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Transverse Emittance reconstruction in presence of space charge and application to the 50 MeV beam of Linac4

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Summary

During the commissioning stage of Linac4 a test bench is planned to be used in order to characterize the 50 MeV beam after the DTL. Among other parameters, it will be possible to measure the transverse emittance using both the 3 monitors and the quadrupole scan method. As the space charge effects are not negligible at this energy, classical techniques of emittance reconstruction become questionable and a different approach based on recursive beam dynamics simulations must be applied.

1. Introduction

The characterization of the transverse emittance is an essential step during the commissioning of a linac as it describes the quality of the beam at that stage and it allows predicting the beam behaviour and potential beam losses downstream.

At the low energy part of the Linac, where the penetration depth is small, the measurement of the transverse emittance is made via a slit and collector system. However, after the DTL at 50 MeV, the use of this kind of devices becomes difficult and the emittance reconstruction can be performed by means of beam profile measurements.

The emittance can be characterized by profile measurements either at one monitor varying the gradient of a quadrupole upstream (Quadrupole scan method) or measuring the beam size at different locations along the line with fixed magnetic settings (3 monitor method). With these two techniques the calculation of the emittance at the input of the line is done by inverting the matrixes describing the transverse beam transport. These methods are valid if:

- The emittance is constant along the line.
- The transfer matrixes are well known.
- The space charge effects are negligible.

In case of a beam in high space charge regime one could implement the space charge effect as additional matrixes multiplication (like in Trace3D), but unfortunately these special matrixes are beam size dependent as the space charge effects are. This means that one has to make assumptions on the beam envelopes itself to reconstruct the transverse emittance.

In this note a different complementary approach is proposed, the so-called “forward” method. This method allows reconstructing the emittance with minimum assumptions on the beam characteristics and represents a very efficient technique in presence of space charge.

2. Description of the 3 monitor method

For sake of completeness, the techniques to reconstruct the emittance based on matrixes inversion are reported in the following sections. By means of the 3 monitor method, the emittance is determined applying a linear transformation from the measurement of the beam profile at 3 different positions along the line [1] backwards to the input of the line.

Knowing the transfer matrix **R**, product of transfer matrixes of the individual elements in the line **R**= R_N*R_{N-1}*…R_2*R_1, we can obtain the σ matrix from position 0 (reconstruction point) to position 1 (measurement point) this way:

$$\sigma(1) = \mathbf{R}\sigma(0)\mathbf{R}^{\mathbf{T}}.$$

Examples of these matrixes are:

- the matrix of a drift with length L:

$$\mathbf{R}_{\mathbf{drift}} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}.$$

- the matrix of a focusing quadrupole with quadrupole constant k and length l:

$$\mathbf{R}_{\mathbf{focus}} = \begin{pmatrix} \cos\sqrt{k}l & \frac{1}{\sqrt{k}}\sin\sqrt{k}l \\ -\sqrt{k}\sin\sqrt{k}l & \cos\sqrt{k}l \end{pmatrix}.$$

- the matrix of a quad defocusing in the horizontal plane with quadrupole constant k and length l:

$$\mathbf{R}_{\mathbf{defocus}} = \begin{pmatrix} \cosh\sqrt{k}l & \frac{1}{\sqrt{k}}\sinh\sqrt{k}l \\ -\sqrt{k}\sinh\sqrt{k}l & \cosh\sqrt{k}l \end{pmatrix}.$$

The sigma matrix in 2-D space (i.e. x-x') is related to the twiss parameters in the following form:

$$\sigma = \epsilon \cdot \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

Measuring the beam profile at position 1, the element of the sigma matrix $\sigma_{11}(1) = x^2_{max}(1)$ is determined. The equation relating the measured beam profile to the sigma matrix at the input of the line is:

$$x^2_{max}(1) = \sigma_{11}(1) = R_{11}^2\sigma_{11}(0) + 2R_{11}R_{22}\sigma_{12}(0) + R_{12}^2\sigma_{22}(0).$$

Therefore 3 equations of this type are needed to solve the system of equations, this means having the beam profile measurement in 3 different positions. The emittance can be finally calculated solving:

$$\epsilon = \sqrt{\det\,\sigma}.$$

In presence of space charge the accuracy of this approach becomes questionable, as shown in the next chapter.

3. Application of the 3 monitors method to reconstruct the emittance

In order to test the validity of this method at 50 MeV after the DTL, a first simulation of the test bench (as described in Section 6) with zero current was launched. The optics was tuned to get a waist of the beta function in the horizontal plane, as shown in Figure 3.1. The code used for the simulations is Travel [2].

Table I shows the comparison between the simulated parameters at the first monitor and the ones reconstructed by the inversion of the transfer matrixes taking the beam size at the monitor position.

Table I: 3 monitor method results for zero current (horizontal plane)

	Simulated value	**Reconstructed value**
Alpha x	2.29	2.29
Beta x [mm/mrad]	1.21	1.21
RMS Emittance x [pi.mm.mrad. Norm]	0.284	0.284

The procedure applied is valid when no space charge effects are involved.

When the line is run at full current (62.5 mA) the beta function changes due to the extra defocusing force the beam experiences along the line, as shown in Figure 3.1. The emittance along the line is affected as well, being not constant anymore, as shown in Figure 3.2. Adding these two effects, the beam envelope is very different for the two cases (see Figure 3.3). Applying the same procedure as before, the estimated values (reported in the next Table) are quite far from the reference ones.

Table II: 3 monitors method results for full current (horizontal plane)

	Simulated Value	**Estimated value**	**Error (%)**
Alpha x	2.14	1.74	-19
Beta x [mm/mrad]	1.73	1.59	-8
RMS Emittance x [pi.mm.mrad. Norm]	0.2854	0.3136	+10

The same happens to the vertical plane, once the quadrupoles are set to have the vertical waist at the central monitor.

Table III: 3 monitors method results for full current (vertical plane)

	Simulated Value	**Estimated value**	**Error (%)**
Alpha y	2.21	1.80	-20
Beta y [m.rad]	1.78	1.60	-11
RMS Emittance y [pi.mm.mrad. Norm]	0.287	0.318	+10

In both planes the error is about 10% for the emittance and 20% for the Twiss parameters: the emittance is overestimated whereas the Twiss parameters are underestimated.

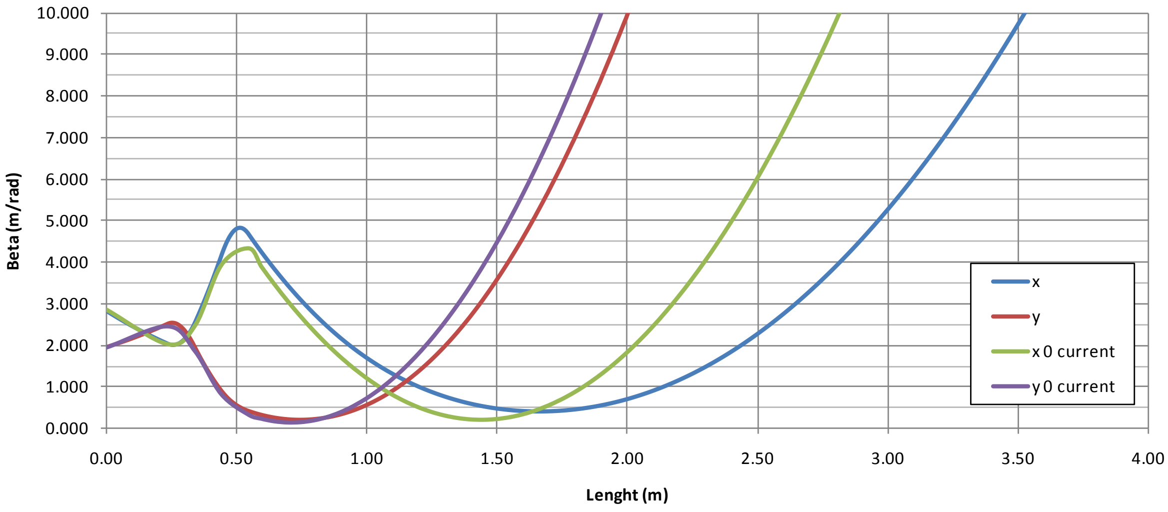

Figure 3.1: Beta function along the test bench for zero and full current.

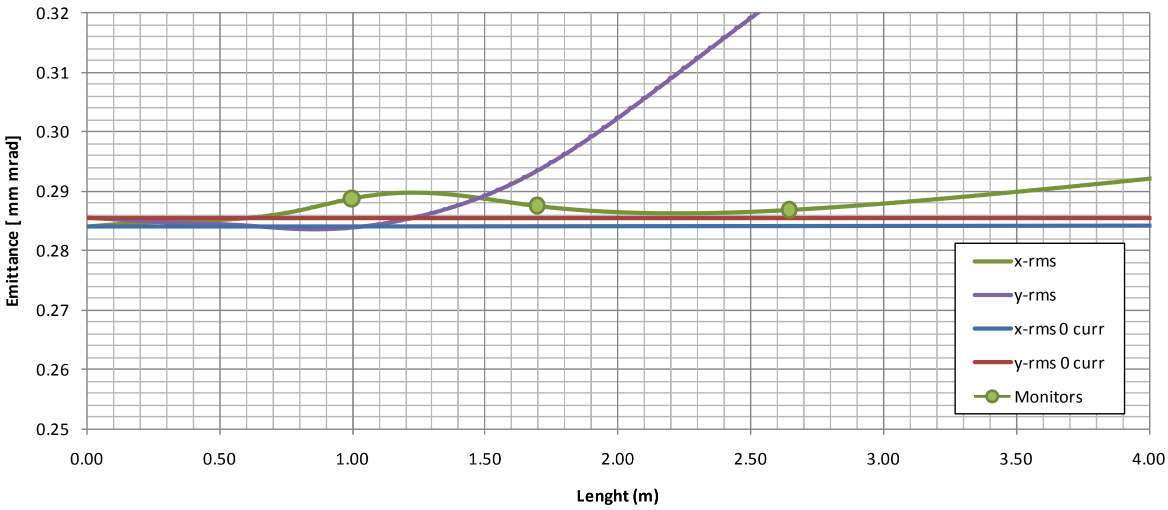

Figure 3.2: RMS normalized emittance and position of the monitors for zero and full current.

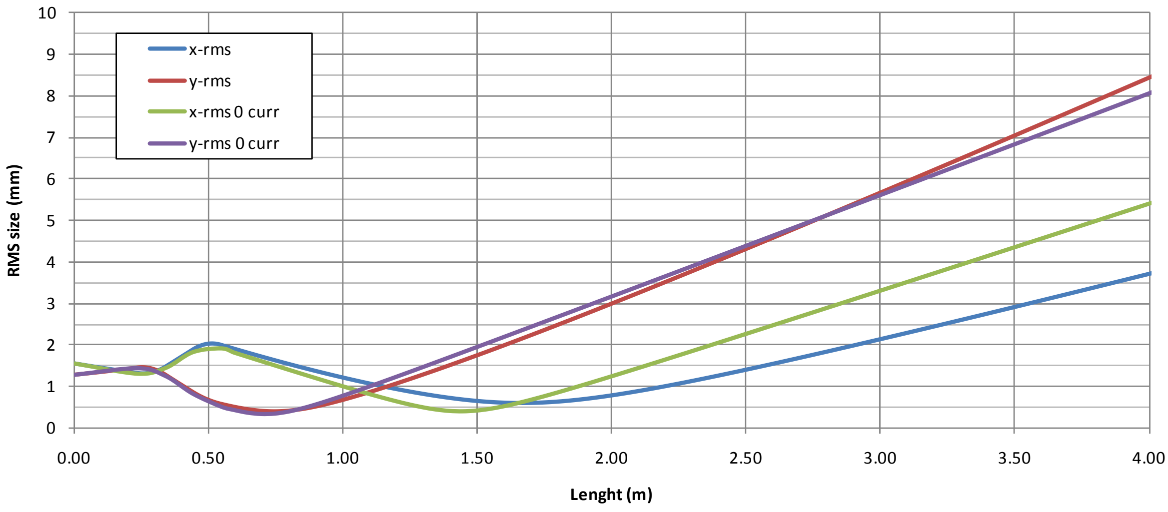

Figure 3.3: RMS envelopes for zero and full current.

4. Quadrupole Scan Method Description

The emittance can be characterized by varying the quadrupole gradient and gathering the information at one monitor. The same linear transformations described in the previous chapter are valid for this method. To simplify the calculations only the second quadrupole of the doublet is used for the scan and therefore the point of estimation (s0) is placed between the two quadrupoles. In this case we obtain for every monitor the beam profile as a function of the gradient of the second quadrupole, the Twiss parameters and the emittance.

To have an idea of the effect of the space charge to the horizontal beam profiles, in the following pictures we show the comparison between the horizontal RMS with zero and full current varying the gradient of the second quadrupole.

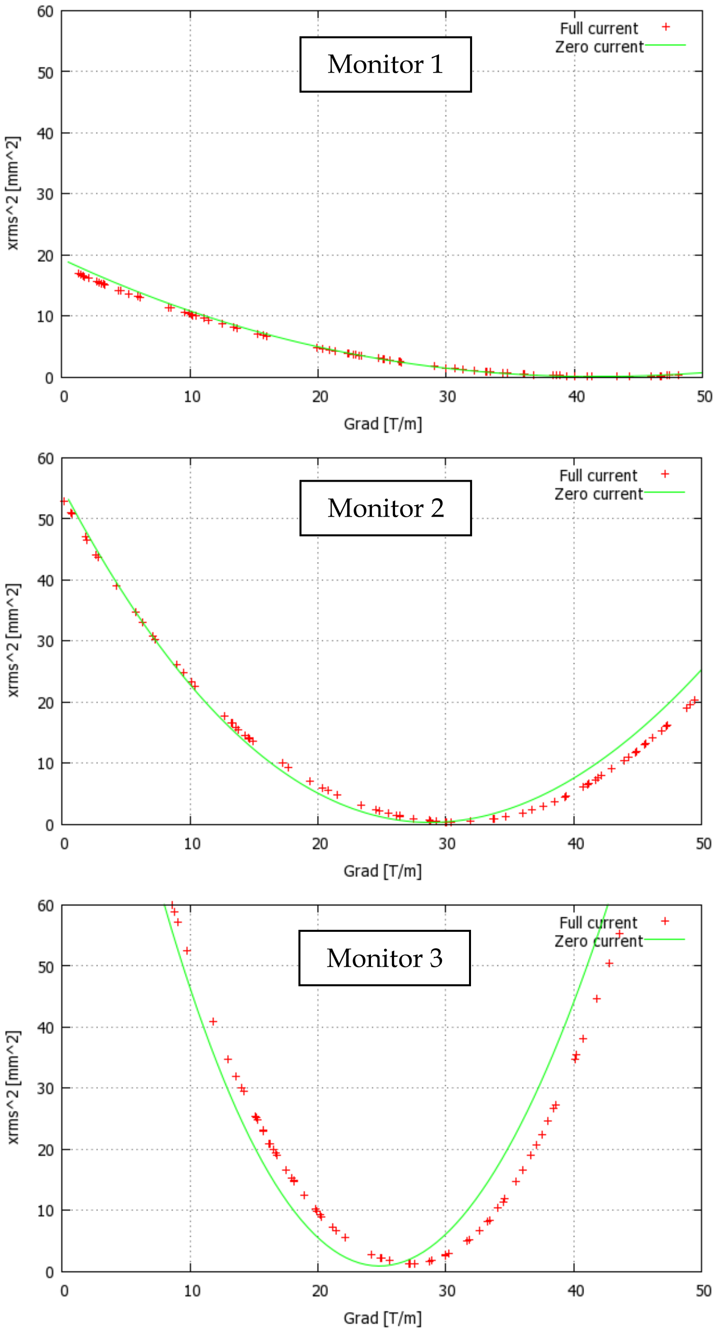

Figure 4.1: s_{11} as a function of the quadrupole gradient for each monitor for a set of twiss parameters and emittance given.

The differences in the curves come again from the defocusing effect of the space charge and the emittance increase. Applying the matrix technique backwards, the accuracy in the emittance reconstruction is about 15-20% (see Table below). This error takes into account also the fact that the solution is not obtained solving a 3 equations system but fitting the data (also with certain error) to a function generated via the matrix formalism [3].

Table IV: Quadrupole scan method results for full current (horizontal plane)

		Monitor 1		**Monitor 2**		**Monitor 3**	
Parameter	*Ref.*	*Estimated*	*Error (%)*	*Estimated*	*Error (%)*	*Estimated*	*Error (%)*
Alpha	-8.99	-7.47	16.91	-8.03	10.68	-8.41	6.45
Beta [mm/mrad]	4.17	3.30	20.86	3.54	15.11	3.69	11.51
RMS Emitt x [π.mm.mrad]	0.285	0.353	23.86	0.332	16.49	0.333	16.84

In the same way it turned out for the 3 monitors method, the Twiss parameters obtained are below the real value and the estimated emittance is above.

5. The forward method

In view of the low accuracy given by traditional methods of emittance reconstruction, a new technique has been developed to overcome the limitations in presence of space charge. The idea is to use the inversion of the matrixes as a first approximation of the solution and then to apply a more accurate complementary procedure not dependent on the beam size or transfer matrixes, what we called "forward method".

The forward method consists of fitting the experimental data to simulated beam envelope curves obtained from multiparticle simulations with space charge. The general procedure is the following:

1. Measure beam size at 3 locations or with different optics.
2. First deduction of the emittance and the Twiss parameters using the matrix formalism.
3. Vary the Twiss parameters and the transverse emittance around the estimated value. The longitudinal emittance it is assumed to be known.
4. Find the solution which minimizes the sum of squared residuals, being the residual the difference between the experimental data and the value given by the simulation.

For the 3 monitors method the final step will be carried out summing the residuals at the 3 positions whereas for the quadrupole scan method for each monitor this will be done minimizing the sum of the residuals for different gradients values.

The precision of the method and the time consumption depend on the numbers of runs and the range of variation chosen for the initial parameters. For this reason and trying to optimize the process one hundred beams were run, changing the initial Twiss parameters and emittance by 20%. Then the matrix technique was applied to obtain the first estimation. As shown in the next plots, it can be state that the Twiss parameters are always underestimated and the emittance overestimated, as it was already predicted in the previous Sections.

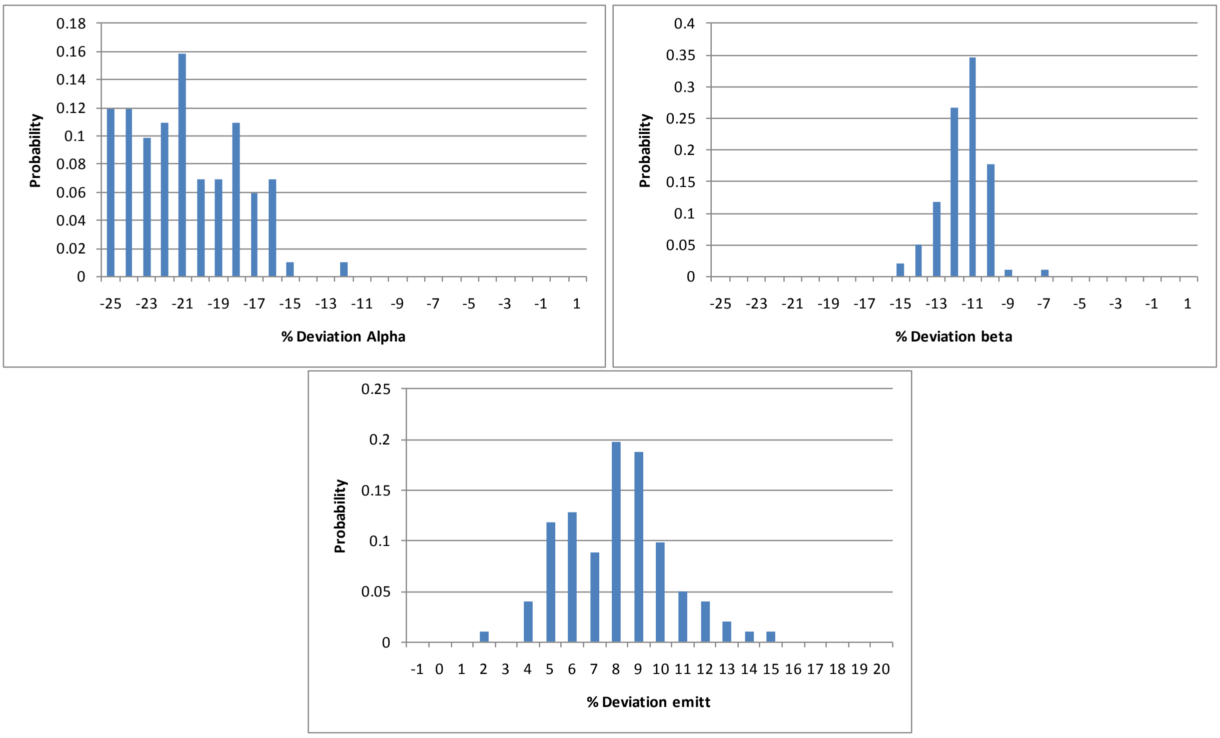

Figure 5.1: Probability distribution of the deviation estimating each of the input parameters using the 3 monitors method.

Therefore the forward method could be speed-up in two ways, for example:

- Setting the range of variation knowing that the Twiss parameters are underestimated and the emittance overestimated.
- Applying an iterative procedure, reducing progressively the variation range, where the initial values of the next step are the output of the previous one.

Due to the time consumption this is assumed to be an off-line method. Another approach would be to have a database covering a wider range of input the parameters (α, β and ε) for different currents and longitudinal emittances, and perform only the least squared method comparison.

Forward method measuring at three locations

Applying the forward method to the 3 monitors, a series of runs were launched using 20% variation to the input parameters obtained in Section 3. The best case found by the least squared method is summed up in the following table.

Table V: Forward method results for full current (horizontal plane)

	Reference Value	**First step (Section 3)**	**Forward Method**	**Final Error**
Alpha x	2.14	1.74	2.05	4.4%
Beta x [mm/mrad]	1.73	1.59	1.70	1.7%
RMS Emitt x [π.mm.mrad]	0.285	0.317	0.287	<1%

The error in the reconstruction decreases considerably and the error in the emittance calculation is less than 1%. In this case, the whole process took about 2 hours where most of the time was consumed by the runs (CPU time) and the range was applied symmetrically to the first estimation.

Forward method varying the optics

The objective of this section is to determine which one of the three monitors is the most effective to reconstruct the emittance and to validate the forward method for quadrupole scan method.

The difference between the RMS value at the monitors with or without space charge, for a set of Twiss parameters and emittance, are shown in the next Figure.

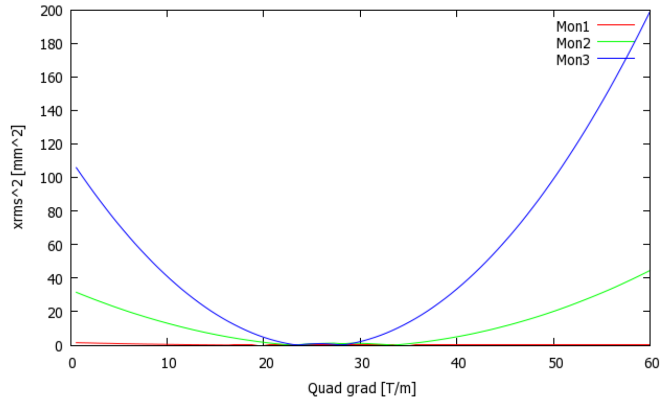

Figure 5.2: Difference between the functions with and without space charge at each monitor.

As expected, the further is the monitor the bigger is the difference between the simulation with and without space charge. From this point of view, the first monitor would be the best candidate to be used for the emittance reconstruction.

Another comparison between the monitors response has to be made in terms of sensitivity to the variation of the Twiss parameters and the emittance. This is reported in the next Figures showing the partial derivative of the RMS size at the monitors with respect to each parameter for the zero current case.

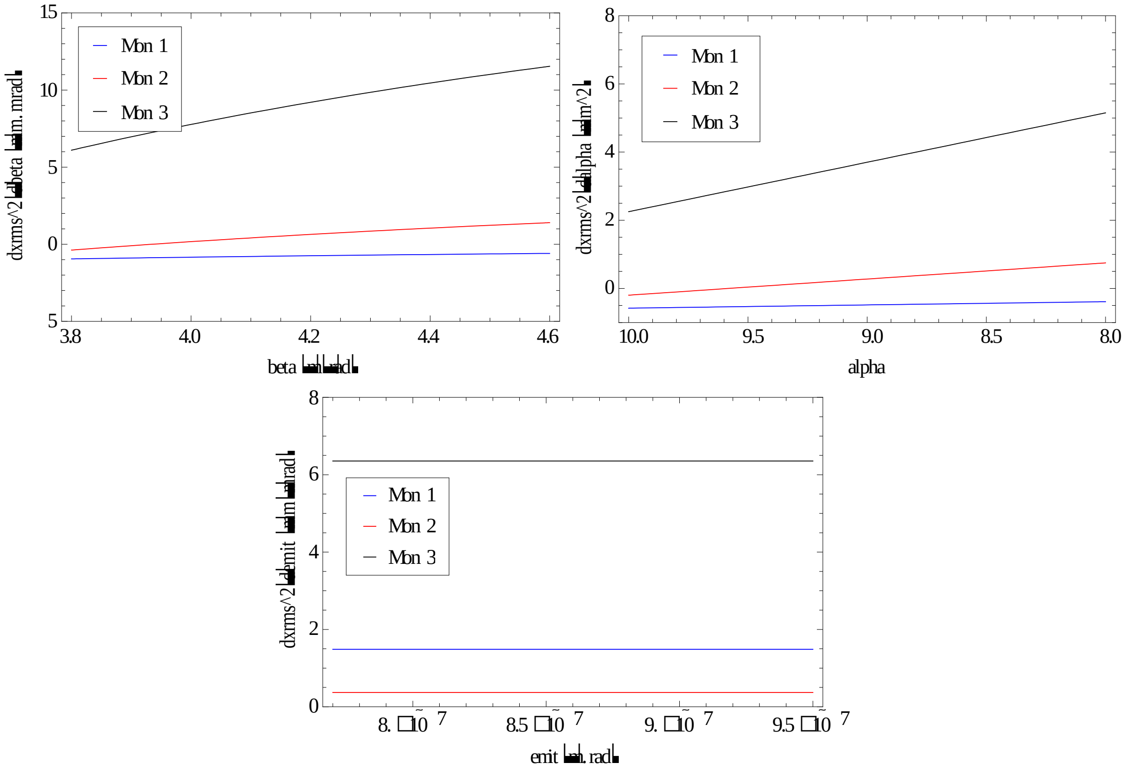

Figure 5.3: Sensitivity to the input parameters at the monitors.

The first monitor is the least sensitive to the variations of the input parameters at the previous defined input point (between the doublet). Consequently, applying the same variation range in the forward method for the 3 monitors, at monitor 1 the samples to be analyzed via the least squared method would be closer to the real value and therefore a more accurate solution will be found for the same number of simulations. In other words, the further away we are from the quadrupole, the more samples are needed to reach an accurate result.

For these reasons the forward method is applied to the quadrupoles scan at the first monitor. 30% uncertainty was applied both to the Twiss parameters and the horizontal emittance obtained in Section 4. The results are summed up in the following Table.

Table VI: Forward method results for full current (horizontal plane)

	Reference Value	**First step (Section 4)**	**Forward Method**	**Final Error**
Alpha x	-8.99	-7.47	-9.34	4%
Beta x [mm/mrad]	4.17	3.30	4.21	1%
RMS Emittance x [π.mm.mrad. Norm]	0.285	0.353	0.285	<1%

The difference between the forward method applied to the 3 monitors and the quadrupole scan methods is that for the first one we run only one set of simulations, whereas for the latter one, a set of simulations for each gradient value is needed. Therefore, the time consumption is multiplied by the number of data collected to reconstruct the curve of Figure 4.1.

6. Implementation of the test bench in Linac4

The test bench for the transverse emittance measurement consists of a doublet and 3 SEM-grids. The two quadrupoles will be like PIMS type , 0.105 m length and an aperture of 20 cm radius. The line will be placed after the intertank section between the DTL and the CCDTL. The position of the doublet is described in the figure below.

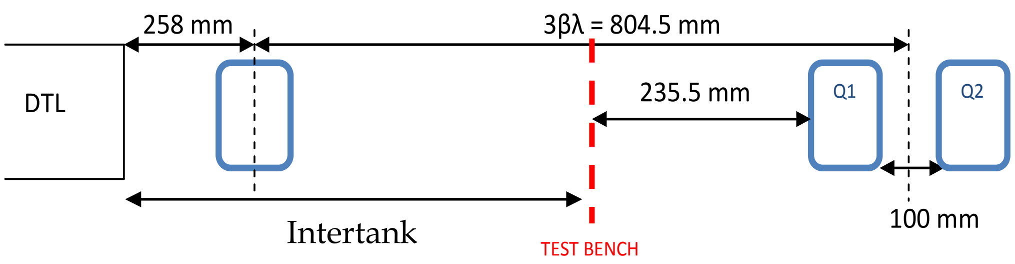

Figure 6.1: Scheme of the location of the test bench after the DTL.

With this configuration and varying the quadrupole strength, the beam dynamics for each plane is set in order to get the optimum position of the SEM-grids. The procedure consists of placing the central monitor near the beam waist and choosing the position of the other two monitors trying to obtain the 60 degrees phase advance between monitors for the theoretically best sampling.

As a first approach to deduce the position of the first and third monitors, the equation of the phase advance in a drift [4] can be applied:

The beta function in a drift is modified due to the space charge forces acting against the focusing forces and therefore the position of the monitors must be refined calculating the beam phase advance along the line. This leads to an asymmetry of the lateral monitors with respect to the central one.

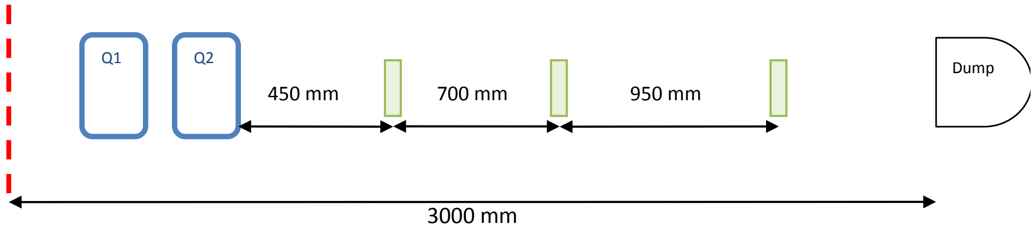

Figure 6.2: Monitors position in the test bench.

However the 60 degrees phase advance is not the only criterion determining the position of the monitors, the final set-up being also a trade-off between:

- The value of the beta functions at the waist. It must be measurable within reasonable SEM-grid resolution.

- The emittance variation among the monitors which should be as small as possible.

The quadrupoles settings for measuring both planes are shown in the table below along with the phase advance between the monitors obtained with these configurations. The horizontal phase space for these values is shown in Figure 6.3.

Table VII: Proposed quadrupole settings for measuring the emittance in both planes at 50MeV

Quad	**Horizontal plane**	**Vertical plane**
Q1	35 T/m	-13 T/m
Q2	-30 T/m	24 T/m

Table VIII: Phase advance between consecutive monitors

Phase advance	**Horizontal plane**	**Vertical plane**
1,2	59.5 deg	59.6 deg
2,3	59.6 deg	59.4 deg

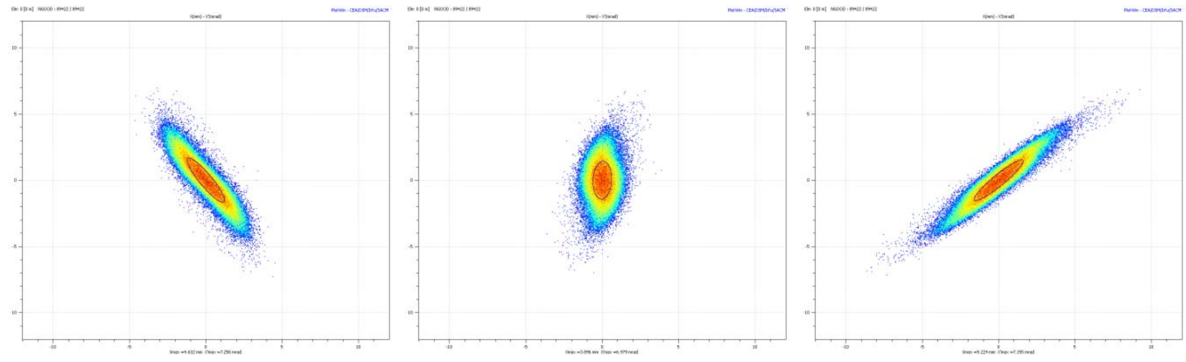

Figure 6.3: Beam horizontal phase space at three monitors after the optimization.

In order to carry out the emittance reconstruction using the quadrupole scan method, the scanning range of the second quadrupole has to be set. Taking into consideration the physical limit of the quadrupole, 50 T/m, and the beam pipe radius, 33mm, the working range of the quadrupole, without losses, has been set between 0 and 50 T/m for both configurations. The position of the dump will be at about 3 meters from the beginning of the test bench.

Figure 6.4 shows the maximum excursion of the beam envelope in the test bench, for the horizontal and vertical configurations, varying the second quadrupole inside the working range.

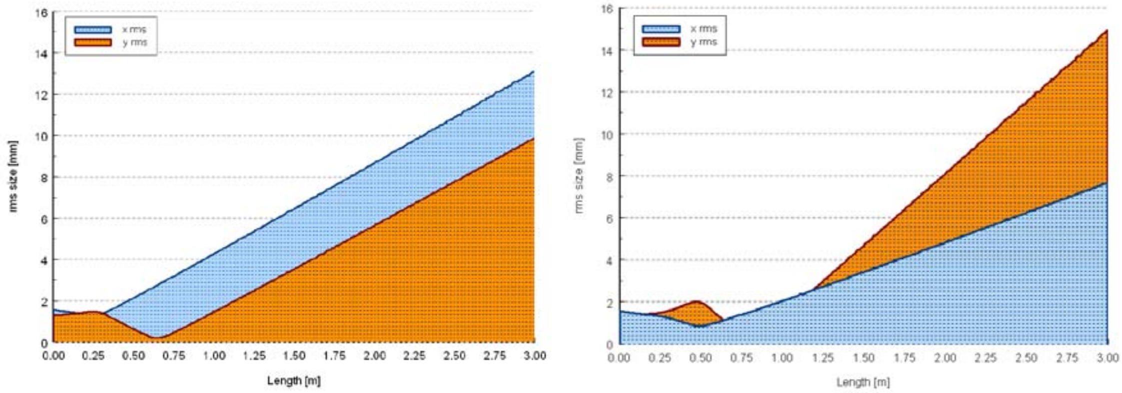

Figure 6.4: Maximum RMS size of the beam for the vertical (left) and horizontal (right) plane measurements.

Emittance reconstruction at 30 MeV

Once installed, the same test bench design for 50 MeV is going to be used as well to measure the emittance at 30 MeV switching off the RF of DTL tank3. In this scenario the beam is focused but it arrives to the test bench debunched, as shown in the following Figure.

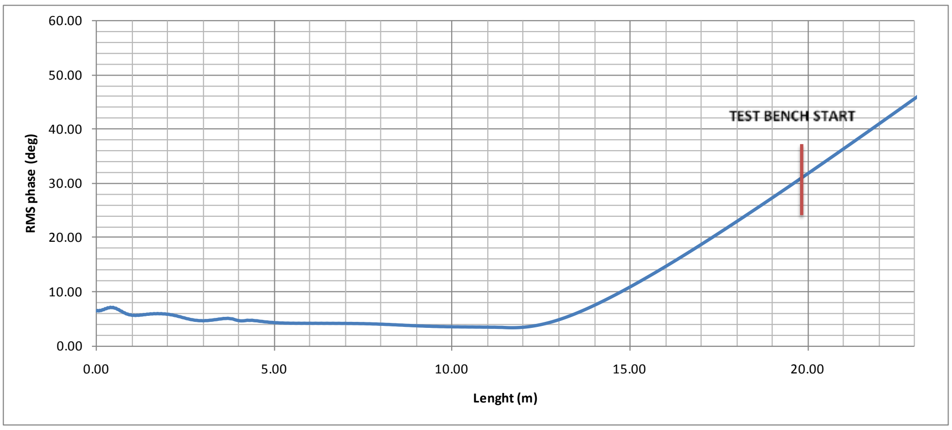

Figure 6.4: RMS phase of the beam along the DTL and the test bench.Tank3 switched off.

A beam dynamics similar to the one obtained for 50 MeV can be achieved tuning the doublet in the test bench in addition to the EMQ in the intertank section. Table IX shows the quadrupoles settings for this measurement.

Table IX: Quadrupole settings for measuring the emittance in both planes at 30MeV

Quad	Horizontal plane.	Vertical plane.
EMQ Intertank	-14.80 T/m	-14.80 T/m
Q1	30 T/m	-10 T/m
Q2	-24.5 T/m	-20 T/m

As the beam is longitudinally debunched at the test bench the effect of the space charge is almost negligible and the reconstruction of the emittance via the matrix formalism has accuracy better than 2%. Figure 6.5 represents the RMS emittance in x and y, from the beginning of the DTL up to the test bench, and table X the error estimating the input parameters with the three monitors method.

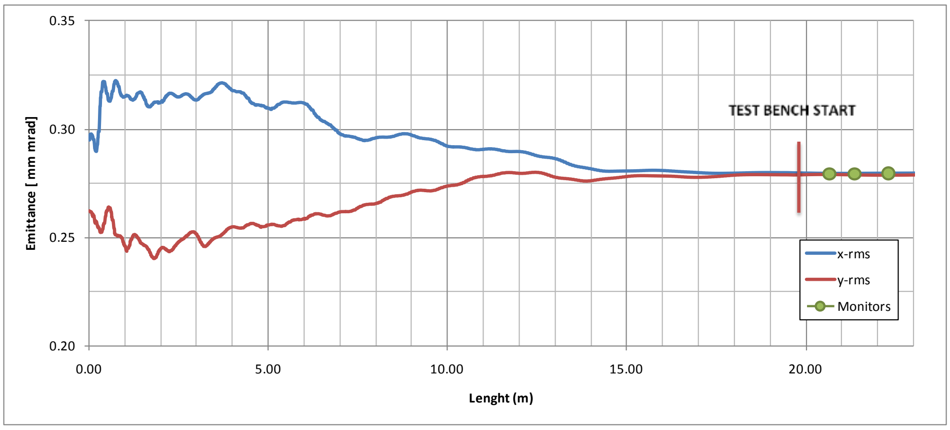

Figure 6.5: RMS emittance along the DTL and the test bench.

Table X. Results of applying the 3 monitor method in both planes at 30 MeV

	Horizontal plane			**Vertical plane**		
	Reference Value	*Estimated value*	*Error (%)*	*Reference Value*	*Estimated value*	*Error (%)*
Alpha	1.62	1.59	-1.9	1.44	1.41	-2.0
Beta [m.rad]	1.54	1.52	-1.3	1.40	1.39	-0.7
RMS Emittance [pi.mm.mrad. Norm]	0.279	0.283	+1.5	0.280	0.283	+1.0

7. Finite profile measurement resolution

So far, the emittance reconstruction has been performed assuming infinite resolution of the diagnostics. In reality the monitors installed will be SEM grids (secondary electron emission grids) and the accuracy of the measurement will be given by the wire spacing.

The beam data from the simulations has been arranged with different bins, or separation between wires from 1.0 mm to 0.2 mm. Then the RMS value for each case is calculated from the data using,

and compared with the real RMS beam size. The results of the binning to each monitor are shown in the tables below and the error between the real rms value and the rms value of the binning measurement is also reported .

The separation between the wires should be at least half the RMS size of the beam to achieve accurate results, therefore, the central monitor, where the beam size is smaller, will be the most critical one and it will require smaller wire separation. The thickness of the wire is not taken into account.

Table XI: Error introduced by the resolution of the SEM-grids for different wire spacing

Monitor 1			
Wire spacing [mm]	*Estimated RMS [mm]*	*Real RMS value [mm]*	*Error (%)*
1	1.35	1.22	10.6
0.5	1.26	1.22	2.7
0.3	1.23	1.22	1.0
0.2	1.23	1.22	0.4
Monitor 2			
Wire spacing [mm]	*Estimated RMS [mm]*	*Real RMS value [mm]*	*Error (%)*
1	0.84	0.61	38
0.5	0.68	0.61	11
0.3	0.63	0.61	4.1
0.2	0.62	0.61	1.8
Monitor 3			
Wire spacing [mm]	*Estimated RMS [mm]*	*Real RMS value [mm]*	*Error (%)*
1	1.72	1.61	6.3
0.5	1.64	1.61	1.6
0.3	1.63	1.61	0.6
0.2	1.62	1.61	0.3

If the RMS values for 0.2 mm wire spacing are taken as input for the algorithm of the emittance reconstruction, the calculated value for the 3 monitor measurement is 0.32 π.mm.mrad (12% error in the estimation), this adds only 2% error to the error, as we had 10% without the resolution issue.

8. Conclusions

For Linac4 at 50 MeV the classical methods to reconstruct the emittance give emittance values with an accuracy of 10-20% and therefore another technique has to be applied. A complementary method has been studied and successfully applied proving to be valid in general in presence of space charge, and in particular to reconstruct the emittance in Linac4 at 50 MeV.

9. References

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