\mu[\gamma^\mu, \gamma^\nu]\psi'_\nu, \quad (21)$$

where $\psi'_\mu \to e^{\varphi}\psi_\mu$, and the relation between $m'_{3/2}$ and $m_{3/2}$ is

$$m'_{3/2} = e^{\varphi} m_{3/2}, \quad (22)$$

so for $\langle\varphi\rangle < 0$, which is the physical case in several of the backgrounds discussed in order to allow for dynamical breaking of local SUSY, the gravitino mass in the conformal supergravity scenario will be smaller than the corresponding one in the normal SUGRA.

In the NMSSM [10] the neutralino field can be written as

$$\chi = N_{11}\tilde{B} + N_{12}\tilde{W}^3_0 + N_{13}\tilde{H}^1_0 + N_{14}\tilde{H}^2_0 + N_{15}\tilde{S}, \quad (23)$$

where N_{ij} are the elements of the 5×5 neutralino diagonalizing matrix. If N_{11} dominates the sum $\sum_{i=1,5} N_{i1}^2 = 1$, then the lightest neutralino is binolike. On the other hand, if N_{15} is dominant then the neutralino is singlinolike. The latest data of the LHC experiments, indicating a Higgs boson mass in the ballpark of 125 GeV [17], combined with other experimental data from B physics and direct dark matter searches, seem to disfavor the singlino case [18]. Thus, in the following it will be assumed that the lightest neutralino is mainly bino. In this case, the dominant two body decay channels for gravitino are $\tilde{G} \to \gamma\chi$ and $\tilde{G} \to Z\chi$. Nevertheless, even in the singlinolike case those channels, especially the $\gamma\chi$, are dominant, mainly due to the large available phase space. Therefore, our assumption that the light neutralino is mainly bino is sufficiently generic.

We note in passing at this point that, as shown in Ref. [19], a time-dependent (cosmological) dilaton (which can run with the cosmic time before BBN) can reduce considerably the neutralino relic density, thereby increasing the cosmologically allowed available parameter space of SUSY even beyond the LHC reach. In this article we ignore such effects, focusing on the gravitino interactions exclusively, and assuming that the dilaton in our case has been stabilized to its vacuum expectation value at the scale of SUSY breaking (or at least it is approximately constant during a cosmological epoch). However, even for stabilized dilatons, in the Einstein frame, the neutralino-pair annihilation processes are affected by the conformal couplings; in particular they are enhanced for large $e^{\varphi} > 1$. Such enhancement may reduce the relic abundances already in the constant dilaton case. We postpone a comprehensive study of such effects on gravitino decays and the neutralino dark matter abundance for a future study.

Below we shall consider two cases: one, in which the neutralino is the stable dark matter candidate and the gravitino is heavier, thus unstable, and the other, in which the gravitino is cosmologically stable and thus constitutes the dark matter candidate. We commence our discussion from the former case.

In such a case, the formula for the decay width $\tilde{G} \to \gamma\chi$ without the dilaton effects reads as [20]

$$\Gamma_{\gamma\chi} = \frac{1}{16\pi}\frac{\overline{|\mathcal{M}_\gamma|^2}}{m_{3/2}}\mathcal{F}(m_{3/2}, m_\chi, 0), \quad (24)$$

where the spin average amplitude squared is

$$\overline{|\mathcal{M}_\gamma|^2} = \frac{B_\gamma^2}{6M_P^2}\frac{1}{m_{3/2}^2}(m_{3/2}^2 - m_\chi^2)^2(3m_{3/2}^2 + m_\chi^2) \quad (25)$$

and the kinematical factor is defined as

$$\mathcal{F}(m_0, m_1, m_2) = \frac{1}{m_0^2}[(m_0^2 - (m_1 + m_2)^2) \times (m_0^2 - (m_1 - m_2)^2)]^{1/2}. \quad (26)$$

On the other hand, taking into account the dilaton effects, i.e., considering the corresponding process in the conformal SUGRA model in the Einstein frame, the corresponding width becomes

$$\Gamma'_{\gamma\chi} = \frac{1}{16\pi} \frac{\overline{|\mathcal{M}'_\gamma|^2}}{cm_{3/2}} \mathcal{F}(cm_{3/2}, m_\chi, 0), \quad (27)$$

where

$$\overline{|\mathcal{M}'_\gamma|^2} = \frac{B_\gamma^2}{12M_P^2} \frac{1}{m_{3/2}^2} c^2 (m_{3/2}^2 - m_\chi^2)^2 \times (6c^4 m_{3/2}^2 + (c^2+1)m_\chi^2). \quad (28)$$

Above it was defined $c = e^\varphi$. Notice that putting $c = 1$ we recover the result of Eq. (24). The factor B_γ is related to the bino ($\tilde{B}$) and neutral wino ($\tilde{W}_0^3$) components of the neutralino, that is, $B_\gamma = N_{i1}\cos\theta_W + N_{i2}\sin\theta_W$, where θ_W is the electroweak mixing angle.

For the channel $\tilde{G} \to Z\chi$ in the standard (dilaton-free) SUGRA the width reads as

$$\Gamma_{Z\chi} = \frac{1}{16\pi} \frac{\overline{|\mathcal{M}_Z|^2}}{m_{3/2}} \mathcal{F}(m_{3/2}, m_\chi, M_Z), \quad (29)$$

where

$$\overline{|\mathcal{M}_Z|^2} = \frac{B_Z^2}{6M_P^2} \frac{1}{m_{3/2}^2}[3m_{3/2}^6 - m_{3/2}^4(5m_\chi^2 + M_Z^2) + 12m_{3/2}^3 m_\chi M_Z^2 + m_{3/2}^2(m_\chi^4 - M_Z^4) + (m_\chi^2 - M_Z^2)^3], \quad (30)$$

where $B_Z = -N_{i1}\sin\theta_W + N_{i2}\cos\theta_W$. In the case of dilaton the same width becomes

$$\Gamma'_{Z\chi} = \frac{1}{16\pi} \frac{\overline{|\mathcal{M}'_Z|^2}}{cm_{3/2}} \mathcal{F}(cm_{3/2}, m_\chi, M_Z), \quad (31)$$

where

$$\overline{|\mathcal{M}'_Z|^2} = \frac{B_Z^2}{24M_P^2} \frac{1}{m_{3/2}^2} c^2[-2(c^2+1)M_Z^6 + M_Z^4(6(c^2+1)m_\chi^2 + (-6c^4 + c^2 + 1)m_{3/2}^2) + 2(m_{3/2} - m_\chi)^2(m_{3/2} + m_\chi)^2(6c^4 m_{3/2}^2 + (c^2+1)m_\chi^2) + M_Z^2(-6(c^2+1)m_\chi^4 + (-6c^4 + c^2 + 1)m_{3/2}^4 + 6c(12c^4 - 3c^2 - 1)m_{3/2}^3 m_\chi + 3(-2c^4 + c^2 + 1)m_{3/2}^2 m_\chi^2)]. \quad (32)$$

It is worth noticing that $\Gamma_{Z\chi}$ goes to $\Gamma_{\gamma\chi}$ in the limit $M_Z \to 0$, and the same holds for Γ'.

We start our numerical analysis discussing models where dark matter consists of neutralinos and the gravitino is unstable. In this case decays of the gravitinos to neutralinos can be an important constraint, affecting significantly the BBN predictions. In Fig. 1 we present the ratio $R = \Gamma'/\Gamma$ for the processes $\tilde{G} \to \gamma\chi$ and $\tilde{G} \to Z\chi$ as a function of e^φ. To make this figure we use the numerical values $m_{3/2} = 300$ GeV, $m_\chi = 150$ GeV, and for these the two body decay widths for the dominant channels (involving γ) are 1.4×10^{-32} GeV and 3.3×10^{-33} GeV for the $\gamma\chi$ and $Z\chi$, respectively. This yields a gravitino lifetime $\sim 4.7 \times 10^7$ s.

We first concentrate in the region of Fig. 1 where $c > 1$, for which although dynamical generation of gravitino mass and thus breaking of local SUSY may *not* occur [11,14], nevertheless, as mentioned above, one may assume a more or less conventional mechanism [15] for SUSY breaking and the generation of a gravitino mass term $m_{3/2}$, (21) and (22). In this case, there is an enhancement of the ratio $R = \Gamma'/\Gamma$, where the prime denotes the width for the conformal SUGRA case, where dilaton effects are taken into account. An important observation concerns the fact that for $c < 1$, which is the case where dynamical (metastable) breaking of local SUSY mass is possible, according to the arguments of Ref. [11] reviewed above [cf. discussion leading to Eq. (14)] the decay of gravitino to photons and neutralinos is kinematically *forbidden*. This case will face important constraints from BBN which will be discussed below.

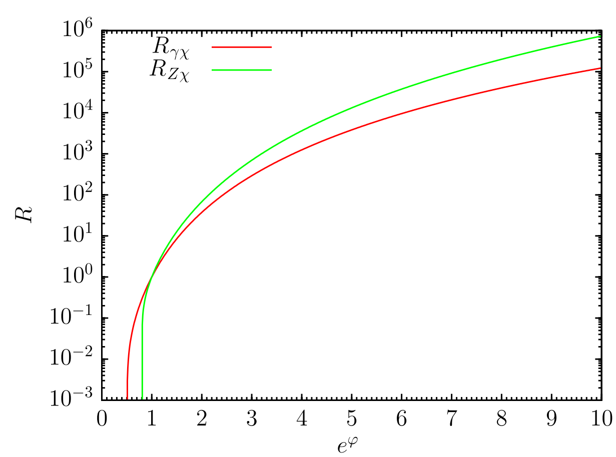

FIG. 1 (color online). The ratio $R = \Gamma'/\Gamma$ for the channels $\tilde{G} \to \chi\gamma$ and $\tilde{G} \to \chi Z$, as a function of e^φ, for the neutralino dark matter case.

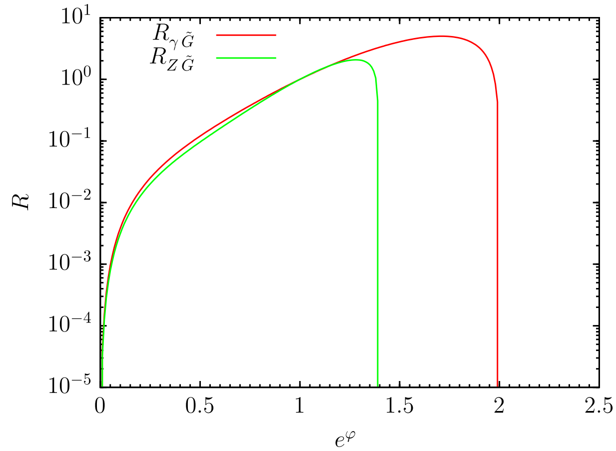

FIG. 2 (color online). The ratio $R = \Gamma'/\Gamma$ for the channels $\chi \to \tilde{G}\gamma$ and $\chi \to \tilde{G}Z$, as a function of e^{φ} for the gravitino dark matter case.

Concerning the BBN constraints, we know that they become important for the lifetime of the unstable particle $\tau \gtrsim 10^2$ s [21]. Since $\tau' = \tau/R$ and the gravitino lifetime without the dilaton effects is $\mathcal{O}(10^7)$ s, one observes that with $R \gtrsim 10^5$ one avoids all the important BBN constraints practically for any gravitino mass. Using Fig. 1 we understand that this happens for $e^{\varphi} \gtrsim 9$ or 10. For smaller values of e^{φ} the BBN appears to become important, since the lifetime of the gravitino approaches its original value 10^7 s as $e^{\varphi} \to 1$, but its abundance is still enhanced by the factor e^{φ}. On the other hand, values for $e^{\varphi} < 1$ enhance dramatically the gravitino lifetime, leading to the *exclusion* of this range of the scale factor, being incompatible with the BBN constraints. We have checked numerically different values of gravitino mass up to TeV, but our results remain basically the same. That is, one finds that for e^{φ} in the range above 8 or 10, the BBN constraints can be avoided.

In addition, we discuss also the complementary case where the gravitino is the lightest supersymmetric particle. In this case one studies the reverse processes $\chi \to \tilde{G}\gamma$ and $\chi \to \tilde{G}Z$, from the point of view of the constraints induced by BBN. To compute the corresponding decay widths we use the fact that the amplitudes squared $\overline{|\mathcal{M}_{\gamma,Z}|^2}$, both for the standard and the dilaton cases, are the same as before, due to the assumed *CPT* invariance and unitarity. On the other hand, one has to interchange m_χ and $m_{3/2}$ in $\mathcal{F}$ and in the denominators of Eqs. (24), (27), (29), and (31). Doing so, we plot in Fig. 2 the ratio $R = \Gamma'/\Gamma$ for these reverse processes. The numerical values we use are $m_{3/2} = 150$ GeV, $m_\chi = 300$ GeV, exactly the reverse case of Fig. 1. With this choice, the neutralino lifetime is now $\mathcal{O}(10^7)$ s. We thus see that the kinematically allowed region happens for values of $c = e^{\varphi} < 2$. This region depends on the choice neutralino and gravitino masses, but the attained values of R are not sensitive to this. One observes that in this case $R \lesssim 10$. Thus in this case the dilaton effects cannot be used to relax the BBN constraints.

IV. CONCLUSIONS AND OUTLOOK

In this article we have discussed the effects of stabilized dilatons on processes involving unstable particles, including gravitinos, that may affect BBN in conformal SUGRA models, incorporating the NMSSM in their spectra. We have found that, in the case where the gravitino is unstable, and the neutralino plays the role of dark matter, there are regions of the scale (dilaton) factor $e^{\varphi} > 8$ in which the BBN constraints can be avoided altogether. Moreover, in such regimes the neutralino dark matter abundances may be diluted thereby avoiding the cosmological and particle physics constraints on SUSY matter at current colliders, including LHC. Unfortunately, this case seems not to favor dynamical SUGRA breaking (due to quantum-gravity instabilities), and therefore one has to assume more or less conventional breaking of SUSY and its communication to the gravitational sector. On the other hand, if the gravitino is cosmologically stable, playing the role of dark matter, the BBN constraints are very restrictive in the full (kinematically allowed) range of the scale (dilaton) factor e^{φ}.

We have not discussed in detail time-dependent dilaton effects that involve running scale factors up to BBN, which are known to reduce significantly the dark matter abundances. Such effects, when combined with the dilaton effects on the gravitino decays considered here, may change significantly the cosmology and phenomenology of such conformal SUGRA models. We plan to return to these interesting issues in a forthcoming publication.

ACKNOWLEDGMENTS

The work of N. E. M. is supported in part by STFC (UK) under the research Grant No. ST/J002798/1 and by the London Centre for Terauniverse Studies (LCTS), using funding from the European Research Council via the Advanced Investigator Grant No. 267352; this latter grant also supported visits by V. C. S. to the CERN TH Division, which he thanks for its hospitality and where this work was initiated. The work of V. C. S. is supported by Marie Curie International Reintegration Grants No. SUSYDM-PHEN, No. MIRG-CT-2007-203189.

[1] S. Ferrara, R. Kallosh, A. Linde, A. Marrani, and A. Van Proeyen, Phys. Rev. D **82**, 045003 (2010).
[2] S. Ferrara, R. Kallosh, A. Linde, A. Marrani, and A. Van Proeyen, Phys. Rev. D **83**, 025008 (2011).
[3] F. L. Bezrukov and M. Shaposhnikov, Phys. Lett. B **659**, 703 (2008).
[4] F. Bezrukov and M. Shaposhnikov, J. High Energy Phys. 07 (2009) 089.
[5] F. Bezrukov, A. Magnin, M. Shaposhnikov, and S. Sibiryakov, J. High Energy Phys. 01 (2011) 016.
[6] J. Garcia-Bellido, J. Rubio, M. Shaposhnikov, and D. Zenhausern, Phys. Rev. D **84**, 123504 (2011).
[7] M. B. Einhorn and D. R. T. Jones, J. High Energy Phys. 03 (2010) 026.
[8] L. Alvarez-Gaume, C. Gomez, and R. Jimenez, Phys. Lett. B **690**, 68 (2010).
[9] L. Alvarez-Gaume, C. Gomez, and R. Jimenez, J. Cosmol. Astropart. Phys. 03 (2011) 027.
[10] C. Panagiotakopoulos and K. Tamvakis, Phys. Lett. B **446**, 224 (1999); M. Maniatis, Int. J. Mod. Phys. A **25**, 3505 (2010); For a review see U. Ellwanger, C. Hugonie, and A. M. Teixeira, Phys. Rep. **496**, 1 (2010) and references therein.
[11] J. Ellis and N. E. Mavromatos, Phys. Rev. D **84**, 085016 (2011).
[12] S. Deser and B. Zumino, Phys. Rev. Lett. **38**, 1433 (1977).
[13] I. Antoniadis, C. Bachas, J. R. Ellis, and D. V. Nanopoulos, Nucl. Phys. **B328**, 117 (1989).
[14] I. L. Buchbinder and S. D. Odintsov, Classical Quantum Gravity **6**, 1955 (1989).
[15] See, for instance, H. P. Nilles, Phys. Rep. **110**, 1 (1984) and references therein.
[16] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).
[17] G. Aad *et al.* (ATLAS Collaboration), Phys. Lett. B **716**, 1 (2012); S. Chatrchyan *et al.* (CMS Collaboration), Phys. Lett. B **716**, 30 (2012).
[18] K. Kowalska, S. Munir, L. Roszkowski, E. M. Sessolo, S. Trojanowski, and Y.-L. S. Tsai, arXiv:1211.1693.
[19] A. B. Lahanas, N. E. Mavromatos, and D. V. Nanopoulos, PMC Phys. A **1**, 2 (2007); Phys. Lett. B **649**, 83 (2007); A. B. Lahanas, Phys. Rev. D **83**, 103523 (2011); A. B. Lahanas and V. C. Spanos, J. High Energy Phys. 06 (2012) 089.
[20] J. R. Ellis, K. A. Olive, Y. Santoso, and V. C. Spanos, Phys. Lett. B **588**, 7 (2004).
[21] M. Kawasaki, K. Kohri, and T. Moroi, Phys. Rev. D **71**, 083502 (2005); R. H. Cyburt, J. Ellis, B. D. Fields, F. Luo, K. A. Olive, and V. C. Spanos, J. Cosmol. Astropart. Phys. 10 (2009) 021.