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THE TOTAL $\gamma\gamma$ CROSS-SECTION FOR ALL Q^2

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A B S T R A C T

The total cross-section for the production of hadrons in $\gamma\gamma$ scattering, and its breakdown into flavour contributions, are calculated as a function of the invariant mass of one of the photons, the other being on-shell, in a vector meson dominance model. Knowledge of the vector meson mass spectrum is not required, although when available it may be used to further check the over-all consistency of the model. The predicted cross-section is compared with data from PETRA.

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1. - INTRODUCTION

We would like to reconsider in this paper the behaviour of the total cross-section for the production of hadrons in photon-photon collisions, as a function of the invariant mass of one of the photons¹⁾⁻³⁾. Hadron production in $\gamma\gamma$ scattering

$$\gamma(q) + \gamma(k) \rightarrow X \quad (1)$$

is observed indirectly through the annihilation process⁴⁾⁻⁶⁾

$$e^+ + e^- \rightarrow e^+ + e^- + X \quad (2)$$

In the type of experiments considered in this paper, one of the photons is arranged to be nearly on-shell ($k^2 \approx 0$) and the other, with a large virtual mass, $Q^2 = -q^2$, is in the deep inelastic limit, i.e., $Q^2, k \cdot q \rightarrow \infty$, with $x = Q^2/2k \cdot q$ finite. In the usual interpretation, the high mass photon is said to probe the structure of the quasi-real photon.

The first set of results from experiments⁴⁾⁻⁶⁾ of this sort suggests that a photon has an underlying quark-constituent structure which shows up and is dominant at high energies, as well as a vector meson-like structure which dominates at low energies²⁾. The first structure characteristic can be understood within the context of QCD while the second follows immediately from a vector meson dominance assumption.

There is a theoretical necessity to unify these two descriptions, especially in view of the fact that vector mesons themselves are understood to be composites of quarks and antiquarks. It is the purpose of this paper to show that this is possible within the so-called generalized vector meson dominance model⁷⁾. In this model the information that vector mesons are made up of quarks and antiquarks is taken approximately into account through the assumption that their couplings to the photon are given by local averages, about the corresponding vector meson masses, of the imaginary part of the photon vacuum polarization amplitude, whose form is taken from QCD. In formulae

$$\frac{e^2 m_n^2}{f_n^2} = \frac{1}{\pi} \int_{m_n^2 - \Delta m_n^2/2}^{m_n^2 + \Delta m_n^2/2} ds \operatorname{Im} \Pi(s) \cong \frac{1}{\pi} \operatorname{Im} \Pi_{\text{QCD}}(m_n^2) \frac{dm_n^2}{dn} \quad (3)$$

where m_n^2/f_n is the photon vector meson coupling in units of the electric charge e , m_n , the mass of the n^{th} vector meson and $\text{Im } \Pi(s)$ the imaginary part of the photon vacuum polarization amplitude $\Pi(s)$.

It will be shown that with the help of this assumption, a vector meson dominance model (VMD) is able to interpolate smoothly between the low and high energy regions, in agreement both with QCD, asymptotically, and with experiment for all values of Q^2 ³⁾. We give predictions for the total $\gamma\gamma$ cross-section into hadrons, averaged over the variable x , i.e., $\sigma_{\gamma\gamma}(Q^2)$, as well as for the various flavour components contributing to it.

The paper is organized as follows: in Section 2, we state the input assumptions and define our model; in Section 3, we exhibit our results and compare them with experiment; Section 4 is slightly more theoretical and in it we try to motivate the scaling assumption used in Section 2 and indispensable for obtaining our results.

2. - THE VMD MODEL FOR $\sigma_{\gamma\gamma}(Q^2)$

Let $\sigma_{\gamma\gamma}(v, Q^2)$, with $v = (k+q)^2 = Q^2(1/x - 1)$, be the total cross-section for the process in Eq. (1) at total centre-of-mass energy \sqrt{v} . According to the vector meson dominance model, $\sigma_{\gamma\gamma}(v, Q^2)$ is given by

$$\sigma_{\gamma\gamma}(v, Q^2) = \sum_{n, n'=0}^{\infty} \left(\frac{em_n^2}{f_n} \right)^2 \left(\frac{em_{n'}^2}{f_{n'}} \right)^2 \frac{\sigma_{nn'}(v)}{(m_n^2 + Q^2)^2 m_{n'}^4} \quad (4)$$

where $\sigma_{nn'}(v)$ is the total cross-section for the scattering of a vector meson of mass m_n against another of mass $m_{n'}$, at centre-of-mass energy \sqrt{v} . An important simplifying assumption has been used in Eq. (4) to reduce a four-fold summation to two. The assumption is that of dominance of diffractive scattering in hadron-hadron collisions. In the VMD framework it assures that in the forward direction the same vector meson couples to the same incoming and outgoing photon. The assumption of dominance of diffractive scattering also allows one to replace $\sigma_{nn'}(v)$ by a v independent cross-section $\sigma_{nn'}$.

Substituting for $e^2 m_n^2 / f_n^2$ from Eq. (3) into Eq. (4) allows one to rewrite the latter as a double integral³⁾

$$\sigma_{\gamma\gamma}(Q^2) = \frac{1}{\pi^2} \int_0^{\infty} \int_0^{\infty} ds ds' \frac{\text{Im}\Pi(s) \text{Im}\Pi(s')}{s' (s + Q^2)^2} s \sigma_{\gamma\gamma'}(s, s') \quad (5)$$

where we use $\sigma_{VV'}(s,s')$ for the continuum version of $\sigma_{nn'}$. The v dependence will be neglected from now on. Note that since $\text{Im } \Pi(s)$ may be supposed to be given by QCD, the integrals in Eq. (5) may be evaluated without knowledge of the vector meson mass spectrum. $\sigma_{VV'}(s,s')$ will now be defined. We shall assume that $\sigma_{VV'}(s,s')$ has the form

$$\sigma_{VV'}(s,s') = \frac{B_0}{s + s' + M_0^2} \quad (6)$$

and refer to Section 4 for its theoretical justification. The parameters B_0 and M_0^2 are determined by comparing our results for $\sigma_{\gamma\gamma}(Q^2)$ with QCD, for $Q^2 \rightarrow \infty$, and with the experimental value of $\sigma_{\gamma\gamma}(Q^2)$ at $Q^2 = 0$, respectively. This way we find

$$B_0 = 4 \pi^3 \quad (7)$$

and

$$M_0^2 = 3.7 \text{ GeV}^2 \quad (8)$$

It is interesting to note that if we fit a mass spectrum of the form

$$m_{ni}^2 = m_{oi}^2 (1 + n b_i)^{1+\lambda_i} \quad (9)$$

to experiment, for each of the vector meson families ($i \equiv \rho, \omega, \phi, \psi, \Upsilon, \dots$), then M_0^2 is numerically equal to $2b_i^{1+\lambda_i} \rho m_{oi}^2$. Note further in this connection that the ρ family makes the dominant contribution to $\sigma_{\gamma\gamma}(Q^2)$. This observation will allow us, when we come to consider the flavour contributions to $\sigma_{\gamma\gamma}(Q^2)$, to put for the corresponding parameter M_{oi}^2 in the cross-section $\sigma_{V_i V_i'}(s,s')$,

$$M_{oi}^2 = 2 b_i^{1+\lambda_i} m_{oi}^2 \quad (10)$$

To complete the definition of our model we give the QCD expression for $\text{Im } \Pi(s)$ which is to be used in Eq. (5):

$$\text{Im } \Pi(s) = \frac{e^2}{4\pi} \sum_i q_i^2 \theta(s - 4M_i^2) \left(1 + \frac{2M_i^2}{s}\right) \left(1 - \frac{4M_i^2}{s}\right)^{1/2} \cdot \left\{ 1 + \frac{4}{3} \alpha_s(s) h[V_i(s)] \right\} \quad (11)$$

M_i and q_i are, respectively, the mass and charge (in units of e) of the quark of flavour i ; $v_i(s) = (1 - 4M_i^2/s)^{1/2}$ and $h(v)$ is Schwinger's function⁸⁾. It is well approximated by

$$h(v) \simeq \frac{\pi}{2v} - \frac{3+v}{4} \left(\frac{\pi}{2} - \frac{3}{4\pi} \right) \quad (12)$$

With the help of the equations given in this section, the determination of $\sigma_{\gamma\gamma}^{(i)}(Q^2)$ from Eq. (5) and of its various flavour contributions,

$$\sigma_{\gamma\gamma}^{(i)}(Q^2) = \frac{1}{\pi^2} \int_0^\infty \int_0^\infty ds ds' \frac{\text{Im}\Pi_i(s) \text{Im}\Pi_i(s')}{s' (s+Q^2)^2} s \sigma_{v_i v_i'}(s, s') \quad (13)$$

is a matter of integration. We carried this out numerically for various values of quark masses, varying from typical current to typical constituent quark mass values. The results are reported in the next section.

3. - PREDICTIONS AND COMPARISON WITH EXPERIMENT

It is convenient both theoretically and for the case of numerical computation to rewrite Eq. (13) [or Eq. (5)] as a single integral

$$\sigma_{\gamma\gamma}^{(i)}(Q^2) = -4\pi^2 \int_0^\infty ds \frac{s \text{Im}\Pi_i(s) \Pi_i(-s - M_{oi}^2)}{(s+Q^2)^2 (s+M_{oi}^2)} \quad (14)$$

We have made use of Eqs. (6) and (7) as well as the dispersion integral representation of the photon vacuum polarization amplitude

$$\Pi_i(p^2) = \frac{p^2}{\pi} \int_0^\infty ds \frac{\text{Im}\Pi_i(s)}{s(s-p^2)} \quad (15)$$

It is clear that, apart from the method described in the last section of evaluating $\sigma_{\gamma\gamma}^{(i)}(Q^2)$ using the QCD expression for $\text{Im}\Pi_i(s)$ [see Eq. (11)], we can check further the consistency of our approach by using the VMD expression for $\text{Im}\Pi_i(s)$, i.e.,

$$\text{Im } \Pi_i(s) = \pi \sum_{n=0}^{\infty} \frac{e^2 m_{ni}^2}{f_{ni}^2} \delta(s - m_{ni}^2) \quad (16)$$

and the mass spectrum in Eq. (9) fitted to experiment in order to determine the parameters (b_i, λ_i) . We have used both methods and found that they agree for a limited range of quark masses. This is shown in Fig. 1 for the case of the sum of the contributions of the u and d quarks, corresponding, in the second method, to the sum of the contributions of the ρ and ω families. The corresponding value of the quark mass is $M_u = M_d = 125$ MeV. We note that $\sigma_{\gamma\gamma}^{(i)}(Q^2)$ depends strongly on the value of the quark mass only in the first, but not in the second, method of calculation. The dependence arises essentially from the much lower threshold, $4M_i^2 < m_{oi}^2$, in the first method. For comparison, we have also plotted in Fig. 1 the result of the quark model calculation using a quark mass $M_u = M_d = 8$ MeV. Figure 2, on the other hand, illustrates the weak dependence on the quark mass of the VMD calculation for the cases of the ρ and ψ families, as examples.

Using the second method of calculation, (i.e., VMD inputs) we compare in Fig. 3 the various flavour contributions $\sigma_{\gamma\gamma}^{(i)}(Q^2)$ with themselves, as well as with their sum, $\sigma_{\gamma\gamma}(Q^2)$. The contribution of the T family is negligible, being of the order of 10^{-2} nb for $Q^2 \sim 0$, and is therefore not shown in this figure. The contribution indicated as ρ - ω interference arises from the crossed term $q_\rho^2 q_\omega^2$ in the sum of the 4th power of quark charges for the ρ and ω families

$$q^4 = \frac{1}{2} (q_\rho^4 + 6 q_\rho^2 q_\omega^2 + q_\omega^4) = q_u^2 + q_d^2 \quad (17)$$

where $(q_\rho, q_\omega) = (1/\sqrt{2}, 1/3\sqrt{2})$ and $(q_u, q_d) = (2/3, -1/3)$. The data points are from Refs. 4). Note that the interference term is necessary in order to get agreement with experiment.

We should perhaps state that the method of determining the parameters (b_i, λ_i) for the ψ and T families is different from that used for the ρ , ω and ϕ families. For the ψ and T families one simply fits Eq. (9) to experiment. For the ρ , ω and ϕ families, on the other hand, we assume the existence of the first excited states ρ' (1.57)⁹⁾, ω' (1.57) and ϕ' (1.68)¹⁰⁾. With these values for m_{1i} ($i = \rho, \omega, \phi$) and with f_{oi} calculated from the experimental leptonic widths using the formula

$$\Gamma(V_{oi} \rightarrow e^+ e^-) = \frac{4\pi\alpha^2}{3} \frac{m_{oi}}{f_{oi}^2}$$

one obtains from Eqs. (3) and (9) two equations for determining (b_i, λ_i) for these families, for given values of the u, d and s quark masses. However, as stated previously, and as can be seen from Fig. 2, the dependence of $\sigma_{\gamma\gamma}^{(i)}(Q^2)$ on the quark mass is not strong when one uses VMD inputs.

Finally, some comments on the effects of QCD radiative corrections; they contribute in the order of 10 - 15% to the cross-sections $\sigma_{\gamma\gamma}^{(i)}(Q^2)$, more for the lighter quark families than for the heavier ones. We did not include these corrections in the plots of Figs. 1-3.

The important feature of the model discussed in this paper is its unified nature. The simple addition of a VMD $\gamma\gamma$ cross-section, valid at low energies, to one calculable from QCD, and valid only at high energies, is claimed to involve double counting and is to be avoided. Instead, a QCD quark structure, underlying both the small and large Q^2 behaviour of $\sigma_{\gamma\gamma}(Q^2)$, is implemented through the properties of vector mesons considered as bound states of quarks and antiquarks. Consequently, although the formula for $\sigma_{\gamma\gamma}(Q^2)$ is derived entirely within the VMD model, one can use in its computation QCD inputs throughout. Knowledge of the vector meson mass spectrum is not required. However, in the case where the mass spectrum is available, as for instance in the case of the ψ and T families, one can also employ VMD inputs and further check in this way the over-all consistency of the entire approach. Alternatively, if the spectrum is only poorly known, as in the case of the ρ , ω and ϕ families, one has here a means of checking the viability of an ansatz for it.

4. - ASYMPTOTIC SCALING AND THE VMD REPRESENTATION

We wish to return in this section to Eq. (6) which we rewrite here for ease of reference

$$\sigma_{VV'}(s, s') = \frac{B_0}{s + s' + M_0^2} \quad (6)$$

and comment both on its effect on our predictions and more generally on its relation to the generalized VMD model. Our first observation is about the inputs which allow us to determine the parameters B_0 and M_0^2 respectively. B_0 is determined by comparing our prediction for $\sigma_{\gamma\gamma}^{(i)}(Q^2)$, for large Q^2 , with the QCD expression^{1),2)}

$$\sigma_{\gamma\gamma}^{(i)}(Q^2) \Big|_{\text{QCD}} \xrightarrow{Q^2 \rightarrow \infty} 4\pi^3 \left(\frac{q_i^2 \alpha}{\pi} \right)^2 \frac{\ln Q^2}{Q^2} \quad (18)$$

This asymptotic is obtainable in the VMD model³⁾, as will be shown later, under certain specific assumptions about asymptotic scaling of total cross-sections of photon induced hadronic processes. In fact, if all that is required is the agreement with the leading QCD behaviour, one can set $M_0^2 = 0$ in Eq. (6) and obtain the following scaling law

$$\sigma_{VV'}^{(0)}(s, s') = \sigma_{VV'}(s, s') \Big|_{M_0^2=0} = \frac{B_0}{s + s'} \quad (6')$$

for the cross-section $\sigma_{VV'}^{(0)}(s, s')$. The role of M_0^2 is thus that of a fixed mass scale which is used to break the above scaling law and to parametrize the contributions of non-leading terms by means of the expansion

$$\begin{aligned} \sigma_{VV'}(s, s') &= \sigma_{VV'}^{(0)}(s, s') \sum_{n=0}^{\infty} \left(-\frac{M_0^2}{s + s'} \right)^n \\ &\equiv \frac{B_0}{s + s' + M_0^2} \end{aligned} \quad (6'')$$

We stress that while Eq. (6') is crucial and absolutely necessary if one wants to reproduce the QCD asymptotics, the particular model of breaking of the scaling law in Eq. (6'') was adopted for the sake of simplifying the numerical computations only. A logarithmic modification of Eq. (6') seems more realistic but leads to rather complicated formulae. The essential fact is that with Eq. (6') alone, that is with the neglect of non-leading terms, it is not possible to reproduce the low Q^2 behaviour of $\sigma_{\gamma\gamma}(Q^2)$. The asymptotically non-leading terms actually turn over to dominate when $Q^2 \rightarrow 0$. The parameter M_0^2 essentially sets the dimensional scale of the real photon cross-section $\sigma_{\gamma\gamma}(Q^2 = 0)$.

Our next observation is that the properties of scaling in mass of $\sigma_{\gamma\gamma}(Q^2)$ and $\sigma_{VV'}(s, s')$ are not ad hoc assumptions in the VMD model, introduced to re-obtain known QCD results. On the contrary, the VMD representation of photon-induced hadronic cross-sections is itself an exact representation of the action of the scale operator in momentum space, $D = Q^2 d/dQ^2$, in the limit when mass thresholds and QCD radiative corrections are neglected. In fact, Eq. (5) may also be written as the product of the action of two representations of the scale transformation operator

$$\sigma_{\gamma\gamma}(Q^2) = (1 + D) \frac{1}{\pi} \int_0^{\infty} ds \frac{\overline{\sigma}_{\gamma\gamma}(s)}{s + Q^2} \quad (19)$$

where

$$\bar{\sigma}_{\gamma\gamma}(s) = \text{Im} \Pi(s) \frac{1}{\pi} \int_0^{\infty} \frac{ds'}{s'} \text{Im} \Pi(s') \bar{\sigma}_{\gamma\gamma'}(s, s') \quad (20)$$

The integral in Eq. (19) is a Hilbert transform which, in the space of homogeneous functions $f_{\lambda}(s) = \text{const. } s^{\lambda-1}$, $0 < |\lambda| < 1$, acts just like the scale operator D ¹¹⁾

$$H f_{\lambda}(Q^2) = \frac{1}{\pi} \int_0^{\infty} ds \frac{f_{\lambda}(s)}{s+Q^2} = \Gamma(\lambda) \Gamma(1-\lambda) f_{\lambda}(Q^2) \quad (21)$$

Consequently, if $\bar{\sigma}_{\gamma\gamma}(s)$ were of the special form

$$\bar{\sigma}_{\gamma\gamma}(s) = a_{\lambda} s^{\lambda-1} \quad (22)$$

for some fixed λ , with $0 < |\lambda| < 1$, then $\sigma_{\gamma\gamma}(Q^2)$ would be proportional to it, i.e.,

$$\sigma_{\gamma\gamma}(Q^2) = \Gamma(1+\lambda) \Gamma(1-\lambda) \bar{\sigma}_{\gamma\gamma}(Q^2) \quad (23)$$

In the λ plane, the point $\lambda = 0$ is therefore of special significance for the VMD transformation in Eq. (19). Only for this value of λ is the transformation exactly self-reproducing. Note that for $\lambda = 0$, Eq. (22) says that $\bar{\sigma}_{\gamma\gamma}(s)$ scales with the square of the mass, a result of which Eq. (6') is the obvious generalization.

The important point is now that even when this scaling law is broken and $\bar{\sigma}_{\gamma\gamma}(s)$ is a sum

$$\bar{\sigma}_{\gamma\gamma}(s) = \frac{1}{2\pi i} \int_{\lambda_0 - i\infty}^{\lambda_0 + i\infty} d\lambda a(\lambda) s^{\lambda-1}; \quad \lambda_0 > 0 \quad (24)$$

with the simultaneous eigenfunctions of D and H as basis, the equality between $\sigma_{\gamma\gamma}(Q^2)$ and $\bar{\sigma}_{\gamma\gamma}(Q^2)$ holds asymptotically if $\lambda = 0$ is the leading singularity of $a(\lambda)$. This is easy to see, for, by substituting (24) into (19) one gets

$$\sigma_{\gamma\gamma}(Q^2) = \frac{1}{2\pi i} \int_{\lambda_0 - i\alpha}^{\lambda_0 + i\alpha} d\lambda a(\lambda) \Gamma(1+\lambda) \Gamma(1-\lambda) (Q^2)^{\lambda-1}; \lambda_0 > 0, \quad (25)$$

and assuming that $a(\lambda)$ has an N^{th} order pole at $\lambda = 0$ [i.e., $a(\lambda) \xrightarrow{\lambda \rightarrow 0} a_0/\lambda^N$], one finds for large Q^2

$$\sigma_{\gamma\gamma}(Q^2) \cong \bar{\sigma}_{\gamma\gamma}(Q^2) \xrightarrow{Q^2 \rightarrow \infty} \frac{a_0}{(N-1)!} \frac{(\ln Q^2)^{N-1}}{Q^2} \quad (26)$$

In the special instance $N = 2$ [i.e., a double pole of $a(\lambda)$ at $\lambda = 0$] one reproduces the QCD result in Eq. (18). This special case is also reproduced by substituting (6') into Eq. (20), for then one gets exactly

$$\bar{\sigma}_{\gamma\gamma}(s) = B_0 \text{Im} \Pi(s) \frac{\Pi(-s)}{-s} \quad (27)$$

For large s , $\text{Im} \Pi(s)$ tends to a constant and $\Pi(-s)$ to $-\ln s$. Equation (6') is therefore not an ad hoc assumption but part of the asymptotic properties inherent in the VMD representation. Stated heuristically, these scaling properties ensure that the photon behaves as much as possible like a hadron. This is clearly part of the implications of vector meson dominance.

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FIGURE CAPTIONS

- Fig. 1 : Flavour contributions to the total $\gamma\gamma$ cross-section. The full curve is the $\rho+\omega$ contribution, obtained by using VMD parameters. The $u+d$ contribution, calculated by inserting Eq. (11) into Eq. (5), for $M_u = M_d = 125$ MeV (dashed curve and $M_u = M_d = 8$ MeV (dotted curve).
- Fig. 2 : The quark mass dependence of the flavour contributions to $\sigma_{\gamma\gamma}(Q^2)$, evaluated with VMD inputs in the ρ (full curves) and in the ψ (dashed curves) cases.
- Fig. 3 : The total $\gamma\gamma$ cross-section (full curve) and its various flavour contributions. Data points are taken from Ref. 4).

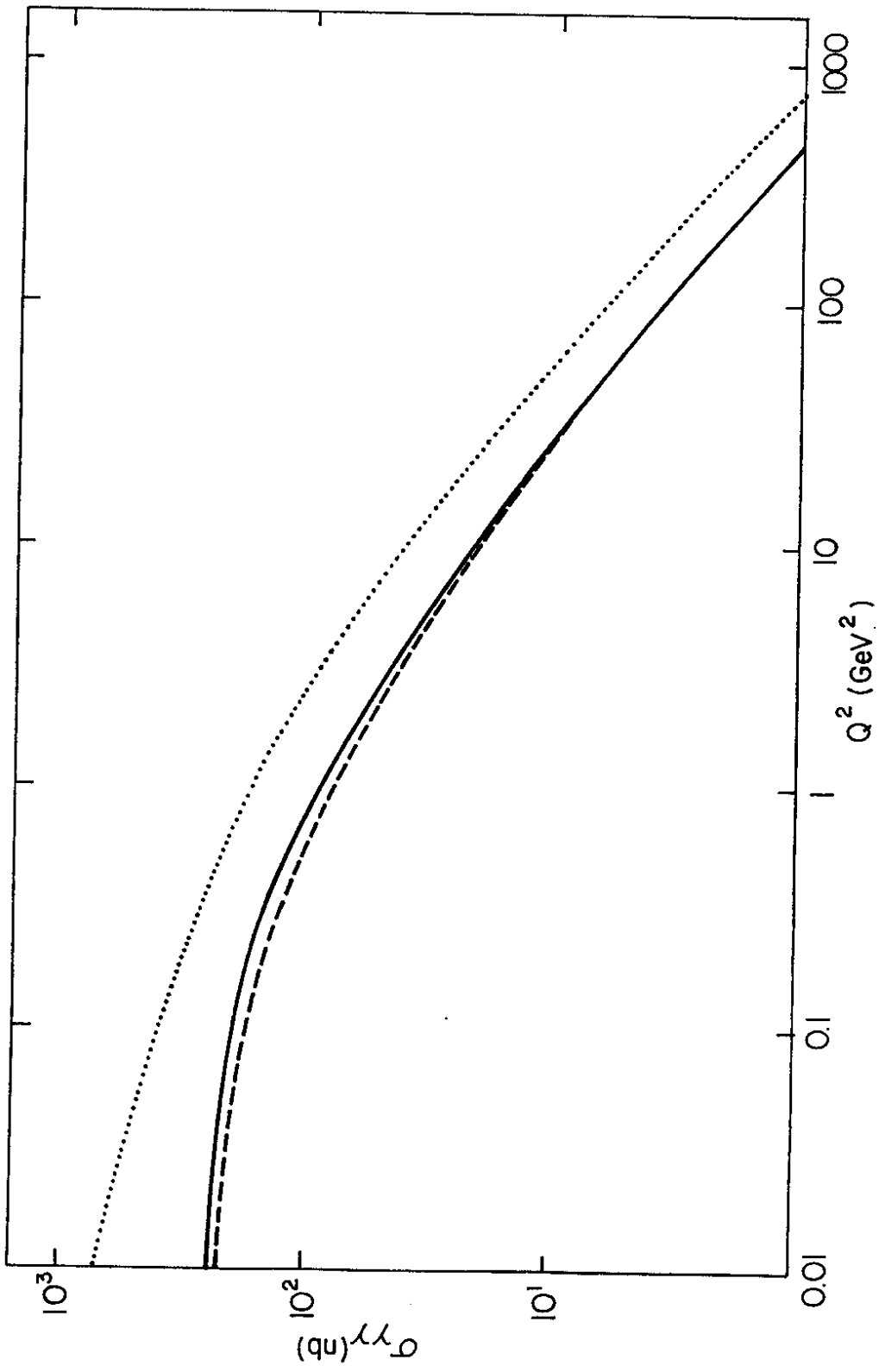


Fig. 1

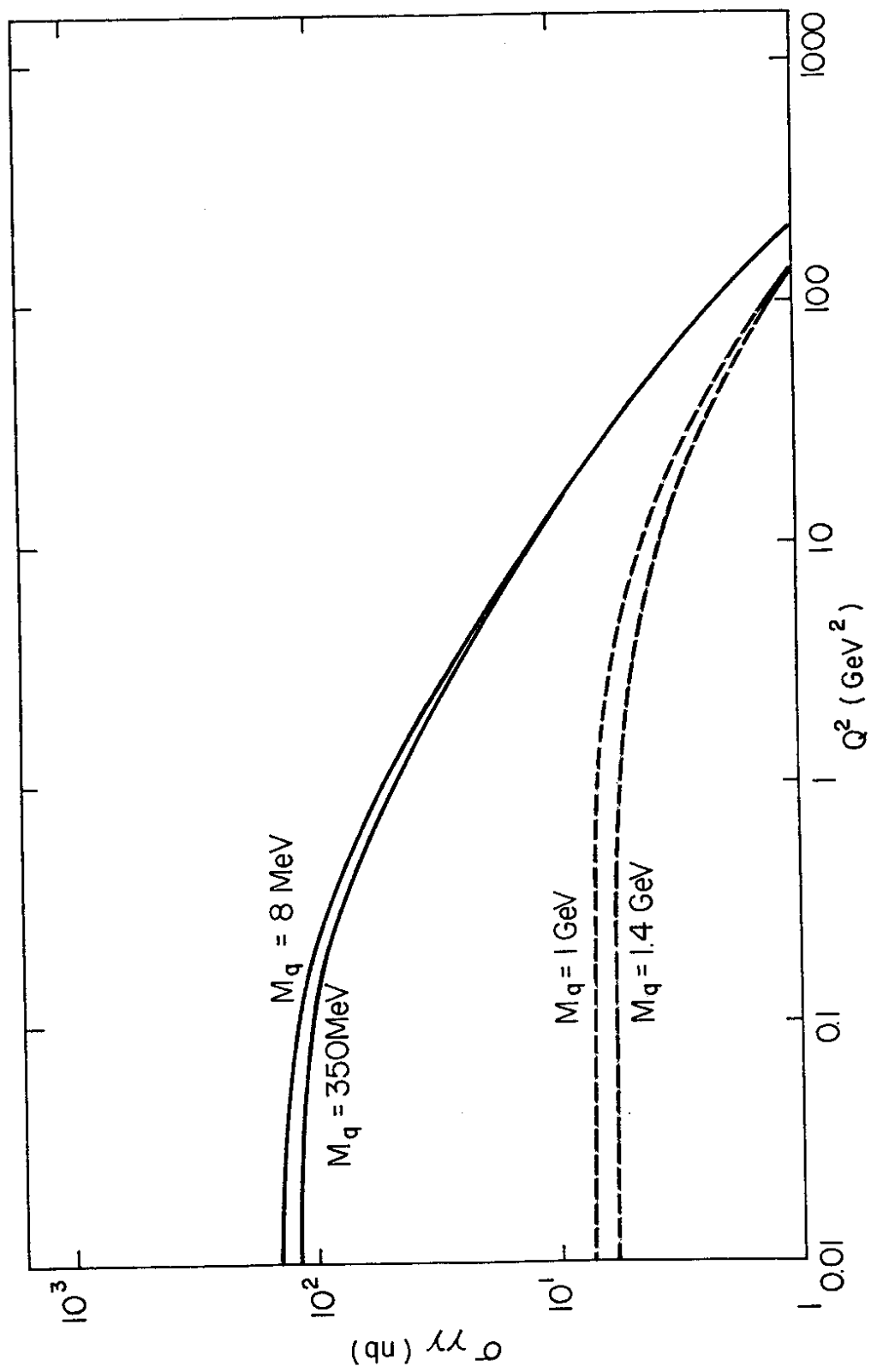


Fig. 2

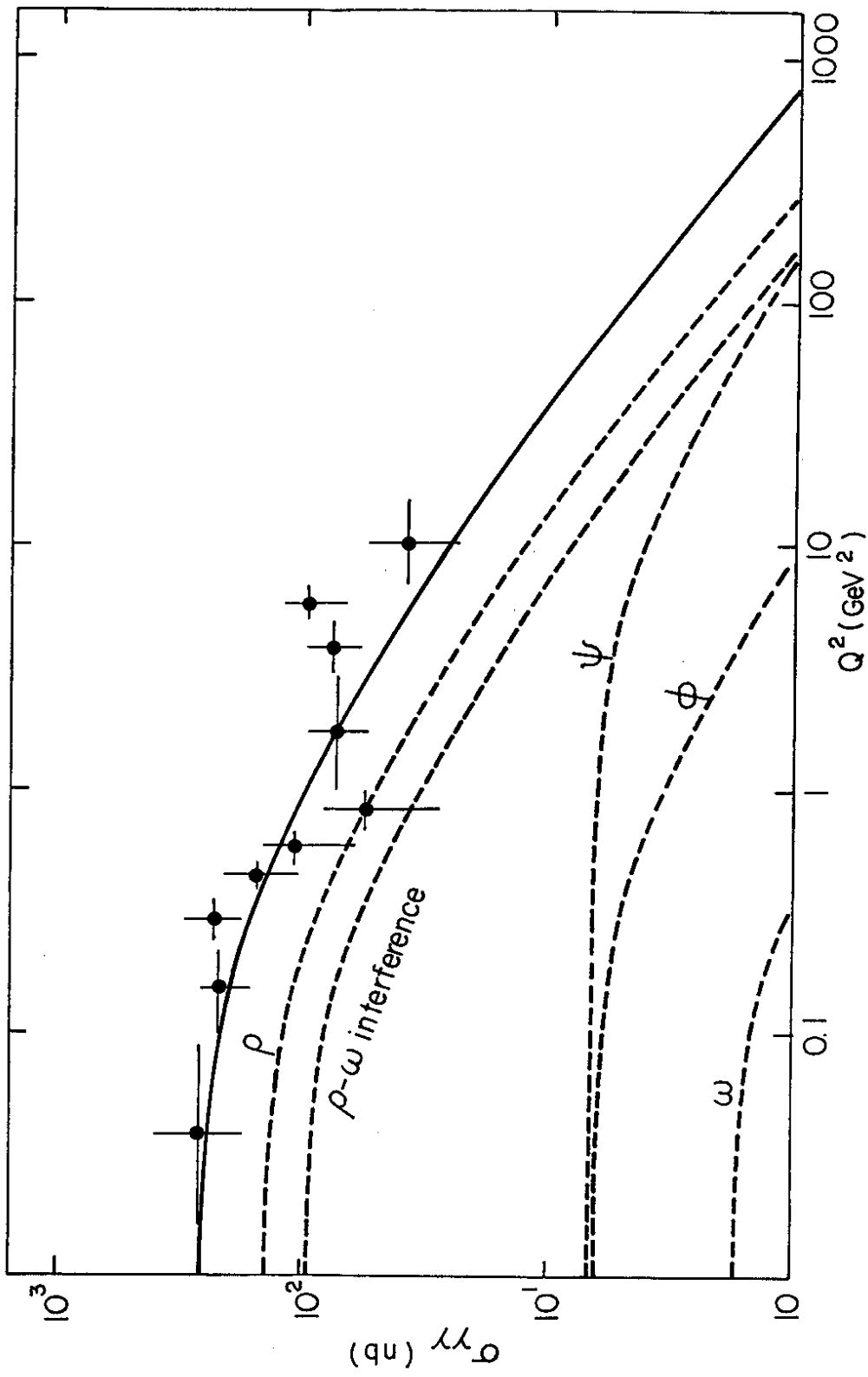


Fig. 3

