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EFFECTIVE LAGRANGIANS FOR SUPERSYMMETRIC GAUGE THEORIES *)

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A B S T R A C T

An effective Lagrangian approach for supersymmetric gauge theories is developed. Both exact and anomalous Ward identities are taken into account. We use this approach to discuss the questions of spontaneous breaking of supersymmetry and chiral symmetries, in pure $N = 1$ supersymmetric Yang-Mills theory and a supersymmetric QCD-like theory.

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1. - INTRODUCTION

In recent years supersymmetric (SUSY) gauge theories [1] have been advocated as grand unification theories which may be free from the hierarchy or fine tuning problem [2]. To make contact with phenomenology supersymmetry must be broken, presumably at a scale of around 1 TeV. Three different ways of breaking global supersymmetry have been proposed so far.

A) - Soft breaking

In this most common approach one introduces soft mass terms into the supersymmetric Lagrangian. The rôle of supersymmetry is to keep these mass parameters finite (no quadratic divergences).

This approach, as well as other approaches in which the symmetry is broken (even spontaneously) at the tree level [3], are unsatisfactory both from the aesthetical as well as the more conceptual point of view. All the relevant mass scales are introduced by hand rather than explained and the hierarchy problem is not really solved.

B) - Spontaneous breaking

Since we are dealing with asymptotically free gauge theories whose coupling grows towards the infra-red, various condensates may be formed. In particular, one may hope that a condensate which breaks supersymmetry is formed [4]. Since in this case a global continuous symmetry (SUSY) is broken, a Goldstone fermion will appear in the spectrum. It is this type of breaking that we will consider in this lecture.

C) - Explicit dynamical breaking

For completeness let me mention this type of approach which was recently discussed, although to my mind the existence of such a mechanism has not yet been demonstrated in a convincing satisfactory way. The idea is that, due to some non-perturbative effects (instantons ?!), an anomaly appears in the divergence of the supersymmetric current [5]. Much in the same way that the anomaly in the chiral $U(1)$ current in QCD is responsible for the axial $U(1)$ non-conservation, this anomaly should provide a mechanism for SUSY breaking. In QCD, as a result, the would-be Goldstone boson η' acquires a mass of the order of the scale of QCD, i.e., ~ 1 GeV. For the SUSY case, if such a mechanism is indeed operative, we expect that the would-be Goldstone fermion will acquire a mass of the order of the scale at which the appropriate gauge coupling becomes strong. At very high energy where the coupling is small, the whole effect would be negligible (e^{-1/g^2}) and the theory would be effectively supersymmetric. At lower energy, when the coupling becomes of order one, the breaking would be noticeable.

As mentioned above, in this lecture I will discuss the possibility of spontaneous breaking of SUSY. During the last year we have learned that it is rather difficult to break global supersymmetry spontaneously. This is due to the fact that the Hamiltonian is part of the SUSY algebra

$$\{ Q_\alpha , \bar{Q}_{\dot\beta} \} = 2 (\sigma^\mu)_{\alpha\dot\beta} P_\mu \tag{1.1}$$

where Q is the generator of supersymmetry and p_μ the space-time translation operator. An a priori criterion to check whether SUSY is spontaneously broken was introduced by Witten [6]. One should calculate the index defined as

$$\Delta = n_B^o - n_F^o \tag{1.2}$$

where n_B^o (n_F^o) is the number of bosonic (fermionic) states of zero energy. If $\Delta \neq 0$, SUSY is not broken. Computation of the index for various theories and a more formal discussion of its meaning was carried out by Witten [6] and by Cecotti and Girardello [4]. For all the theories for which the index analysis could be carried out, the outcome was that no spontaneous breakdown of supersymmetry occurs.

The theories which I will consider in this lecture using the effective Lagrangian approach could be analyzed by the index criterion. Our results are in accordance with the index analysis. The advantages of the effective Lagrangian approach are the following.

(i) The index analysis involves some formal steps, in particular with respect to the question of whether the index is independent of the parameters of the theory. At special points in the parameter space, the index may change (phase transitions ?!). The effective Lagrangian approach is perhaps more intuitive and gives some more insight into the physics of the underlying theory.

(ii) In the theories under consideration, besides supersymmetry, there will be other symmetries of interest (e.g., chiral symmetries). The effective Lagrangian approach also investigates the realization of these symmetries. Of particular interest is the inter-relationship between the realizations of supersymmetry and chiral symmetries.

(iii) There is a class of important theories, i.e., chiral gauge theories which cannot be analyzed by the index criterion. The effective Lagrangian approach is, however, easily generalized to chiral theories.

The effective Lagrangian should describe the low-energy physics of the underlying theory which, for the cases we are interested in, will be supersymmetric gauge theories. The effective theory will be constructed and investigated much in the same way as was done for QCD where it has been proved to be very successful [7]. For illustration, I shall discuss the most simple case, i.e., pure $N = 1$ supersymmetric Yang-Mills gauge theory. The investigation is based on work done in collaboration with G. Veneziano [8]. Then I will briefly discuss further work on supersymmetric QCD-like theories.

2. - PURE N = 1 SUPERSYMMETRIC YANG=MILLS GAUGE THEORY

General properties

The pure $N = 1$ supersymmetric Yang-Mills gauge theory is an $SU(N_c)$ gauge theory with one Majorana (Weyl) fermion in the adjoint representation. The Lagrangian is

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{i}{2} \bar{\lambda}^a \not{D}_{ab} \lambda^b$$

$$+ \text{auxiliary fields} + \text{gauge fixing} + \text{ghost term} \qquad (2.1)$$

with repeated indices summed over $(a,b=1,\ldots,N_c^2-1)$. The metric used is $\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$. In Eq. (2.1) λ^a is the spinor field and $F^a_{\mu\nu}$, D_μ are the usual Yang-Mills field strength and covariant derivative, respectively.

A few comments on this theory :

1) The theory is supersymmetric for any N_c. After elimination of the auxiliary field the supersymmetric transformation takes the form

$$\delta \lambda^a = -i(\sigma \cdot F)\xi^a$$
$$\delta A^a_\mu = i\bar{\xi}\gamma_\mu \lambda^a \qquad (2.2)$$

2) The theory is asymptotically free

$$\beta_{1\text{-loop}} \propto -(11N_c - 2N_c) = -9N_c < 0 \qquad (2.3)$$

and is expected to exhibit "colour confinement". The reason for putting the word confinement in parentheses is that in the present theory the fermions are in the adjoint representation and one cannot talk about confinement of triality. In other words, the screening phenomenon always occurs in this kind of theories. Yet, if we introduce into the system an external colour triplet-antitriplet pair of charges, we expect a linear force between them. Moreover, we expect all physical states of the theory to be colourless bound states made up of an indefinite number of constitutent "quarks" and "gluons".

3) To familiarize ourselves with the theory we can carry the large N_c analysis. The topological classification of the diagrams contributing to order $(1/N_c)^n$ is the same as that of QCD diagrams in the limit $N_c \to \infty$ with N_f/N_c fixed [9] (N_f is the number of flavours), i.e., boson and fermion loops count alike. In particular, all planar graphs are leading while a graph of genus h (h "handles") is of order $(1/N_c)^{2h}$ relative to the leading ($h = 0$) diagrams. In spite of this, assuming colour confinement, the leading term of the expansion is closer to that of QCD in the limit $N_c \to \infty$, $N_f/N_c \to 0$ [10] in the sense that it describes colourless narrow bound states whose residual n body interactions go to zero as $(N_c)^{2-n}$. Both "mesons" and "baryons" coexist as $N_c \to \infty$, unlike QCD where only the mesons survive in the limit.

Symmetries, anomalies and order parameters

Into the effective Lagrangian we should build all the correct and relevant Ward identities. In particular, we shall be interested in those associated with exact and anomalous global symmetries. At the classical level this theory has the following symmetries

supersymmetry	$\partial_\mu S_\mu = 0$
superconformal	$\partial_\mu (x\cdot\gamma S_\mu) = \gamma_\mu S_\mu = 0$
scale invariance	$\partial_\mu d_\mu = \partial_\mu (x_\nu \Theta_{\mu\nu}) = 0$
chiral invariance	$\partial_\mu j_\mu^{(\lambda)} = 0$

where S_μ is the supersymmetry current, d_μ the scale current, $\theta_{\mu\nu}$ the energy momentum tensor and $j_\mu^{(\lambda)}$ the λ fermion number current.

I shall assume, following the accepted lore, that supersymmetry is not broken explicitly at the quantum level. This is certainly true in each order of perturbation theory [11]. In this respect the interesting question which we will investigate, is whether SUSY is spontaenously broken or not. As for the other symmetries, we know that they are broken at the quantum level as it is expressed by the appearance of anomalies on the right-hand side of Eqs. (2.3)

$$\partial_\mu (x\cdot\gamma S_\mu) = \gamma\cdot S = \frac{2\beta(g)}{g} F^a_{\mu\nu}\sigma_{\mu\nu}\lambda^a$$
$$\partial_\mu (x_\nu \Theta_{\mu\nu}) = \Theta_{\mu\mu} = \frac{\beta(g)}{2g} F^a_{\mu\nu} F^a_{\mu\nu}$$
$$\partial_\mu j^{(\lambda)}_\mu = -\frac{\beta(g)}{2g} F^a_{\mu\nu}\tilde{F}^a_{\mu\nu} \tag{2.4}$$

In writing down Eqs. (2.4), I have used the fact [12] that the current $j^{(\lambda)}_\mu$, S_μ, $\theta_{\mu\nu}$ belong to a supermultiplet structure $V^{\alpha\dot\alpha}$ ($V^{\alpha\dot\alpha}$ is a general multiplet whose lowest component is $j^{(\lambda)}_\mu$). The anomalies $\partial_\mu j^{(\lambda)}_\mu$, $\gamma\cdot S$, $\theta_{\mu\mu}$ belong to another supermultiplet structure (chiral) which I shall denote by S. The chiral multiplet S is known as the anomaly (or Lagrangian) multiplet. All the anomaly equations (2.4) can be summarized in a supersymmetric way [12]

$$\bar{D}_{\dot\alpha} V^{\alpha\dot\alpha} = D^\alpha S \tag{2.5}$$

Note that the axial current which sits in the multiplet with the supersymmetry current does not satisfy the Adler-Bardeen theorem [13], i.e., the full β function appears as the coefficient of $F\tilde{F}$. This seems to be an unavoidable conclusion if supersymmetry is unbroken explicitly and Eq. (2.5) holds. Because of gauge invariance we have not been able to find any acceptable modification to Eq. (2.5) for $N = 1$ SUSY gauge theories [14]. It is therefore extremely important to check directly in two-loop order, using a supersymmetric regularization, the status of the Adler-Bardeen theorem with respect to the axial current which resides in the supermultiplet $V^{\alpha\dot\alpha}$ [15].

There are two important lowest dimension gauge invariant order parameters in this theory, $F^a_{\mu\nu}F^a_{\mu\nu}$ and $\bar\lambda^a\lambda^a$. These order parameters are associated with supersymmetry and chiral symmetry respectively. If F^2 gets a vacuum expectation value SUSY is broken. This comes about from the fact that [16]

$$\frac{1}{8} Tr\{\gamma^\mu S^\alpha_\mu, \bar{Q}^\alpha\} = \Theta_{\mu\mu} = \frac{\beta(g)}{2g} F^a_{\mu\nu}F^a_{\mu\nu} \tag{2.6}$$

An expectation value for the right-hand side of Eq. (2.6) would imply that Q_α does not annihilate the vacuum. Alternatively, using Lorentz invariance

$$\langle\Theta_{\mu\mu}\rangle_0 = 4\langle\Theta_{00}\rangle = 4\mathcal{E} \tag{2.7}$$

where ε is the vacuum energy. Having non-zero vacuum energy implies breaking of supersymmetry. An explicit calculation of the anticommutator in Eq. (2.6) gives

$$\Theta_{\mu\mu} \underset{N\to 4}{=} (4-N)\left(-\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{3i}{8} \bar{\lambda}^a \not{D} \lambda^a\right) \tag{2.8}$$

The first term is the conventional one while the second, also present in non-SUSY theories [17] with different numerical coefficient, is a rather trivial operator because of the equation of motion. The coefficient of $\frac{3}{8}$ appearing in Eq. (2.8) is just what is needed in order to achieve the exact cancellation of gluonic and fermionic loops. This is consistent with the fact that in deriving Eq. (2.8) we have eliminated the auxiliary field through its equation of motion. In any case the above arguments show that the operator appearing on the right-hand side of Eq. (2.6) is not just F^2, a fermionic free term having been subtracted. This spoils the positivity argument which was advocated [18] to prove that on kinematical grounds $\langle F^2\rangle_0 = 0$ (and therefore SUSY is not broken), and leaves the question of SUSY breaking a dynamical one.

Next I would like to discuss the expectation value of $\langle\bar{\lambda}\lambda\rangle$ with respect to SUSY breaking and chiral symmetry breaking. In the literature there is some confusion with regard to the question of whether $\langle\bar{\lambda}\lambda\rangle \neq 0$ breaks SUSY. The arguments have to do with the following anticommutation relation

$$\langle \bar{\lambda}\lambda \rangle = \frac{1}{4} \langle \{ Q_\alpha , (\bar{\lambda}\not{\partial})_\alpha \} \rangle \tag{2.9}$$

from which it looks as if $\langle\bar{\lambda}\lambda\rangle \neq 0$ implies SUSY breaking. [In Eq. (2.9) we have used the fact that Lorentz symmetry is not broken, hence only scalar operators can have a non-zero vacuum expectation value.] The problem with Eq. (2.9) is that it involves gauge-dependent quantities. Moreover, the operator Q_α which appears is actually not the supersymmetry generator but a combination of the generator and the generator of gauge transformation. Alternatively, we can show that this relation holds only in the Wess-Zumino gauge. We conclude that one should not use Eq. (2.9) naively. In fact we shall see explicitly [8] that in this theory SUSY is preserved even through $\langle\bar{\lambda}\lambda\rangle \neq 0$.

The question of whether $\langle\bar{\lambda}\lambda\rangle \neq 0$ represents in some sense spontaneous chiral symmetry breaking deserves more discussion, due to the explicit breaking by the anomaly. What we can do is give some plausible arguments in favour of a component in the vacuum expectation value of $\bar{\lambda}\lambda$ which is due to spontaneous chiral breaking.

A) - Analogy with QCD

In QCD the anomaly can be switched off by taking the limit $N_f/N_c \to 0$ and in this case the possibility of a spontaneous breaking of the U(1) symmetry is a meaningful one. Unfortunately, in the present SUSY case, no parameter can be adjusted to switch off the anomaly while preserving SUSY [in the present SUSY theory the anomaly is larger than in QCD by a factor $O(N_c)$]. Yet the diagrams contributing to the large N_c limit of the three axial current amplitude can be classified according to whether the three currents are attached to the same fermion loop or not. It is the first type of diagram which, in the QCD case, survives in the large N_c limit and which is responsible for spontaneous chiral symmetry breaking while the other ones, being related to the anomaly, provide a mass for the would-be Goldstone boson (the η'). In the SUSY case both sets of diagrams should be considered simultaneously. However, by analogy, we still expect the first type to induce a spontaneous breaking with a Goldstone boson while the others provide the mass shift through explicit breaking.

B) - More flavours

Consider the SUSY theory under consideration as the $N_f \to 1$ limit of a more general theory with N_f flavours of Majorana fermions in the adjoint representation. This theory is not supersymmetric (if $N_f \neq 1$) but has instead a $U(N_f)$ chiral symmetry broken to $SU(N_f)$ by the strong anomaly. In this case, taking also $N_c \to \infty$, we can easily show that a Coleman-Witten [19] type of result follows, using 't Hooft anomaly equations [20]. One expects $SU(N_f)$ to be broken to $O(N_f)$ with

$$\langle \bar{\lambda}^a_i \lambda^a_j \rangle \sim \delta_{ij} \qquad (i,j = 1,\dots,N_f) \tag{2.10}$$

Notice that only the diagrams of the first type discussed above contribute to the $SU(N_f)$ anomalies and are responsible for the vacuum expectation value of Eq. (2.10). Unless something discontinuous happens as we come down to $N_f = 1$, we expect the same diagrams to give $\langle\bar{\lambda}^a\lambda^a\rangle$ in the SUSY case as well.

C) - Loop expansion

Consider the loop expansion (a non-supersymmetric expansion in powers of N_f). Assuming, as in QCD, that the quarkless theory has non-trivial θ dependence, one finds [21] that this can be cancelled in the full theory with massless fermions only if a Goldstone pole appears at the one-fermion loop level [which gets a mass $O(\sqrt{N_f})$ after resumming all fermion loops].

All the above arguments indicate that one of our low-lying degrees of freedom is the would-be Goldstone boson of chiral symmetry. If SUSY is spontaneously broken, a Goldstone fermion should appear in our low-lying spectrum as well.

Constructing an effective Lagrangian

The effective Lagrangian should describe the low-lying physics of the underlying theory. From the previous discussion it is clear that the effective Lagrangian should be general enough to incorporate the (would-be) Goldstone boson of chiral symmetry and the (would-be) Goldstone fermion of supersymmetry. Moreover, the composite fields which appear in the effective theory should correspond to the relevant order parameters of the underlying theory much in the same way that the Landau theory is the theory of the order parameter of the underlying statistical system. In constructing the effective Lagrangian the following constraints must be kept :

1) only gauge invariant operators appear in the effective theory ;
2) the operators should correspond to the low-lying propagating states ;
3) all the relevant order parameters of the underlying theory should appear ;
4) operators of lowest possible dimension (low energy approximation) should appear ;
5) all Ward identities (corresponding to exact and anomalous symmetries) should be satisfied.

Using these constraints, an effective Lagrangian for the QCD theory was built and proved to be useful [7]. The composite fields which were used were $\bar{\psi}\psi$ (actually $\bar{\psi}_i\psi_j$ for the case of N_f flavours $i,j=1,\ldots,N_f$) and $F\tilde{F}$. In view of the previous discussion, it is obvious that the composite operators should appear in supermultiplet structure. (Supersymmetry is not broken explicitly, hence we impose supersymmetry on the effective theory.) Since we have already established the fact that $\bar{\lambda}\lambda$ and F^2 are important order parameters, it would be reasonable to add to them the other members of the anomaly supermultiplet [22] [Eqs. (2.4) and (2.5)]. The minimal effective theory will therefore be built from the composite anomaly (or Lagrangian) supermultiplet. It will be of the Wess-Zumino type with the components of the supermultiplet given by the composite fields (chiral notation)

$$
\begin{aligned}
&\phi = c\,\bar{\lambda}_R\lambda_L \qquad\qquad \phi^* = c\,\bar{\lambda}_L\lambda_R \\
&\chi = \frac{i}{2}\, c\, F^a_{\mu\nu}\,\sigma_{\mu\nu}\,\lambda^a_L \\
&M = -\frac{c}{2}\,(F^2 + iF\tilde{F}) \qquad\qquad M^+ = -\frac{c}{2}\,(F^2 - iF\tilde{F})
\end{aligned}
\tag{2.11}
$$

where terms that vanish upon use of the equation of motion have been omitted, and

$$C = \frac{\beta(g)}{2g} = -\frac{3g^2 N_c}{32\pi^2} + O(g^4)$$

$$\lambda_L = \frac{1}{2}(1 - i\gamma_5)\lambda \qquad \lambda_R = \frac{1}{2}(1 + i\gamma_5)\lambda \tag{2.12}$$

The composite fields appearing in Eq. (2.11) can be combined to form the chiral anomaly multiplet

$$\begin{aligned} S(x,\theta) &= \phi + 2\theta\chi - \theta^2 M - i(\theta\sigma_\mu\bar\theta)\partial_\mu\phi \\ &\quad - i\theta^2(\bar\theta\bar{\partial}\chi) - \frac{1}{4}\theta^2\bar\theta^2\Box\phi \\ &= W^\alpha W_\alpha \end{aligned} \tag{2.13}$$

where W^α is just the supermultiplet of the underlying gauge theory (the multiplet which starts with λ^a and contains the A_μ^a).

Given a set of chiral fields $\{S_i\}$, the most general Wess-Zumino type model built on these fields is

$$\mathcal{L} = d(\{S_i\},\{S_i^*\})_D + f(\{S_i\})_F + c.c \tag{2.14}$$

where f and d are arbitrary functions. The subscripts D and F refer to taking the D component (the coefficient of $\theta^2\bar\theta^2$) and F component (the coefficient of θ^2) of the corresponding expressions. The first term (the d function) will be referred to as the kinetic part, while the second term is the potential part. In our case we shall use just one chiral field, i.e., the anomaly multiplet S [Eq. (2.13)]. The function f will be completely determined by imposing the anomalous Ward identities which are expressed by Eq. (2.4). The right-hand side of Eq. (2.4) gives the variation of the action under superconformal, dilatation and chiral transformation. To illustrate the procedure, we shall impose the correct chiral transformation on the effective theory. Under the chiral transformation

$$S(x,\theta) \longrightarrow e^{3i\alpha} S(x, e^{-3i\alpha/2}\theta) \tag{2.15}$$

Hence

$$\int d^2\theta\, f(S(x,\theta)) \longrightarrow \int d^2\theta\, f(e^{3i\alpha}S(x, e^{-3i\alpha/2}\theta)) \tag{2.16}$$

Changing the variable to $\theta' = e^{-3i\alpha/2}\theta$ and demanding that the variation be given by the F component of S, we obtain

$$e^{-3i\alpha} f(e^{3i\alpha} S) - f = i\alpha S \tag{2.17}$$

Specializing to infinitesimal transformation we derive the following differential equation

$$-f + S\frac{\partial f}{\partial S} = \frac{1}{3} S \tag{2.18}$$

whose solution is

$$f = \frac{1}{3}\left(S \log S/\mu^3 - S \right) \tag{2.19}$$

The same equation is obtained by considering either the scale transformation or the superconformal transformation (since both are in the same supermultiplet structure with chiral transformation [12]). Alternatively, it is straightforward to check that f of Eq. (2.19) satisfies the correct anomalous Ward identities. Under the chiral transformation [Eq. (2.15)]

$$(S)_F \equiv \int d^4x\, d^2\theta\ S(x,\theta) \longrightarrow \int d^4x\, d^2\theta'\, S(x,\theta') \qquad \theta' = e^{-i3/2}\theta$$

$$\begin{aligned} \delta(S \log S/\mu^3) + h.c &= 3i\alpha \int d^4x\, d^2\theta'\, S(x,\theta') + h.c. \\ &= 3i\alpha \int d^4x\,(-M + M^+) = -3\alpha C \int d^4x\, F\tilde{F} \end{aligned} \tag{2.20}$$

Similarly, under scale transformation

$$S(x,\theta) \longrightarrow e^{3\gamma} S(xe^{\gamma}, \theta e^{\gamma/2}) \tag{2.21}$$

and therefore

$$(S)_F \equiv \int d^4x\, d^2\theta\ S(x,\theta) \longrightarrow \int d^4x'\, d^2\theta'\, S(x',\theta') \qquad x' = xe^{\gamma} \quad \theta' = \theta e^{\gamma/2}$$

$$\begin{aligned} \delta(S \log S/\mu^3)_F + h.c &= -3\gamma \int d^4x'\, d^2\theta'\,(S(x',\theta') + h.c.) \\ &= -3\gamma \int d^4x\,(M + M^+) = 3\gamma c \int d^4x\, F^2 \end{aligned} \tag{2.22}$$

Note that we have used here the naive dimensions. The effective Lagrangian is built out of renormalization group invariant composite operators since already on the tree level it describes physical processes.

The kinetic part of the effective Lagrangian should be invariant under all the classical symmetries. In particular, it should be scale invariant. This fixes uniquely the kinetic part in our case and we obtain

$$\mathcal{L}_{eff.} = \frac{9}{\alpha}\left(S^*S\right)^{1/3}_{D} + \frac{1}{3}\left[S \log S/\mu^3 - S + h.c.\right] \tag{2.23}$$

Note that μ is a renormalization group invariant scale whose physical meaning will become clear upon investigation of the effective theory. All quantities which appear in the effective theory are renormalization group invariant and no explicit g dependence occurs (dimensional transmutation).

Next we turn to the analysis of the effective theory of Eq. (2.23). This is done most easily by writing down L_{eff} in components. The auxiliary fields turn out to be given by

$$M+M^{+} = -cF^2 = -\frac{1}{3}\left\{2\left(\frac{\bar{\chi}_R\chi_L}{\phi} + \frac{\bar{\chi}_L\chi_R}{\phi^*}\right) - \alpha\left(\phi^*\phi\right)^{2/3}\log\left(\frac{\phi^*\phi}{\mu^6}\right)\right\}$$

$$-i(M-M^{+}) = -cF\tilde{F} = \frac{i}{3}\left\{2\left(\frac{\bar{\chi}_R\chi_L}{\phi} - \frac{\bar{\chi}_L\chi_R}{\phi^*}\right) - \alpha\left(\phi^*\phi\right)^{2/3}\log\left(\phi^*/\phi\right)\right\} \tag{2.24}$$

In terms of ϕ and χ alone the effective Lagrangian, Eq. (2.23), takes the form

$$\begin{aligned}\mathcal{L}_{eff} = {} & \frac{1}{\alpha}\left(\phi^*\phi\right)^{-2/3}\left(\partial_\mu\phi^*\partial_\mu\phi + i\bar{\chi}\not{\partial}\chi\right)\\ & -\frac{1}{3}\left(\frac{\bar{\chi}_L\chi_R}{\phi^*} + \frac{\bar{\chi}_R\chi_L}{\phi}\right)\\ & -\frac{\alpha}{9}\left(\phi^*\phi\right)^{2/3}\log\phi/\mu^3\,\log\phi^*/\mu^3\\ & +\frac{2}{9}\left(\frac{\bar{\chi}_L\chi_R}{\phi^*}\log\phi^*/\mu^3 + \frac{\bar{\chi}_R\chi_L}{\phi}\log\phi/\mu^3\right)\\ & +\frac{2}{3\alpha}\left(\bar{\chi}\gamma_\mu\gamma_5\chi\right)\left(\phi^*\overleftrightarrow{\partial}_\mu\phi\right)\left(\phi^*\phi\right)^{-5/3}\end{aligned} \tag{2.25}$$

Equation (2.25) is quadratic in the fermionic fields. This arises from our choice of a factorized D term, Eq. (2.14), while a four-fermi interaction $\chi_L^2\chi_R^2$ will be at most present in the general case [22]. Notice that the non-polynomial form of Eq. (2.23) reflects itself in Eq. (2.25) as non-polynomial terms in ϕ, ϕ^* but not in χ, $\bar{\chi}$. This is a general result [23] independent of the particular choice of the functions f and d, Eq. (2.14). In particular, the scalar potential takes the form

$$V = \frac{\alpha}{9} (\phi^*\phi)^{2/3} \log \phi/\mu^3 \log \phi^*/\mu^3 \qquad (2.26)$$

with minima at $\langle\phi\rangle = \langle\phi^*\rangle = 0$ or $\langle\phi\rangle = \langle\phi^*\rangle = \mu^3$. It is clear that the solution $\langle\phi\rangle = 0$ is not an acceptable one since the potential (as well as other terms in L_{eff}) is not analytic at this point and we cannot expand around it. Indeed, by rescaling the ϕ and χ fields $(\hat{\phi} = 3/\sqrt{\alpha}\ \phi^{1/3},\ \hat{\chi} = \sqrt{2\alpha}\ \phi^{-2/3}\ \chi)$ so that the kinetic terms take the conventional form, one can see that only the $\langle\phi\rangle = \mu^3$ solution makes sense in agreement with our expectation. In particular, the scale of the "non-renormalizable" terms is given by μ.

Expanding the effective Lagrangian around its minimum we find that ϕ describes scalar and pseudoscalar massive bosons and χ a massive (Majorana) fermion. As expected from the fact that $V = 0$ at the minimum, these particles have the same mass

$$m_F = m_B = \frac{1}{3} \alpha \mu \qquad (2.27)$$

and form a Wess-Zumino scalar multiplet. Hence, in spite of $\langle\bar{\lambda}\lambda\rangle \neq 0$, SUSY is not broken (in particular $\langle F^2\rangle = 0$). The would-be pseudoscalar Goldstone has received a mass from the anomaly (as the η' in QCD) but because of SUSY has dragged along a scalar and a fermion.

At this point several remarks are in order.

(i) The fact that $\langle\phi\rangle \neq 0$ does not imply SUSY breaking is intimately related to the fact that $\bar{\lambda}\lambda$ is the lowest component of a chiral multiplet S Eq. (2.13) whose components are gauge invariant composite operators. The lowest component can never be obtained as a commutator of the supersymmetry charge with some other components. Thus the usual Goldstone type argument cannot be applied here. Only when the F component of the supermultiplet (the auxiliary field) gets a non-zero vacuum expectation value can one conclude that supersymmetry is broken.

(ii) At first sight it seems that by adding linear F terms we can achieve a situation where the auxiliary field gets non-zero vacuum expectation value and supersymmetry is broken. This, however, is not the case, since in our effective Lagrangian, Eq. (2.23), such an F term can be swallowed by a redefinition of μ.

(iii) One can check that our result is consistent with the large N_c behaviour of masses and couplings [8].

Summary

We have analyzed the pure $N = 1$ supersymmetric Yang-Mills theory. The expected spectrum is that of a massive supermultiplet of composite hadrons (of which ours is expected to be the lowest one for small values of α) which become weakly interacting in the large N_c limit. Supersymmetry is not broken since only the lowest member of the scalar multiplet $(\phi = \bar{\lambda}\lambda)$ has developed a vacuum expectation value.

3. - FURTHER WORK

As discussed in the Introduction it would be interesting to apply the effective Lagrangian approach to other more realistic SUSY theories, in particular those for which the index criterion cannot give any information. Right now, together with Bars, Nilles and Veneziano, we are studying such theories. Although this work is not yet finished, let me give some details which might be of interest, and share with you our present understanding (and confusion).

The first interesting theory is a supersymmetric QCD-like theory. This is an $SU(N)$ gauge theory with M matter multiplets residing in the $f+\bar{f}$ representation (f denotes here the fundamental representation and all fermions are taken to be left-handed). Let me denote the different (chiral) multiplets in the following way

$$W = (\lambda^a, A^a_\mu) \qquad \text{gauge multiplet}$$

$$\left.\begin{aligned} P_1^{\alpha,i} &= (\varphi^{\alpha,i}, \psi^{\alpha,i}) \\ P^i_{2\alpha} &= (\gamma^i_\alpha, \chi^i_\alpha) \end{aligned}\right\} \text{matter multiplets} \qquad \begin{aligned} \alpha &= 1,\ldots,N \\ i &= 1,\ldots,M \end{aligned} \tag{3.1}$$

We consider the gauge supersymmetric theory where all non-gauge couplings are put to zero. This is in accordance with the fact that the theory is asymptotically free (as long as M is not too large) and the gauge coupling grows towards the infra-red.

$$\beta \propto -(3N-M)g^3 + O(g^5) \tag{3.2}$$

In discussing the dynamics of the theory at low energies we can therefore forget all couplings besides the gauge coupling. The classical internal symmetry group (flavour) is

$$\underbrace{SU(M) \times SU(M) \times U_V(1) \times U_A(1) \times U_R(1)}_{\text{like in QCD}}$$

The reason for the appearance of the extra $U_R(1)$ symmetry is related to the fact that the SUSY theory includes a gauge fermion λ on top of the fermions which are present in QCD.

$$j_A = \bar{\psi}\gamma_\mu\gamma_5\psi + \bar{\chi}\gamma_\mu\gamma_5\chi + \varphi^*\overleftrightarrow{D}_\mu\varphi + \eta^*\overleftrightarrow{D}_\mu\eta$$

$$j_R = \bar{\lambda}\gamma_\mu\gamma_5\lambda + \varphi^*\overleftrightarrow{D}_\mu\varphi + \eta^*\overleftrightarrow{D}_\mu\eta \qquad (3.3)$$

where we have used a Majorana four-component notation for the Weyl fermions. Note the appearance of the scalar terms in the currents which are needed to ensure the invariance of the Yukawa gauge terms in the underlying theory. Moreover, the j_R corresponds to a so-called R symmetry, i.e., a symmetry which does not commute with supersymmetry (λ carries a charge under j_R while A_μ does not). The current which sits in the multiplet together with the supersymmetry current is

$$C_\mu = \tfrac{3}{2}\, j_R - \tfrac{1}{2}\, j_A \qquad (3.4)$$

It is well known that quantum effects lead to a breaking of some axial U(1) currents. Since the "space" of the axial U(1) currents in this theory is two-dimensional, and since there is only one gauge group (and therefore only one $F\tilde{F}$), we can always find one conserved axial U(1) current. At the one-loop level it is straightforward to identify this current

$$j = M j_\lambda - N j_A \qquad (3.5)$$

where here $j_\lambda = \bar{\lambda}\gamma_\mu\gamma_5\lambda + \phi^*\overleftrightarrow{D}_\mu\phi + \eta^*\overleftrightarrow{D}_\mu\eta$. Note that this is an R current (does not commute with SUSY). We expect this symmetry to be spontaneously broken since we were not able to satisfy the 't Hooft anomaly equation [20] with respect to this current.

We have followed the steps described in the previous section to construct an effective Lagrangian for this theory. The composite supermultiplets that we consider are

$$\begin{aligned} S &= \text{Lagrangian multiplet} \\ T^{ij} &= P_1^{\alpha i} P_{2\alpha}^{j} \\ U^{ij} &= D^2 T^{ij} \end{aligned} \qquad (3.6)$$

which include all the relevant order parameters which may control the low-lying physics. At this point we have excluded the possibility that the chiral symmetry (apart from the R symmetry) is realized by massless "baryons" made out of three spin $\frac{1}{2}$ preons, since we were not able to find a set of "baryons" which satisfy the appropriate 't Hooft equations (actually we could not find any acceptable solution even allowing for baryons made out of one spin $\frac{1}{2}$ preon and spin 0 preons). Imposing both the R invariance, the SU(M)×SU(M) invariance and the anomalous Ward identities, we end with the following form of the effective potential term

$$\mathcal{L}_{pot.} = S\left[(-M+N)\log S/\mu^3 + \log \det T/\mu^2\right]_F \tag{3.7}$$

This expression is clearly correct to the one-loop order. There are, however, some delicate points concerning the question of whether this is indeed the full answer. Note that the composite multiplet U^{ij}, Eq. (3.6), whose lowest component includes $\psi^{i\alpha}\chi^j_\alpha$, does not appear in L_{pot}. This already indicates that the physics of the present theory probably does not allow for the $\psi\chi$ type of condensate (which would break both chiral symmetry and supersymmetry). The form of L_{pot} given in Eq. (3.4) is in accordance with the renormalization group equation (at least up to the one-loop order). As discussed in the previous section, μ is a renormalization group invariant scale. If we take the β function which is included in S to the one loop, use (again to one loop) $\mu = \Lambda e^{-1/\beta_1 g^2}$ and substitute it into L_{pot} we recover the usual solution (op to the one-loop) of the renormalization group equation. In particular, L_{pot} starts as $S_o|_F$ (the subscript 0 indicates that the β function inside S is taken to zeroth order) which is the Lagrangian multiplet, and then logarithmic corrections appear.

An important constraint on L_{pot} is the so-called decoupling limit. If we give a (supersymmetric) mass to a whole matter multiplet, L_{pot} should reduce in the limit $m \to \infty$ to the same form as Eq. (3.7) with $M \to M-1$. A mass term appears in the effective theory as an additional term

$$\mathcal{L}_{mass} = mt \tag{3.8}$$

where t is that particular multiplet among the T^{ij}'s which corresponds to the massive matter multiplet of the underlying theory. As long as we keep L_{mass} (for all matter multiplets) in our effective theory, we can analyze the theory with the conclusion that the chiral symmetry is broken by $\varphi\eta$ (lowest component of T) acquiring non-zero vacuum expectation values while supersymmetry remains unbroken. Also the decoupling limit works correctly. Note again that whenever it is the lowest component of the multiplet which acquires non-zero vacuum expectation values, SUSY remains unbroken.

However, the limit of the zero mass theory looks singular. In particular, the minimum of the potential corresponds to zero vacuum expectation value for (lowest component of) S. This is actually a general result independent of the particular choice of the functions f and d, Eq. (2.14), and is a consequence of imposing the R symmetry and the anomalous Ward identities on the effective Lagrangian. Such a solution (zero vacuum expectation value for S) is non-analytic since the potential term, Eq. (3.9), cannot be expanded around this point. The fact that $m = 0$ might be a singular point gets some support from the fact that for $m = 0$ in the underlying theory there are always some directions in which the potential is flat [6]. In particular, one may expect that the index, which was computed to be non-zero for the $m \neq 0$ theory, can actually change its value.

At this point we have not finished our analysis of the theory in the $m = 0$ limit. In particular, the option that supersymmetry is after all broken spontaneously is open, although it is perhaps somewhat hard to believe such a drastic change in the behaviour of the theory as $m \to 0$. It is also possible that one encounters some kind of "phase transition" as one changes m toward zero. In particular, the nature of the order parameter which gets non-zero vacuum expectation values may change. For example, we can write down an effective theory which does seem to have a smooth $m \to 0$ limit, by taking $X \equiv S^{N-M} \det T$ to be our order parameter (i.e., the effective Lagrangian will be a function of the superfields S and X). Note that in this case a non-zero vacuum expectation value for X does not break the conserved U(1) R symmetry of the theory. It is important to remember that if some global symmetries remain unbroken, then the appropriate 't Hooft anomaly equations [20] should be saturated by the massless "baryons" one finds in the theory. (If the symmetry is spontaneously broken, the Goldstone boson would saturate the appropriate equations.) This puts an extra constraint on the effective theory.

It would be extremely interesting to finish the analysis of this theory as well as of non-chiral theories and to find whether supersymmetry can be broken spontaneously and discover the relathioship between the realization of chiral symmetry and supersymmetry.

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