Simple theory of slow extraction

from a synchrotron

Sandro Rossi

3rd-order resonance slow extraction for medical synchrotrons

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NORMALISED SYSTEM OF COORDINATES

Calculations are greatly simplified by using normalised coordinates: the betatron oscillations are represented by a circular motion in the normalised phase plane, such that their amplitude and phase can be easily evaluated. The distance along the equilibrium orbit is denoted by *s* and the radial or vertical component of the displacement from the reference orbit is denoted by *x*, the betatron oscillation is expressed by:

$$x(s) = a\sqrt{\beta(s)}\cos[\mu(s) + \delta]$$

where β(s) is the betatron amplitude function, and

$$\mu(s) = \int \frac{ds}{\beta(s)}$$

the betatron phase function with *a* and δ arbitrary constants. This expression is reduced to a harmonic oscillation if we introduce the normalised variables:

$$X(s) = \frac{x(s)}{\sqrt{\beta(s)}}$$

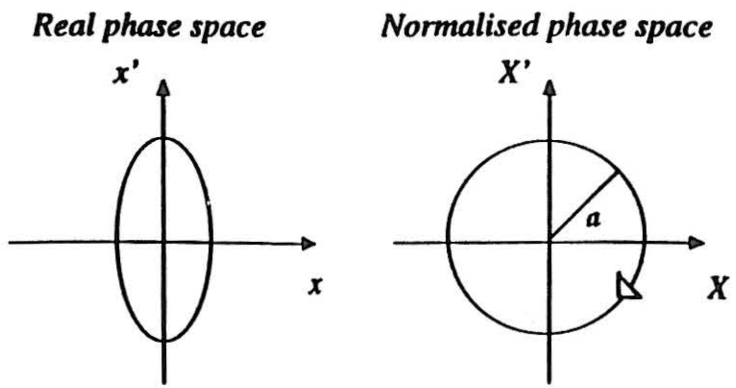
$$\mu(s) = \int \frac{ds}{\beta(s)}$$

In this new system the betatron oscillation is described by:

$$X = a\cos(\mu + \delta)$$

and the trajectory in phase space becomes a circle; deriving X with respect to the variable μ:

$$X' = \frac{dX}{d\mu} = -a\sin(\mu + \delta)$$



The emittance is the same in both systems and is given by:

$$E_x = \frac{\hat{x}^2}{\beta_x}\pi = \hat{X}^2\pi$$

The normalising matrix N is expressed by:

$$N = \frac{1}{\sqrt{\beta}}\begin{pmatrix} 1 & 0 \\ \alpha & \beta \end{pmatrix}$$

while its inverse is:

$$N^{-1} = \frac{1}{\sqrt{\beta}}\begin{pmatrix} \beta & 0 \\ -\alpha & 1 \end{pmatrix}$$

The matrix for the transformation in the real phase plane (x, x') from an azimuth s_1 to an azimuth s_2 is:

$$M_{12} = \begin{pmatrix} \sqrt{\frac{\beta_2}{\beta_1}}(\cos\mu + \alpha_1 \sin\mu) & \sqrt{\beta_1\beta_2}\sin\mu \\ -\left(\frac{(1+\alpha_1\alpha_2)\sin\mu + (\alpha_2 - \alpha_1)\cos\mu}{\sqrt{\beta_1\beta_2}}\right) & \sqrt{\frac{\beta_1}{\beta_2}}(\cos\mu - \alpha_2 \sin\mu) \end{pmatrix}$$

where $\mu = \mu(s_2) - \mu(s_1)$; $\beta_1 = \beta(s_1)$, etc. The matrix for the transformation in the normalised phase plane is:

$$\overline{M_{12}} = \begin{pmatrix} \cos\mu & \sin\mu \\ -\sin\mu & \cos\mu \end{pmatrix}$$

FIELD IN A SEXTUPOLE

From Maxwell equations, assuming that the magnetic field has, to a good approximation, only transverse components and being interested only in the field inside the vacuum pipe, we can derive the vector potential "A" for a magnet with $2n$ poles in Cartesian coordinates:

$$A = \sum_n A_n f_n(x,z)$$

with f_n an homogeneous function in x and z of order n:

$$f_n(x,z) = (x + iz)^n$$

The real terms correspond to regular multipoles, the imaginary ones to skew multipoles as summarised in the following table.

Vector potential solutions in Cartesian coordinates

Multipole	n	Regular	Skew
quadrupole	2	x^2-z^2	$2xz$
sextupole	3	x^3-3xz^2	$3x^2z-z^3$
octupole	4	$x^4-6x^2z^2+z^4$	$4x^3z-4xz^3$

For our calculations, it is useful to relate A_n to the field, for example, in the median plane using the Taylor expansion:

$$B_z(z=0)=\frac{\partial A}{\partial x}=\sum_{n=1}^{\infty} n A_n x^{n-1}=\sum_{n=1}^{\infty}\frac{1}{(n-1)!}\left(\frac{d^{(n-1)}B_z}{dx^{(n-1)}}\right)_0 x^{(n-1)}$$

so that:

$$A_n=\frac{1}{n!}\left(\frac{d^{(n-1)}B_z}{dx^{(n-1)}}\right)_0$$

In particular, for a regular sextupole:

$$A=A_3\left(x^3-3xz^2\right)$$

so that:

$$B_z=\frac{\partial A}{\partial x}=3A_3\left(x^2-z^2\right)=\frac{1}{2}\left(\frac{d^2 B_z}{dx^2}\right)_0\left(x^2-z^2\right)$$

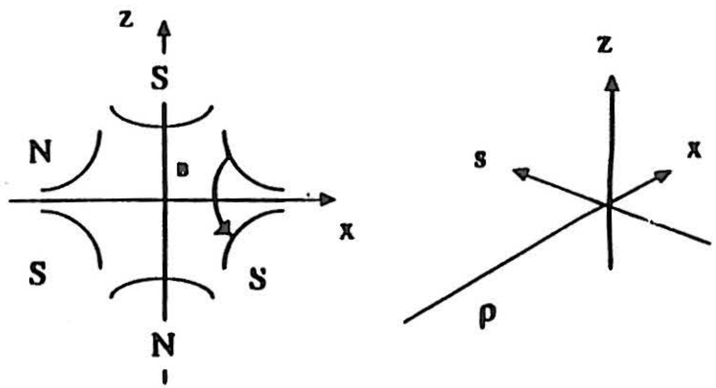
$$B_x=-\frac{\partial A}{\partial z}=6A_3 xz=\left(\frac{d^2 B_z}{dx^2}\right)_0 xz$$

Sign conventions

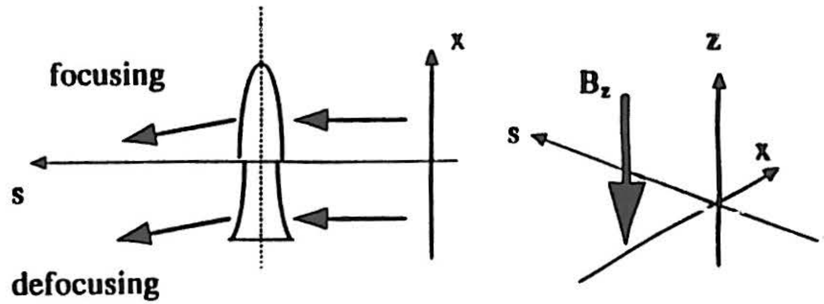
The following diagram shows the conventions adopted, i.e.:

- F-sextupole, if it focuses particle with x positive;
- D-sextupole, if it defocuses particles with x positive.

Focusing sextupole and coordinate system



F-sextupole



Let us now calculate the kick given by a focusing sextupole to a particle moving at a distance x from the central orbit on the median plane supposing the length ℓ_s of the sextupole to be negligible (thin-lens approximation):

$$\Delta x' = -\frac{B_z(z=0)\ell_s}{|B_0\rho|}$$

where the minus sign is due to the fact that a particle at a positive x is bent by the B field toward the origin of axis. Substituting the expression for the field:

$$\Delta x' = -\frac{1}{2}\frac{\ell_s}{|B_0\rho|}\left(\frac{d^2 B_z}{dx^2}\right)_0 x^2$$

In normalised coordinates at the sextupole position, we have:

$$\Delta X_s' = -\sqrt{\beta_s}\frac{1}{2}\frac{\ell_s}{|B_0\rho|}\left(\frac{d^2 B_z}{dx^2}\right)_0 (\sqrt{\beta_s}X)^2$$

$$= -\beta_s^{3/2}\frac{1}{2}\frac{\ell_s}{|B_0\rho|}\left(\frac{d^2 B_z}{dx^2}\right)_0 X^2 = S X^2$$

$$\boxed{\Delta X_S' = S X^2}$$

with S being the normalised sextupole strength:

$$\boxed{S = -\beta_s^{3/2}\frac{1}{2}\frac{\ell_s}{|B_0\rho|}\left(\frac{d^2 B_z}{dx^2}\right)_0 = -\frac{1}{2}\beta_s^{3/2}\ell_s m_{CAS} = -\frac{1}{2}\beta_s^{3/2}\ell_s K2_{MAD}}$$

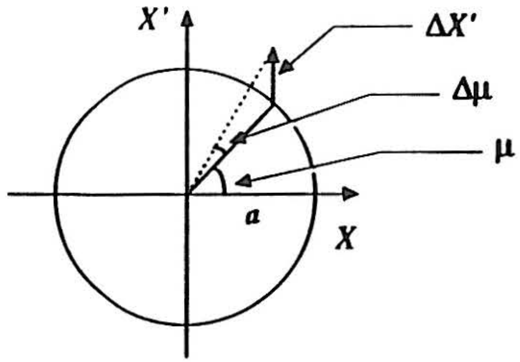
and $K2_{MAD}$ [m^{-3}]:

$$K2_{MAD}\ [\mathrm{m}^{-3}] = \frac{1}{3.33556 \cdot p_0\ [\mathrm{GeV/c}]}\left(\frac{d^2 B_z}{dx^2}\right)_0\ [\mathrm{Tm}^{-2}].$$

LOCKING ACTION AND ADDITION OF SEXTUPOLES

Let us go into detail analysing the effect of a sextupole treated as a thin lens. As shown in the following diagram the phase shift given by the sextupole is:

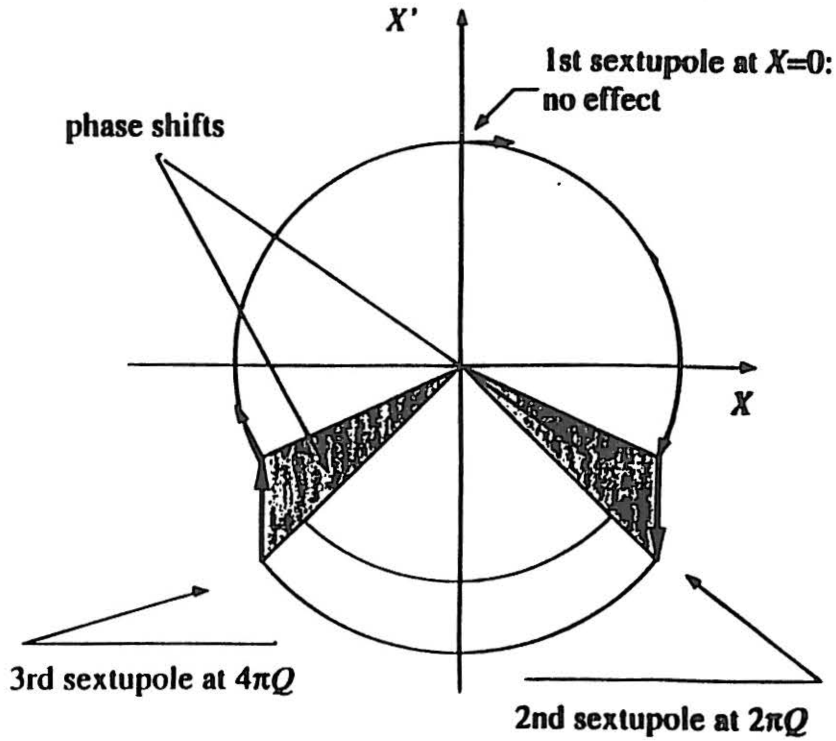
$$\Delta\mu = \frac{\Delta X'}{a}\cos\mu = \frac{S}{a}X^2\cos\mu = Sa\cos^3\mu = \frac{Sa}{4}(\cos 3\mu + 3\cos\mu)$$



Supposing that we are close to a third integer tune, in three turns the second term in the last equation averages zero, while the first term is constant, since its phase is an integer multiple of 2π. From the figure it also appears that:

$$\frac{\Delta a}{a} = \frac{\Delta X'}{a}\sin\mu = \frac{S}{a}X^2\sin\mu = Sa\cos^2\mu\sin\mu = \frac{Sa}{4}\sin 3\mu$$

where the last equality is valid close to the resonance. When the particle is exactly on resonance, the phase advances by an integral number of 2π and the particle 'locks' onto the resonance, repeating again and again the same trajectory in phase space. The following figure is a pictorial representation of the situation, corresponding to the "fixed points - corners" of the last stable triangle.



If we are close to resonance from the expression of phase shift, we have in the smooth approximation where ϕ is the azimuthal angle:

$$\Delta\mu = 2\pi\Delta Q \approx \frac{Sa}{4}(\cos 3Q\phi)$$

Smooth approx ⇒ $\mu = \frac{2\pi Q}{2\pi}\phi = Q\phi$

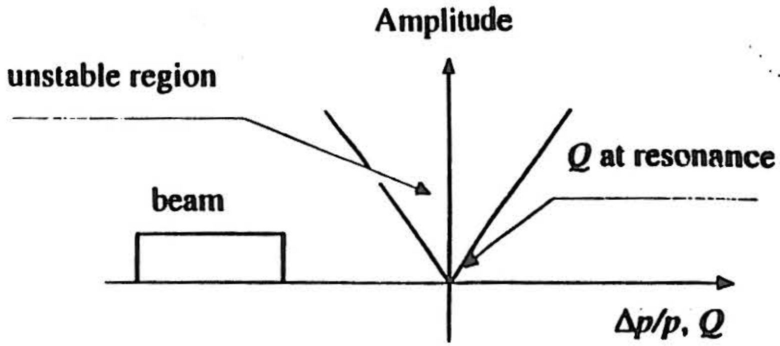
from which we deduce that the tune of the particle wanders within a band around the unperturbed tune Q_0:

$$Q_0 - \frac{Sa}{8\pi} < Q < Q_0 + \frac{Sa}{8\pi}$$

If the third integer tune value is not inside this band, the particle cannot be locked and it is stable, in other words at a "tune distance" ΔQ from Q_0 the particles with amplitude:

$$a < \frac{8\pi|\Delta Q|}{S}$$

are stable. This fact gives the phase space stable triangles that will be discussed later. Replacing the inequality by an equality, we obtain the amplitude of the unstable fixed points and figure out that in the amplitude-tune space the resonance is a straight line as shown in the following picture. The slope of the resonance line is related to the spread in momentum through the chromaticity.



Now, suppose that we have an azimuth distribution of sextupoles that can be expressed as a Fourier series in ϕ:

$$S(\phi) = \sum_s S_s e^{js\phi}$$

introducing it into the equation of phase displacement and integrating around the machine circumference:

$$\frac{e^{j3Q\phi} + e^{-j3Q\phi}}{2}$$

$$\Delta\mu = \sum_s \int \frac{S_s a}{4} \cos(3Q\phi) e^{js\phi}\, d\phi =$$

$$= \sum_s \int_0^{2\pi} \frac{S_s a}{8}\left[e^{j(3Q+s)\phi} + e^{j(s-3Q)\phi}\right] d\phi \quad \neq 0 \text{ only if } s = \pm 3Q$$

we notice that the integral is large and finite if:

$$3Q = s$$

$$S_{3Q} = \int S(\phi)\, e^{j3Q\phi}\, d\phi = \sum_i S_i\, e^{j3\mu_i}$$

that is in the addition of the sextupoles we have to consider the third harmonic of the sextupole distribution in betatron phase around the machine. Actually, since in the expression of S it also appears as a power of the beta function, and periodicities in the lattice structure can also drive the resonance together with multipole field patterns.

Equating the sum of sextupoles to a virtual sextupole we have:

$$S_{virtual} \exp\left(j3\mu_{X,virtual}\right) = \sum_i S_i \exp\left(j3\mu_{X,i}\right)$$

where $\mu_{X,i}$ is the betatron phase location of the i-th sextupole.

By separating real and imaginary parts we obtain:

Equivalent sextupole phase and strength

$$\tan\left(3\mu_{X,virtual}\right) = \frac{\sum_i S_i \sin\left(3\mu_{X,i}\right)}{\sum_i S_i \cos\left(3\mu_{X,i}\right)} \qquad (1)$$

$$S^2_{virtual} = \left(\sum_i S_i \cos\left(3\mu_{X,i}\right)\right)^2 + \left(\sum_i S_i \sin\left(3\mu_{X,i}\right)\right)^2 \qquad (2)$$

that is betatron phase and strength of a single equivalent sextupole in the ring.

First consideration

From the previous equation (2) it is easy to show that for a distribution of sextupoles of equal strength and spaced by $\Delta\mu$ we have:

- *cancellation* of the driving term if $\Delta\mu = \pi/3$;
- *reinforcement* of the driving term if $\Delta\mu = 2\pi/3$.

Second consideration

If the sextupoles are placed in a region of finite horizontal dispersion (D_x) they also affect the chromaticity $\Delta Q'_{x,z}$ of the machine:

$$\Delta Q'_x \equiv \frac{\partial Q_x}{\partial \frac{\Delta p}{p}} = \frac{\ell_s}{4\pi}\left[K2^{SF}_{MAD} \sum_{n=1}^{NF} (\beta_x D_x)_n + K2^{SD}_{MAD} \sum_{n=1}^{ND} (\beta_x D_x)_n\right] \qquad (3)$$

$$\Delta Q'_z \equiv \frac{\partial Q_z}{\partial \frac{\Delta p}{p}} = -\frac{\ell_s}{4\pi}\left[K2^{SF}_{MAD} \sum_{n=1}^{NF} (\beta_z D_x)_n + K2^{SD}_{MAD} \sum_{n=1}^{ND} (\beta_z D_x)_n\right] \qquad (4)$$

In general, it would be better to have the possibility of acting on the resonance and the chromaticity independently:

- *to drive the resonance, without affecting the chromaticity*
  - ♦ put the sextupoles in a zero dispersion region;
- *to act on the chromaticity, without driving the resonance*
  - ♦ arrange the correction sextupoles with a phase difference $\Delta\mu = \pi/3$ between two consecutive ones.

Third consideration

Consider a lattice with a superperiodicity of 2 and with two sextupoles with the same absolute strength (S or $K2_{MAD}$), but diametrically opposing [$\Delta\mu = Q\pi = (n \pm 1/3)\pi$] each other:

from (2) we have:

$$S^2_{virtual} = \left(S_1 \cos(0) + S_2 \cos(3Q\pi)\right)^2 + \left(S_1 \sin(0) + S_2 \sin(3Q\pi)\right)^2$$

$$n \text{ even: } \quad S_{virtual} = S_1 - S_2$$

$$n \text{ odd: } \quad S_{virtual} = S_1 + S_2$$

from (3) and (4) we have:

$$\Delta Q_x' = \frac{\ell_s}{4\pi} \beta_x D_x \left[K2^{S1}_{MAD} + K2^{S2}_{MAD} \right]$$

$$\Delta Q_z' = -\frac{\ell_s}{4\pi} \beta_z D_x \left[K2^{S1}_{MAD} + K2^{S2}_{MAD} \right]$$

so that if *n* is even:

- $S_1 = -S_2$ ====> *only driving the resonance*
- $S_1 = S_2$ ====> *only correcting chromaticity*

no independent action is possible with *n* odd.

PHASE SPACE TRAJECTORIES

Let us consider the effect of a single sextupole supposing that the unperturbed fractional tune is close to one-third of an integer and that the sextupole represents a small perturbation so that we can add up the effects turn by turn independently. We calculate the increments ΔX and $\Delta X'$ on three turns starting from the exit of the sextupole.

The total displacement and deflection after three turns:

$$\begin{pmatrix} \Delta X \\ \Delta X' \end{pmatrix}$$

are the sum of:

- 3 unperturbed turns plus a kick:

$$\begin{pmatrix} \cos 3\mu^* & \sin 3\mu^* \\ -\sin 3\mu^* & \cos 3\mu^* \end{pmatrix} \begin{pmatrix} X_0 \\ X_0' \end{pmatrix}$$

$$+$$

$$\begin{pmatrix} 0 \\ S(\cos 3\mu^* X_0 + \sin 3\mu^* X_0')^2 \end{pmatrix}$$

- 2 unperturbed turns, a kick and another turn:

$$\begin{pmatrix} \cos 2\mu^* & \sin 2\mu^* \\ -\sin 2\mu^* & \cos 2\mu^* \end{pmatrix} \begin{pmatrix} X_0 \\ X_0' \end{pmatrix}$$

$$+$$

$$\begin{pmatrix} 0 \\ S(\cos 2\mu^* X_0 + \sin 2\mu^* X_0')^2 \end{pmatrix}$$

$$+$$

$$\begin{pmatrix} \cos \mu^* & \sin \mu^* \\ -\sin \mu^* & \cos \mu^* \end{pmatrix} \begin{pmatrix} \cos 2\mu^* X_0 + \sin 2\mu^* X_0' \\ -\sin 2\mu^* X_0 + \cos 2\mu^* X_0' + S(\cos 2\mu^* X_0 + \sin 2\mu^* X_0')^2 \end{pmatrix}$$

- 1 unperturbed turn, a kick and 2 other turns:

$$\begin{pmatrix} \cos \mu^* & \sin \mu^* \\ -\sin \mu^* & \cos \mu^* \end{pmatrix} \begin{pmatrix} X_0 \\ X_0' \end{pmatrix}$$

$$+$$

$$\begin{pmatrix} 0 \\ S(\cos \mu^* X_0 + \sin \mu^* X_0')^2 \end{pmatrix}$$

$$+$$

$$\begin{pmatrix} \cos 2\mu^* & \sin 2\mu^* \\ -\sin 2\mu^* & \cos 2\mu^* \end{pmatrix} \begin{pmatrix} \cos \mu^* X_0 + \sin \mu^* X_0' \\ -\sin \mu^* X_0 + \cos \mu^* X_0' + S(\cos \mu^* X_0 + \sin \mu^* X_0')^2 \end{pmatrix}$$

Adding the contributions for:

$$\mu^* = 2\pi\left(n \pm \frac{1}{3} + \delta Q\right)$$

with n integer and $|\delta Q| << \frac{1}{3}$, we get:

Spiral step and spiral kick

$$\Delta X = \varepsilon X_0' + \frac{3}{2} S X_0 X_0'$$

$$\Delta X' = -\varepsilon X_0 + \frac{3}{4} S\left(X_0^2 - X_0'^2\right)$$

where $\varepsilon = 6\pi\,\delta Q$.

Along the separatrices, for $\varepsilon = 0$, from the above equations, we can calculate the increase in amplitude of oscillation $R = \sqrt{(X^2 + X'^2)}$ in 3 turns:

$$\Delta R = \frac{3}{4} S R^2$$

Indicating with R_{ES} the position of the electrostatic septum along the separatrix, we find a relation that connect position of the septum, spiral step ΔR and strength of the sextupole:

$$\frac{1}{R_{ES}} - \frac{1}{R_{ES} + \Delta R} \approx \frac{3}{4} S$$

To identify the trajectories in phase space, we deduce from the above expressions the invariant of motion choosing the time taken by a particle to make three revolutions as unit time:

$$\frac{dX}{dt} = \Delta X = \varepsilon X_0' + \frac{3}{2} S X_0 X_0' = \frac{\partial H}{\partial X'} \quad (1)$$

$$\frac{dX'}{dt} = \Delta X' = -\varepsilon X_0 + \frac{3}{4} S\left(X_0^2 - X_0'^2\right) = -\frac{\partial H}{\partial X} \quad (2)$$

with:

Hamiltonian

$$H = \frac{\varepsilon}{2}\left(X^2 + X'^2\right) + \frac{S}{4}\left(3XX'^2 - X^3\right)$$

The above expressions (1) and (2) identify the invariant H as the Hamiltonian in the conjugate variables X and X'. Different values of H correspond to different trajectories in the phase space.

<u>If S = 0</u>:

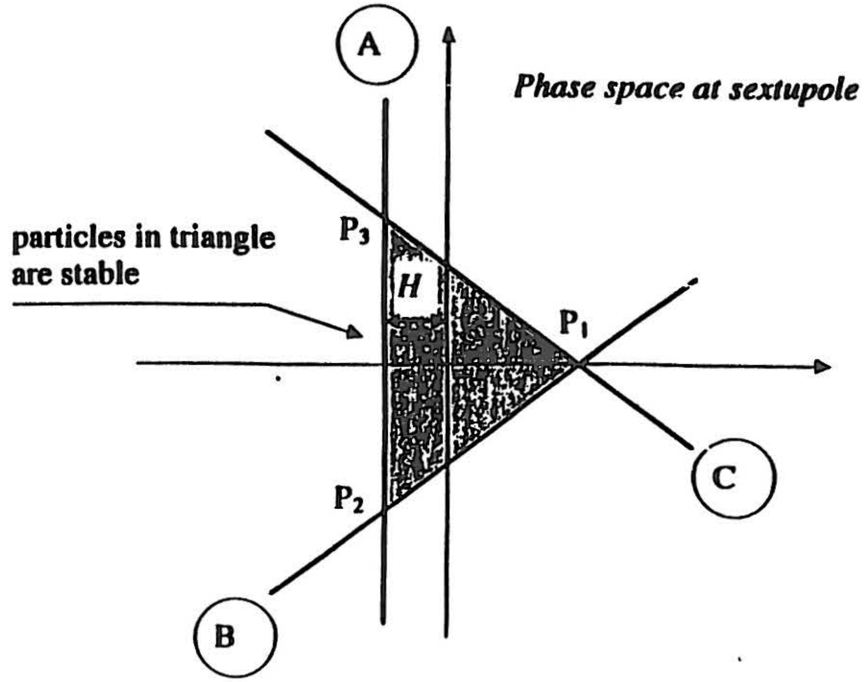
the expression of H becomes the equation of a circle, as it should be in normalised coordinates having eliminated the source of perturbation.

<u>If S ≠ 0</u>:

- the trajectories are symmetric with respect to the X-axis, since X' appears only in quadrature;
- for small values of X and X', the trajectories are slightly deformed circles;
- from expression (1) and (2) equating to zero we derive the coordinates of the fixed points:
  - O(0; 0) fixed point of first order (it repeats itself in one turn);
  - $P_1=\left(\frac{4\varepsilon}{3S};0\right)$
    $P_2=\left(-\frac{2\varepsilon}{3S};-\frac{2\varepsilon}{\sqrt{3}S}\right)$
    $P_3=\left(-\frac{2\varepsilon}{3S};\frac{2\varepsilon}{\sqrt{3}S}\right)$

fixed points of order three (repeat themselves in three turns).

The lines connecting the points P_i define a triangle that limits the stable, phase-space region (inside) from the unstable region (outside). These are called separatrices. The next figure shows the triangle for $\varepsilon/S > 0$.



The equations of the separatrices and the area of the triangle are easily calculated for the present configuration at the sextupole position.

Equation of separatrices

(A) $$X = -\frac{2}{3}\frac{\varepsilon}{S}$$

(B) $$X' = \frac{1}{\sqrt{3}}\left(X - \frac{4}{3}\frac{\varepsilon}{S}\right)$$

(C) $$X' = \frac{1}{\sqrt{3}}\left(-X + \frac{4\varepsilon}{3S}\right)$$

Distance of side of triangle to origin of axis

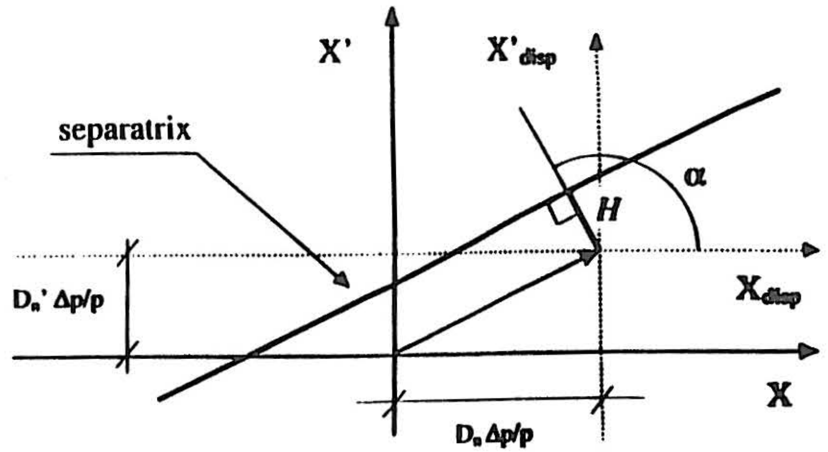
$$H = \frac{2}{3}\frac{\varepsilon}{S} = \frac{4\pi}{S}\delta Q$$

Stable emittance = (area triangle)/π [π m rad]

$$\frac{3\sqrt{3}}{\pi}H^2 = \frac{48\sqrt{3}\pi}{S^2}(\delta Q)^2 = \frac{48\sqrt{3}\pi}{S^2}\left(Q_x'\frac{\Delta p}{p}\right)^2$$

General equation of the separatrix

The equations of the separatrices (A), (B) and (C), deduced earlier, are valid at the sextupole (single or virtual) position in the lattice. Let us generalise these equations with the help of the following picture.



The line equation is given by taking into consideration the following points:

- the anticlockwise rotation of the line by an angle α;
- the displacement of the equilibrium orbit due to the dispersion (D_n, D_n') in the machine and induced by the momentum deviation of the particle ($\Delta p/p$).

Applying the above transformations we get the general expression of the separatrix:

$$\left(X - D_n \frac{\Delta p}{p}\right)\cos\alpha + \left(X' - D'_n \frac{\Delta p}{p}\right)\sin\alpha = H$$

It is more useful to express the equations of the separatrices (A), (B) and (C) as a function of the phase advance (Δμ) between the sextupole position and the observation point, remembering that the phase advance determines a clockwise rotation. The separatrices at the sextupole position are obtained from the preceding equation inserting:

(A) ====> $\alpha = 180°$
(B) ====> $\alpha = 300°$
(C) ====> $\alpha = 420°$

and, rotating by Δμ, we have at the observation point:

(A):

$$-\left(X - D_n \frac{\Delta p}{p}\right)\cos(\Delta\mu) + \left(X' - D'_n \frac{\Delta p}{p}\right)\sin(\Delta\mu) = H$$

(B):

$$-\left(X - D_n \frac{\Delta p}{p}\right)\cos(\Delta\mu + 240°) + \left(X' - D'_n \frac{\Delta p}{p}\right)\sin(\Delta\mu + 240°) = H$$

(C):

$$-\left(X - D_n \frac{\Delta p}{p}\right)\cos(\Delta\mu + 120°) + \left(X' - D'_n \frac{\Delta p}{p}\right)\sin(\Delta\mu + 120°) = H$$

Particle dynamics

We want to analyse the movement of the particles along the separatrices; for this purpose we substitute their expressions in the equations (1) and (2):

$$X = -\frac{2}{3}\frac{\varepsilon}{S}$$

$$\frac{dX'}{dt} = -\varepsilon X_0 + \frac{3}{4} S\left(X_0^2 - X_0'^2\right)$$

$$\blacktriangleright \quad \frac{dX'}{dt} = -\frac{3}{4} S X'^2 + \frac{\varepsilon^2}{S}$$

$$X' = \frac{1}{\sqrt{3}}\left(X - \frac{4}{3}\frac{\varepsilon}{S}\right)$$

$$\frac{dX}{dt} = \varepsilon X'_0 + \frac{3}{2} S X_0 X'_0$$

$$\blacktriangleright \quad \frac{dX}{dt} = \frac{\sqrt{3}}{2} S X^2 - \frac{1}{\sqrt{3}}\varepsilon X - \frac{4}{3\sqrt{3}}\frac{\varepsilon^2}{S}$$

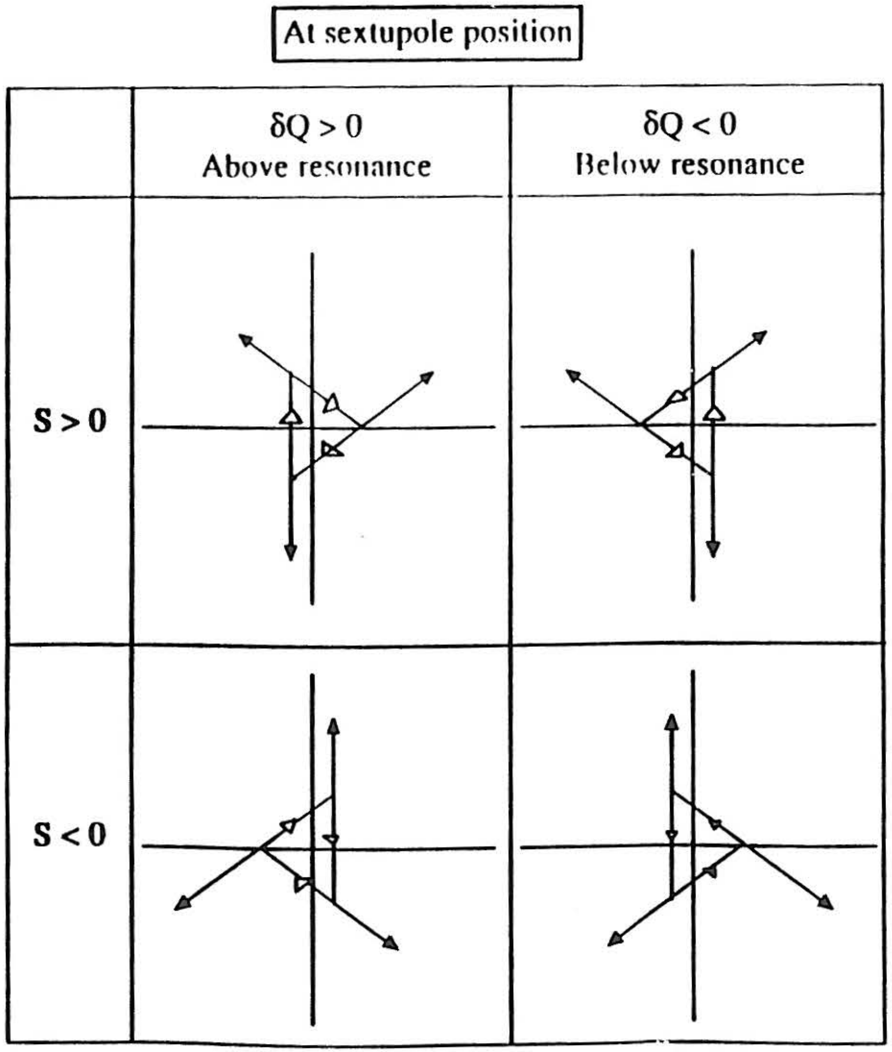
$$X' = \frac{1}{\sqrt{3}}\left(-X + \frac{4}{3}\frac{\varepsilon}{S}\right)$$

$$\frac{dX}{dt} = \varepsilon X'_0 + \frac{3}{2} S X_0 X'_0$$

$$\blacktriangleright \quad \frac{dX}{dt} = -\frac{\sqrt{3}}{2} S X^2 + \frac{1}{\sqrt{3}}\varepsilon X + \frac{4}{3\sqrt{3}}\frac{\varepsilon^2}{S}$$

<u>First consideration</u>

The following figures show the sense of rotation of the particles in three turns and the direction of the outgoing separatrices at the sextupole position, depending on the sign of ε and S.

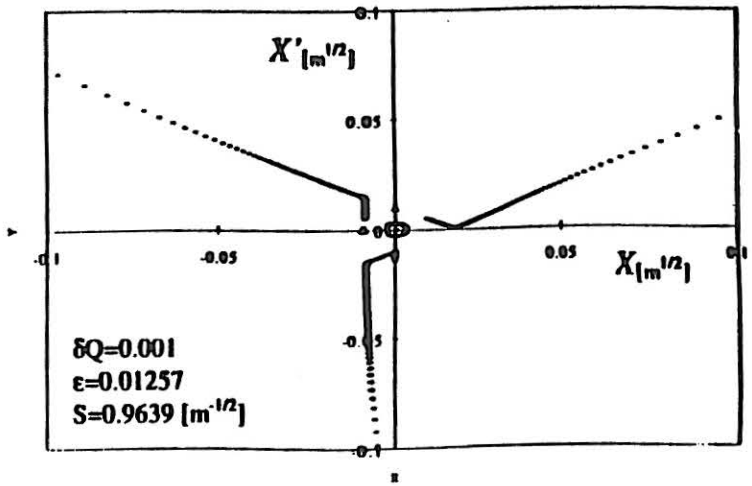


<u>Second consideration</u>

The parabolic expressions obtained so far become zero in correspondence to the point P_i, thus showing that they are indeed fixed points.

<u>Third consideration</u>

The parabolic behaviour also shows that the movement of the particles on the separatrices in between the fixed points becomes slower and slower as we come close to them, thus giving a concentration of particles in the corners of the triangle. On the contrary, the spiral step is increasing quadratically as the particle move away from the fixed points along the separatrices, as shown in the following picture where a stable orbit and the movement on the outgoing separatrices in unit time are indicated.



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