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QCD PREDICTIONS FOR THE Q^2 DEPENDENCE OF QUARK AND GLUON FRAGMENTATION FUNCTIONS USING ALTARELLI-PARISI TYPE EQUATIONS

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ABSTRACT

In the leading logarithmic approximation, the fragmentation functions of quarks and gluons are investigated using Altarelli-Parisi type equations. Using a new method to make the Mellin transformation, the equation is solved. Analytic expressions for the fragmentation functions near $z = 0$ and $z = 1$ are also given. Finally, numerical results for the fragmentation functions D_q^π, D_q^k are presented for different Q^2.

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1. INTRODUCTION

Asymptotic freedom[1),2)] is a remarkable property of QCD, which has made it possible to perform perturbative calculations for some experimentally measured quantities. In the parton picture based on QCD, the quark and gluon distribution functions inside the nucleon have Q^2 dependences given by quantitative analyses[3)]. Kogut and Susskind[4)] give this a physical interpretation. The hadrons are made up of infinite levels of partons; virtual photon probes of higher Q^2 have finer resolution and have yielded a knowledge of the finer structure in the hadron. Altarelli and Parisi[5)] have reformulated the recursion equation of Kogut and Susskind in the following integro-differential equation form which is satisfied by quark and gluon distribution functions

$$\begin{aligned} \frac{\partial q_i(x,t)}{\partial t} &= \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{dy}{y}\left[P_{qq}\left(\frac{x}{y}\right) q_i(y,t) + P_{qG}\left(\frac{x}{y}\right) G(y,t)\right] \\ \frac{\partial G(x,t)}{\partial t} &= \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{dy}{y}\left[P_{Gq}\left(\frac{x}{y}\right) \sum_{j=1}^{2f} q_j(y,t) + P_{GG}\left(\frac{x}{y}\right) G(y,t)\right] \end{aligned} \qquad (1)$$

where $t = \ln Q^2/\Lambda^2$; Λ is a scale parameter with $\Lambda \simeq 0.2$–0.5 GeV/c, $\alpha_s(t)$ is the running coupling constant and f is the number of flavours.

According to the Kogut and Susskind picture, the fragmentation functions[6)] of the quark and gluon are also expected to have Q^2 dependence, since the fragmentation of the N^{th} level partons occurs through the partons of the $(N-1)^{th}$ level and there are different behaviours of the fragmentation functions in different levels. Following Altarelli and Parisi, there are integro-differential equations for the Q^2 dependent fragmentation functions $D^h_{q_i}(z,t)$ and $D^h_G(z,t)$. Several authors discuss it in this way[7)], or in field theory[8)], or in model calculations[9)]. In this paper we shall use Altarelli-Parisi type equations. Particular attention will be paid to solving the equations, to giving boundary behaviour hear $z = 0$ and 1 and to getting the Q^2 dependence using the method for the Mellin transformation given in Ref. 10).

2. ALTARELLI-PARISI EQUATIONS

The integro-differential equations for the Q^2 dependent fragmentation functions $D^h_{q_i}(z,t)$, $D^h_G(z,t)$ are

$$\frac{\partial}{\partial t} D^h_{q_i}(z,t) = \frac{\alpha_s(t)}{2\pi}\int_z^1 \frac{dz'}{z'}\left[P_{qq}\left(\frac{z}{z'}\right) D^h_{q_i}(z',t) + P_{Gq}\left(\frac{z}{z'}\right) D^h_G(z',t)\right] \qquad (2a)$$

$$\frac{\partial}{\partial t} D_G^h(z,t) = \frac{\alpha_s(t)}{2\pi} \int_z^1 \frac{dz'}{z'} \left[P_{qG}\left(\frac{z}{z'}\right) \sum_{j=1}^{2f} D_{q_j}^h(z',t) + P_{GG}\left(\frac{z}{z'}\right) D_G^h(z',t) \right] \tag{2b}$$

where $D_{q_i}^h(z,t)$, $D_G^h(z,t)$ are the mean number of hadrons of type h with momentum fraction z (with component in the direction of p of magnitude zp) per dz in a jet initiated by a quark q_i (gluon G) at a scale t, and

$$\alpha_s(t) = \frac{\alpha_s(t_0)}{1 + b\alpha_s(t_0) \ln \frac{Q^2}{\Lambda^2}} \simeq \frac{1}{bt}$$

$$b = \frac{33 - 2f}{12\pi}$$

$$i = 1, 2, \cdots 2f.$$

The physical explanation of Eq. (2a) is that the Q^2 dependence of fragmentation function $D_{q_i}^h(z,t)$ is due to two processes: the quark can radiate a gluon and then fragment; or it can radiate a gluon which fragments into the hadron. For Eq. (2b), the Q^2 dependence of fragmentation function $D_G^h(z,t)$ is due to these two processes: the gluon can pair-produce a quark which then fragments; or it can pair-produce gluons which fragment into the hadrons.

In the usual way, taking the moment of each side of Eq. (2), we obtain

$$\begin{aligned} \frac{\partial}{\partial t} \langle D_{q_i}^h(t) \rangle_n &= \frac{\alpha_s(t)}{2\pi} \frac{8}{3} A_n^{qq} \langle D_{q_i}^h(t) \rangle_n + \frac{\alpha_s(t)}{2\pi} \frac{8}{3} A_n^{Gq} \langle D_G^h(t) \rangle_n \\ \frac{\partial}{\partial t} \langle D_G^h(t) \rangle_n &= \frac{\alpha_s(t)}{2\pi} \frac{8}{3} A_n^{qG} \sum_{j=1}^{2f} \langle D_{q_j}^h(t) \rangle_n + \frac{\alpha_s(t)}{2\pi} \frac{8}{3} A_n^{GG} \langle D_G^h(t) \rangle_n \end{aligned} \tag{3}$$

where the definition of the moment is

$$\begin{aligned} \langle D_G^h(t) \rangle_n &= \int_0^1 z^{n-1} dz \, D_G^h(z,t) \\ \langle D_{q_i}^h(t) \rangle_n &= \int_0^1 z^{n-1} dz \, D_{q_i}^h(z,t) \end{aligned} \tag{4}$$

and

$$
\begin{aligned}
A_n^{qq} &= \frac{3}{4} + \frac{1}{2n} - \frac{1}{2(n+1)} - \psi(n+1) - c\,, \\
A_n^{qG} &= \frac{3(2+n+n^2)}{16n(n+1)(n+2)}\,, \\
A_n^{Gq} &= \frac{2+n+n^2}{2n(n^2-1)} \\
A_n^{GG} &= \frac{9}{4}\left[\frac{33-2f}{36} + \frac{1}{n-1} - \frac{1}{n} + \frac{1}{n+1} - \frac{1}{n+2} - \psi(n+1) - c\right].
\end{aligned}
\tag{5}
$$

$\psi(n + 1) = \Gamma'(n + 1)/\Gamma(n + 1)$ is the digamma function and c is Euler's constant, $c = 0.5772\ldots.$ Up to a multiplicative factor, the A's are the usual anomalous dimensions in QCD[1]. (We shall neglect mass effects in this paper.)

The solution of Eq. (3) is

$$
\begin{aligned}
\langle D_{q_i}^h(t)\rangle_n &= \left(\langle D_{q_i}^h(t_0)\rangle_n - \frac{1}{2f}\langle D_q^h(t_0)\rangle_n\right) e^{A_n^{qq}\cdot s} \\
&\quad + \frac{1}{2f}\langle D_q^h(t_0)\rangle_n \left[\frac{1}{2}\left(e^{A_n^+ s} + e^{A_n^- s}\right) - \frac{1}{2}\left(A_n^{GG} - A_n^{qq}\right)\frac{e^{A_n^+ s} - e^{A_n^- s}}{A_n^+ - A_n^-}\right] \\
&\quad + \langle D_G^h(t_0)\rangle_n\, A_n^{Gq}\, \frac{e^{A_n^+ s} - e^{A_n^- s}}{A_n^+ - A_n^-}\,, \\
\langle D_G^h(t)\rangle_n &= \langle D_q^h(t_0)\rangle_n\, A_n^{qG}\, \frac{e^{A_n^+ s} - e^{A_n^- s}}{A_n^+ - A_n^-} \\
&\quad + \langle D_G^h(t_0)\rangle_n \left[\frac{1}{2}\left(e^{A_n^+ s} + e^{A_n^- s}\right) + \frac{1}{2}\left(A_n^{GG} - A_n^{qq}\right)\frac{e^{A_n^+ s} - e^{A_n^- s}}{A_n^+ - A_n^-}\right]
\end{aligned}
\tag{6}
$$

where

$$
A_n^{\pm} = \frac{1}{2}\left[\left(A_n^{GG} + A_n^{qq}\right) \pm \sqrt{\left(A_n^{GG} - A_n^{qq}\right)^2 + 8f A_n^{qG} A_n^{Gq}}\,\right]
$$

t_0 is the initial value of t,

$$
s = \frac{16}{33-2f} \ln\frac{t}{t_0}\,,
$$

and we have defined

$$\langle D_q^h(t_0)\rangle_n = \sum_{i=1}^{2f} \langle D_{q_i}^h(t_0)\rangle_n .$$

In order to get the fragmentation functions themselves, we have to make an inverse Mellin transformation as follows:

$$D_{q_i}^h(z,t) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} dn\; z^{-n} \langle D_{q_i}^h(t)\rangle_n$$

$$D_G^h(z,t) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} dn\; z^{-n} \langle D_G^h(t)\rangle_n \tag{7}$$

where the contour $(\gamma - i\infty, \gamma + i\infty)$ is to the right of all singularities of $\langle D_{q_i}^h(t)\rangle_n$, $\langle D_G^h(t)\rangle_n$ in the complex n plane. Formula (7) has another form which we have used in our calculation:

$$D_{q_i}^h(z,t) = \int_z^1 \frac{dy}{y}\left[D_{q_i}^h(y,t_0) - \frac{1}{2f} D_q^h(y,t_0)\right] \tilde{F}_{qq1}\left(\frac{z}{y},t\right) + \int_z^1 \frac{dy}{y}\frac{1}{2f} D_q^h(y,t_0)\, \tilde{F}_{qq2}\left(\frac{z}{y},t\right) + \int_z^1 \frac{dy}{y} D_G^h(y,t_0)\, \tilde{F}_{Gq}\left(\frac{z}{y},t\right) \tag{8a}$$

$$D_G^h(z,t) = \int_z^1 \frac{dy}{y} D_q^h(y,t_0)\, \tilde{F}_{qG}\left(\frac{z}{y},t\right) + \int_z^1 \frac{dy}{y} D_G^h(y,t_0)\, \tilde{F}_{GG}\left(\frac{z}{y},t\right) \tag{8b}$$

where $\tilde{F}(x,t)$ is the inverse Mellin transformation of $F(n,t)$,

$$D_q^h(z,t) = \sum_{i=1}^{2f} D_{q_i}^h(z,t).$$

In our calculation we use two different, but mathematically equivalent, forms of $F(n,t)$ (see Appendix) which are very useful when studying the behaviour of the fragmentation functions near $z = 0$ and $z = 1$, respectively.

3. BEHAVIOUR NEAR THE KINEMATIC BOUNDARY

In this section we shall discuss the behaviour of the fragmentation functions of quark and gluon near $z = 1$ and $z = 0$.

Studying the behaviour of a fragmentation function near $z = 1$ is equivalent to studying the limit of its Mellin transform for n going to infinity. Therefore, we make a $1/n$ expansion of its Mellin transform and calculate the inverse Mellin transform. Assuming that the behaviours of the fragmentation functions of the quark and gluon to hadrons near $z = 0$ are

$$\begin{aligned} z D^h_{q_i}(z,t_0) &= a_i (1-z)^{b_i} \\ z D^h_{q}(z,t_0) &= a (1-z)^{b} \\ z D^h_{G}(z,t_0) &= a_g (1-z)^{b_g}. \end{aligned} \tag{9a}$$

then the behaviours of $zD^h_{q_i}(z,t)$ and $zD^h_G(z,t)$ near $z = 1$ are

$$\begin{aligned} z D^h_{q_i}(z,t) \sim{}& a_i e^{s(\frac{3}{4}-c)} \frac{\Gamma(b_i+1)}{\Gamma(b_i+1+s)} (1-z)^{b_i+s} \\ &+ \frac{2}{5} a_g e^{s(\frac{3}{4}-c)} \frac{\Gamma(b_g+1)}{\Gamma(b_g+2+s)} \frac{(1-z)^{b_g+s+1}}{(c-\frac{21-2f}{20}) + \ln\frac{1}{1-x} + \psi(b_g+2+s)}, \end{aligned}$$

$$\begin{aligned} z D^h_{G}(z,t) \sim{}& a \frac{3}{20} e^{s(\frac{3}{4}-c)} \frac{\Gamma(b+1)}{\Gamma(b+s+2)} \frac{(1-z)^{b+s+1}}{(c-\frac{21-2f}{20}) + \ln\frac{1}{1-x} + \psi(b+2+s)} \\ &+ a_g e^{s\frac{9}{4}(\frac{33-2f}{36}-c)} \frac{\Gamma(b_g+1)}{\Gamma(b_g+1+\frac{9}{4}s)} (1-z)^{b_g+\frac{9}{4}s}. \end{aligned} \tag{9b}$$

For $zD^h_{q_i}(z,t)$, the first term corresponds to the quark radiating a gluon and then fragmenting into hadrons, while the second term corresponds to the process that the quark radiates a gluon which fragments into the hadron. For $zD^h_G(z,t)$, the first (second) term corresponds to pair production of quarks (gluons) which then fragment into hadrons.

Studying the behaviour of the fragmentation function near $z = 0$ is equivalent to studying the behaviour of its Mellin transform near its right-most singularity in the complex n plane.

Assuming the behaviours of fragmentation functions of the quark and gluon near $z = 0$ for $t = t_o$ are

$$
\begin{aligned}
zD^h_{q_i}(z,t) &\sim c_i \\
zD^h_G(z,t) &\sim c_g ,
\end{aligned}
\qquad (10a)
$$

then the behaviours of $zD^h_{q_i}(z,t)$ and $zD^h_G(z,t)$ near $z = 0$ are

$$
\begin{aligned}
zD^h_{q_i}(z,t) \sim & \left(\sum_i c_i\right)\frac{2}{81}\, e^{-s\left(\frac{33}{16}+\frac{1}{72}f\right)} \left(\frac{9s}{4\ln\frac{1}{z}}\right)^{1/2} I_1\!\left(3\sqrt{s\ln\tfrac{1}{z}}\right) \\
& + c_g \frac{4}{9}\, e^{-s\left(\frac{33}{16}+\frac{1}{72}f\right)}\, I_0\!\left(3\sqrt{s\ln\tfrac{1}{z}}\right) \\
zD^h_G(z,t) \sim & \left(\sum_i c_i\right)\frac{1}{18}\, e^{-s\left(\frac{33}{16}+\frac{1}{72}f\right)} \left(\frac{9s}{4\ln\frac{1}{z}}\right)^{1/2} I_1\!\left(3\sqrt{s\ln\tfrac{1}{z}}\right) \\
& + c_g\, e^{-s\left(\frac{33}{16}+\frac{1}{72}f\right)}\, I_0\!\left(3\sqrt{s\ln\tfrac{1}{z}}\right)
\end{aligned}
$$

In the leading logarithmic approximation we obtain the boundary behaviours near $z = 1$ and $z = 0$ (9b), (10b), but in this region, higher order contributions must be considered. This would be a very complicated calculation.

4. NUMERICAL RESULTS

In this section we give numerical results for the fragmentation functions of the parton into pions and kaons.

According to the assumptions in Ref. 11), there are two (three) independent fragmentation functions for pions (kaons). That is:

$$
\begin{aligned}
D_u^{\pi^+} &= D_d^{\pi^-} = D_{\bar{u}}^{\pi^-} = D_{\bar{d}}^{\pi^+} \\
D_u^{\pi^-} &= D_d^{\pi^+} = D_{\bar{d}}^{\pi^-} = D_{\bar{u}}^{\pi^+} \\
&\simeq D_s^{\pi^+} = D_s^{\pi^-} = D_{\bar{s}}^{\pi^+} = D_{\bar{s}}^{\pi^-} \\
D_g^{\pi^0} &= \tfrac{1}{2}\left(D_g^{\pi^+} + D_g^{\pi^-} \right)
\end{aligned}
\qquad (11a)
$$

and

$$
\begin{aligned}
D_u^{K^+} &= D_d^{K^0} = D_{\bar{u}}^{K^-} = D_{\bar{d}}^{\bar{K}^0} \\
D_s^{K^-} &= D_s^{\bar{K}^0} = D_{\bar{s}}^{K^+} = D_{\bar{s}}^{K^0} \\
D_u^{K^-} &= D_d^{\bar{K}^0} = D_u^{K^+} = D_{\bar{d}}^{K^0} \\
&\simeq D_d^{K^-} = D_u^{\bar{K}^0} = D_{\bar{d}}^{K^+} = D_{\bar{u}}^{K^0} \\
&\simeq D_d^{K^+} = D_u^{K^0} = D_{\bar{d}}^{K^-} = D_{\bar{u}}^{\bar{K}^0} \\
&\simeq D_s^{K^+} = D_s^{K^0} = D_{\bar{s}}^{K^-} = D_{\bar{s}}^{\bar{K}^0} .
\end{aligned}
\qquad (11b)
$$

Also, following Field and Feynman[12] we set $\alpha_s = \alpha_v$ and get the following results for the initial values of the D_q functions:

$$
\begin{aligned}
D_u^{\pi^+} &= -3.569849 - 0.741609z + 0.089012z^2 + 0.780270\tfrac{1}{z} + 3.488177z^{0.46} - 0.413852\ln z \\
D_u^{\pi^-} &= -3.153852 - 1.049605z - 0.06499z^2 + 0.780270\tfrac{1}{z} + 3.488177z^{0.46} + 0.263479\ln z \\
D_u^{K^+} &= -1.338995 - 0.393414z + 0.157387z^2 + 0.184108\tfrac{1}{z} + 0.413914z^{0.46} + 0.041315\ln z \\
D_u^{K^-} &= -0.477492 - 0.08541z - 0.035112z^2 + 0.184108\tfrac{1}{z} + 0.413914z^{0.46} + 0.125984\ln z \\
D_s^{K^-} &= -0.828812 + 0.136343z + 0.140448z^2 + 0.184108\tfrac{1}{z} + 0.413914z^{0.46} - 0.503936\ln z
\end{aligned}
\qquad (12)
$$

with $Q_0^2 = 4\ \mathrm{GeV}^2/c^2$ and $\Lambda = 0.4\ \mathrm{GeV}/c$.

Using the assumptions of Ref. 13), the initial values of D_G are

$$z D_G^{\pi} = \frac{1}{2}(1-z)^{1.5}$$
$$z D_G^{K} = \frac{1}{4}(1-z)^{1.5} \tag{13}$$

For small values of z, we use the formula (A1) and compute the contributions of all the singularities of $F(n,t)$ in the complex n plane: $n = 1,0,-1,-2,...$ The inverse Mellin transform of $1/(n+p)^r$ is $x^p\ 1/\Gamma(r)(\ln 1/x)^{r-1}$. The computer is used to sum up all these terms. Finally, we obtain $zD^h_{q_i}(z,t)$ and $zD^h_G(z,t)$ for the small z using Eqs (8a) and (8b).

For large values of z, we use the formula (A2) and expand the integrand in a series in $1/n-1$. Noting that the inverse Mellin transform of $1/(n-1)^r$ is $1/x\Gamma(r)(\ln 1/x)^{r-1}$, we obtain the inverse Mellin transform of each term of the series. After integration over v, $\tilde{F}(z,t)$ is obtained. Also, $zD^h_{q_i}$, zD^h_G are obtained from Eqs (8a) and (8b) for large values of z.

Finally, we check that the two types of calculation give the same results for $z \simeq 0.6$-0.7.

The resulting fragmentation functions are shown in Figs 1-7. For $Q_0^2 = 4(\mathrm{GeV}/c)^2$, $\Lambda = 0.4$ GeV/c, $s = 0.3, 0.2, 0.1$ correspond to $Q^2 = 33.37, 14.56, 7.22\ (\mathrm{GeV}/c)^2$, respectively.

From Figs 1 and 2 we see that when z is close to 1, the values of $zD_u^{\pi^+}$ are larger than $zD_u^{\pi^-}$. This is due to the fact that near $z = 1$, the observed hadron contains the initial quark; for a u quark it is easy to get a $\bar{d}$ quark from vacuum and form a π^+, whereas it is more difficult to form a π^-. Similar situations also happen for $zD_u^{k^+}$ and $zD_u^{k^-}$.

Compared with parton distribution functions, considerably less is known about scaling deviations in fragmentation functions. To my knowledge, this is the first time that numerical results about the Q^2 dependence of light quark and gluon fragmentation functions in leading logarithmic approximation have been obtained. Field theoretic arguments[8)] and model calculations[9)] have shown that $D(z,t)$ should behave similarly to parton distribution functions, i.e., D should fall near $z = 1$ and rise near $z = 0$ as Q^2 grows. From Figs 1-7 we see that this is the case.

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APPENDIX

There are two different forms of F(n,t). Expression (A1) is useful near z = 0 and (A2) is useful near z = 1. The results are

$$
\begin{aligned}
F_{gg1}(n,t) &= e^{A_n^{gg} s} \\
F_{gg2}(n,t) &= \sum_{\ell=0}^{\infty} \frac{1}{\ell!} s^{\ell} (P_n)_{\ell} \\
F_{Gg}(n,t) &= A_n^{Gg} \sum_{\ell=0}^{\infty} \frac{1}{\ell!} s^{\ell} (L_n)_{\ell} \\
F_{gG}(n,t) &= A_n^{gG} \sum_{\ell=0}^{\infty} \frac{1}{\ell!} s^{\ell} (L_n)_{\ell} \\
F_{GG}(n,t) &= \sum_{\ell=0}^{\infty} \frac{1}{\ell!} s^{\ell} (S_n)_{\ell} ,
\end{aligned}
\qquad \text{(A1)}
$$

where

$$
(L_n)_{\ell} = \frac{(A_n^{+})^{\ell} - (A_n^{-})^{\ell}}{A_n^{+} - A_n^{-}}
$$

$$
(P_n)_{\ell} = \frac{1}{2}\left[(A_n^{+})^{\ell} + (A_n^{-})^{\ell} - (A_n^{GG} - A_n^{gg}) \frac{(A_n^{+})^{\ell} - (A_n^{-})^{\ell}}{A_n^{+} - A_n^{-}} \right]
$$

$$
(S_n)_{\ell} = \frac{1}{2}\left[(A_n^{+})^{\ell} + (A_n^{-})^{\ell} + (A_n^{GG} - A_n^{gg}) \frac{(A_n^{+})^{\ell} - (A_n^{-})^{\ell}}{A_n^{+} - A_n^{-}} \right]
$$

all satisfy the following recursive formula

$$
(L_n)_{\ell} = (A_n^{GG} + A_n^{gg})(L_n)_{\ell-1} - (A_n^{GG} A_n^{gg} - 2f A_n^{gG} A_n^{Gg})(L_n)_{\ell-2}
$$

with

$$
\begin{aligned}
(L_n)_0 &= 0 , \quad (L_n)_1 = 1 , \\
(P_n)_0 &= 1 , \quad (P_n)_1 = A_n^{gg} \\
(S_n)_0 &= 1 , \quad (S_n)_1 = A_n^{GG} .
\end{aligned}
$$

$$
\begin{aligned}
F_{qq1}(x,t) &= e^{A_n^{qq} s} \\
F_{qq2}(x,t) &= e^{A_n^{qq} s} + \int_0^s dv \sum_{j=1}^{\infty} U^j \frac{v^j (s-v)^{j-1}}{j!\,(j-1)!}\, e^{(s-v) A_n^{GG} + v A_n^{qq}} \\
F_{qG}(x,t) &= A_n^{qG} \int_0^s dv \sum_{j=0}^{\infty} U^j \frac{v^j (s-v)^{j}}{j!\; j!}\, e^{(s-v) A_n^{GG} + v A_n^{qq}} \\
F_{Gq}(x,t) &= A_n^{Gq} \int_0^s dv \sum_{j=0}^{\infty} U^j \frac{v^j (s-v)^{j}}{j!\; j!}\, e^{(s-v) A_n^{GG} + v A_n^{qq}} \\
F_{GG}(x,t) &= e^{A_n^{GG} s} + \int_0^s dv \sum_{j=1}^{\infty} U^j \frac{v^j (s-v)^{j-1}}{j!\,(j-1)!}\, e^{v A_n^{GG} + (s-v) A_n^{qq}}
\end{aligned}
\tag{A2}
$$

where

$$U = 2f\, A_n^{Gq}\, A_n^{qG}.$$

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Figure Captions

Fig. 1 : The fragmentation function $zD_u^{\pi^+}(z,t)$ for (a) s = 0.3, (b) s = 0.2, and (c) s = 0.1.

Fig. 2 : The fragmentation function $zD_u^{\pi^-}(z,t)$ for (a) s = 0.3, (b) s = 0.2, and (c) s = 0.1.

Fig. 3 : The fragmentation function $zD_u^{k^+}(z,t)$ for (a) s = 0.3, (b) s = 0.2, and (c) s = 0.1.

Fig. 4 : The fragmentation function $zD_u^{k^-}(z,t)$ for (a) s = 0.3, (b) s = 0.2, and (c) s = 0.1.

Fig. 5 : The fragmentation function $zD_s^{k^-}(z,t)$ for (a) s = 0.3, (b) s = 0.2, and (c) s = 0.1.

Fig. 6 : The fragmentation function $zD_G^{\pi}(z,t)$ for (a) s = 0.3, and (b) s = 0.1.

Fig. 7 : The fragmentation function $zD_G^{k}(z,t)$ for (a) s = 0.3, and (b) s = 0.1.

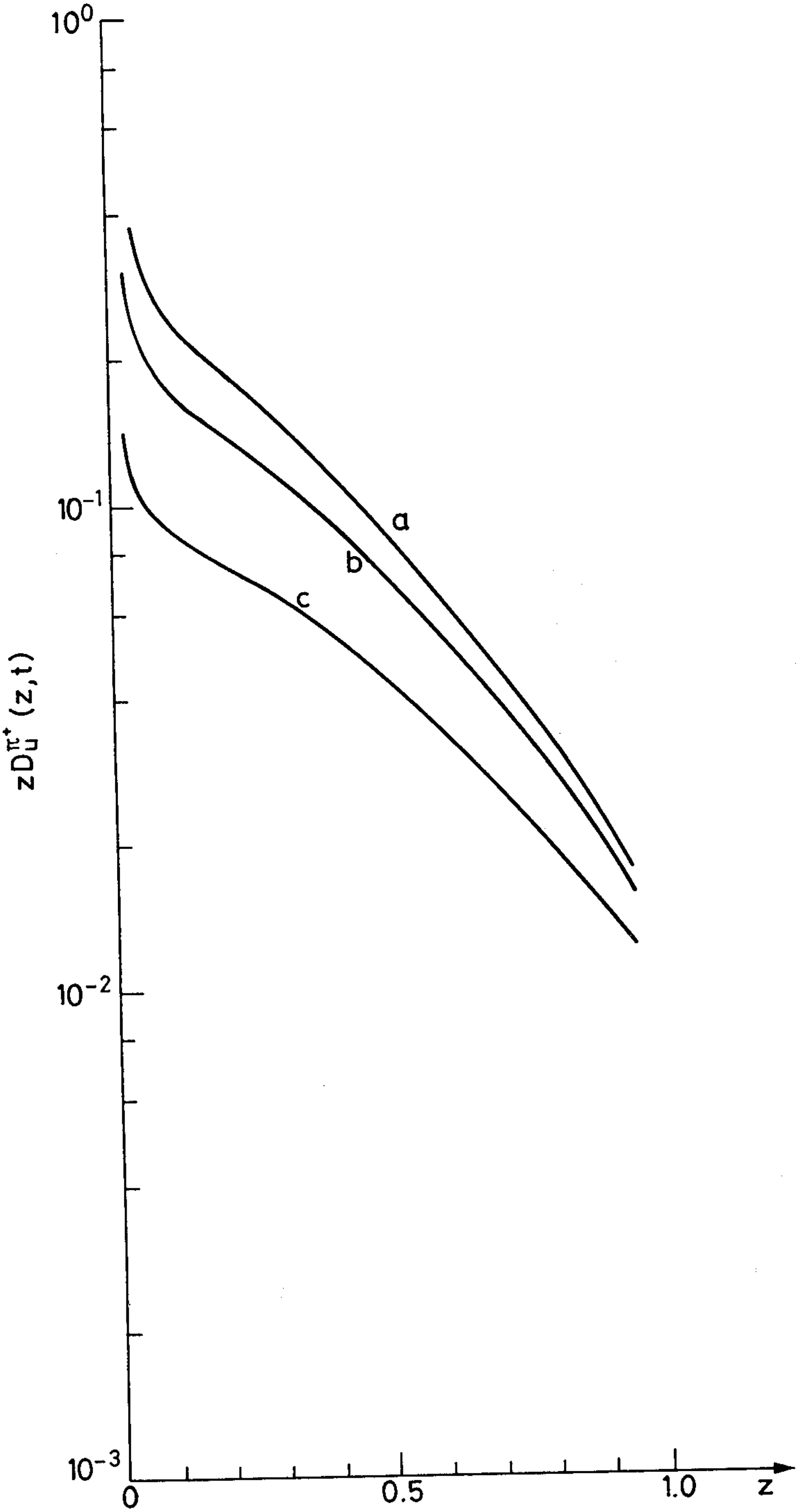

FIG.1

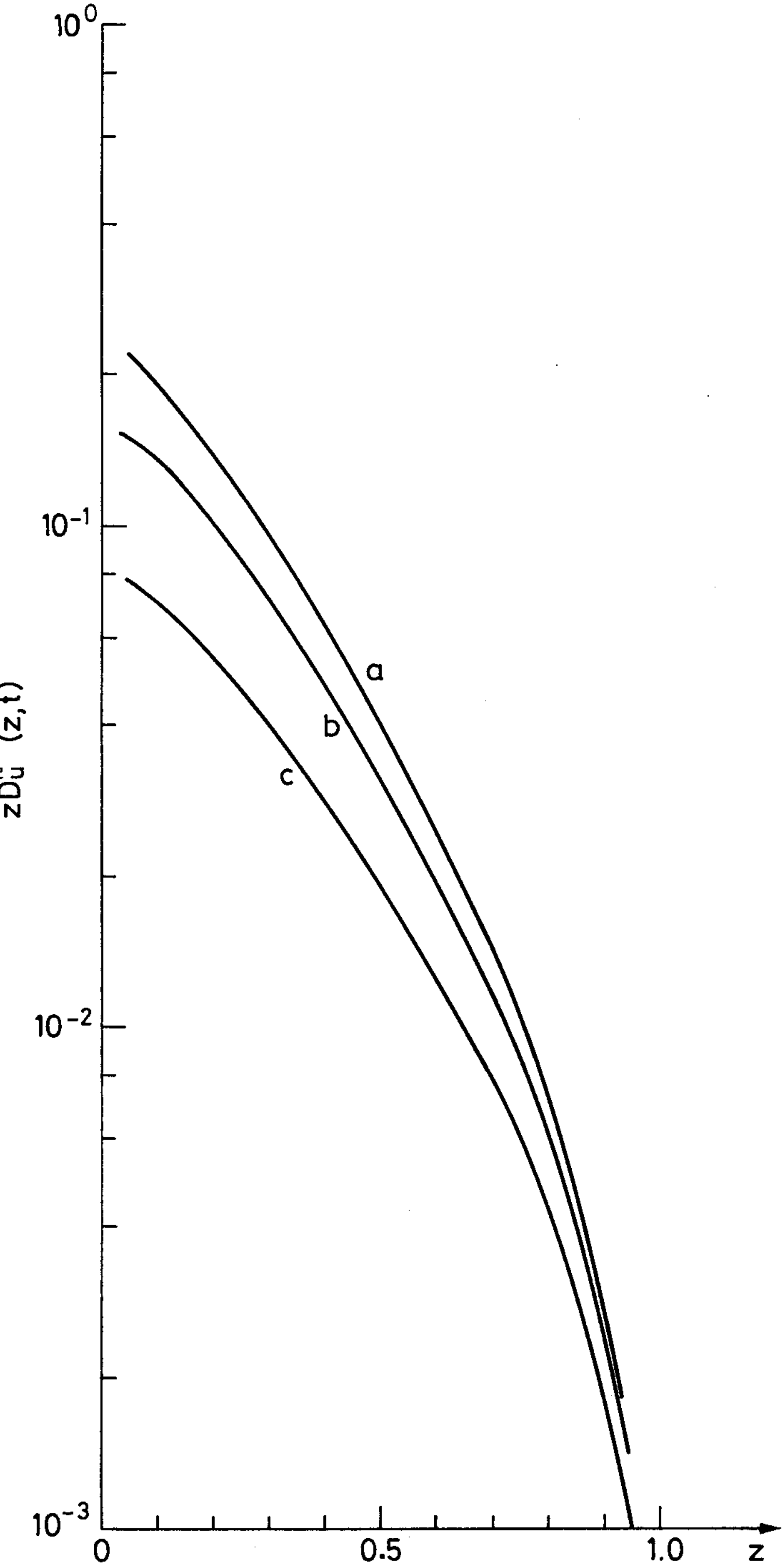

FIG.2

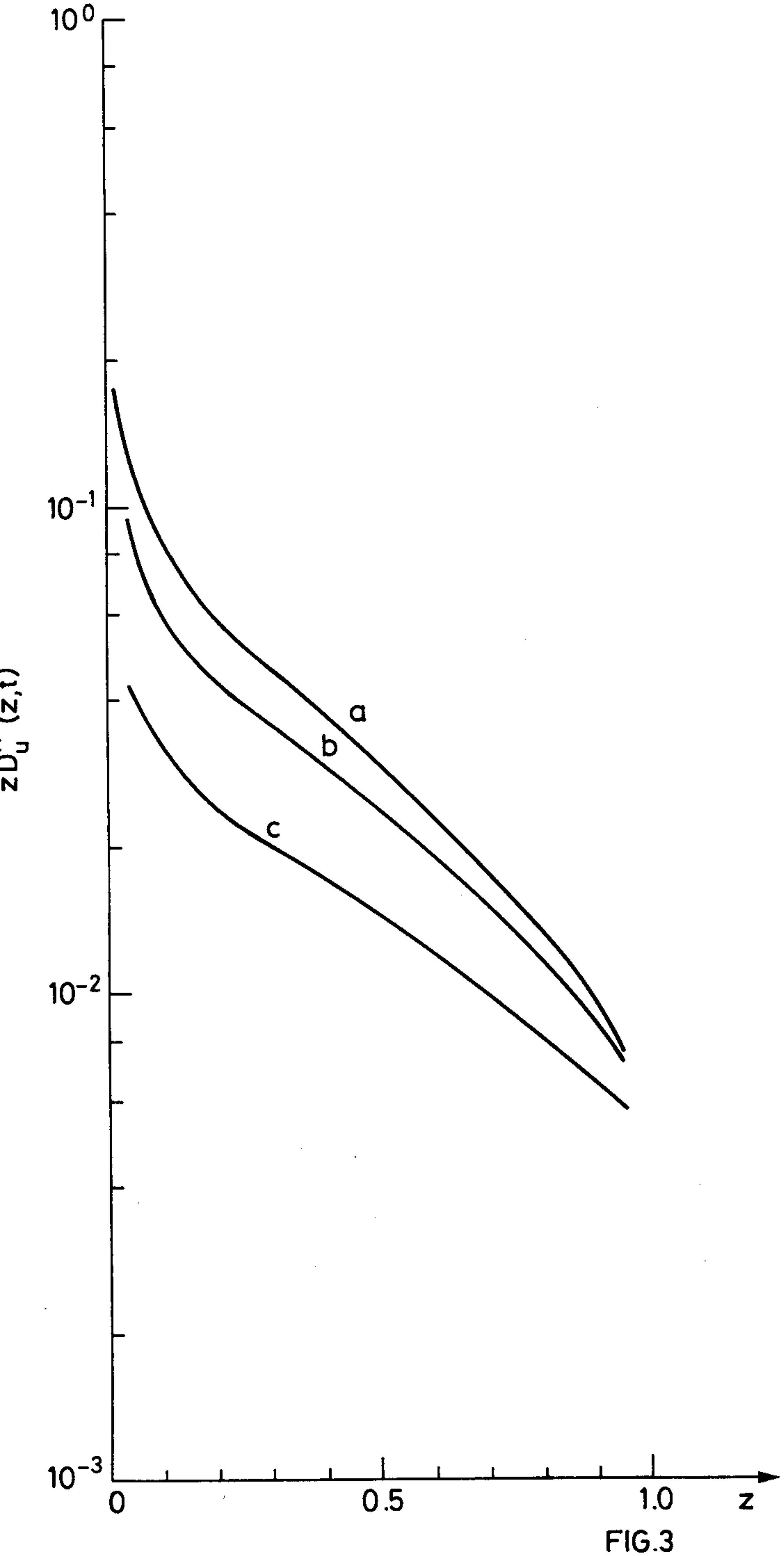

FIG.3

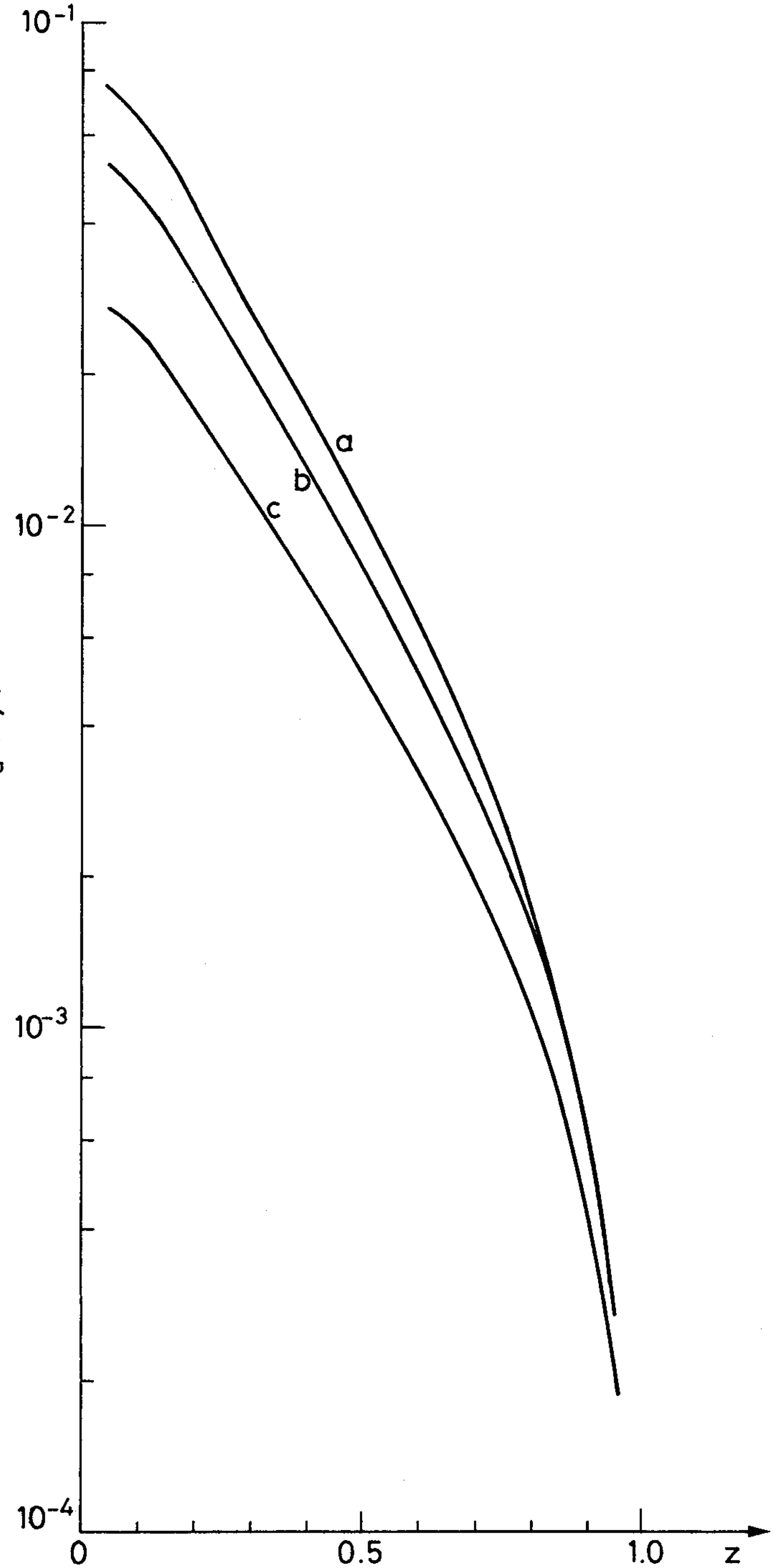

FIG.4

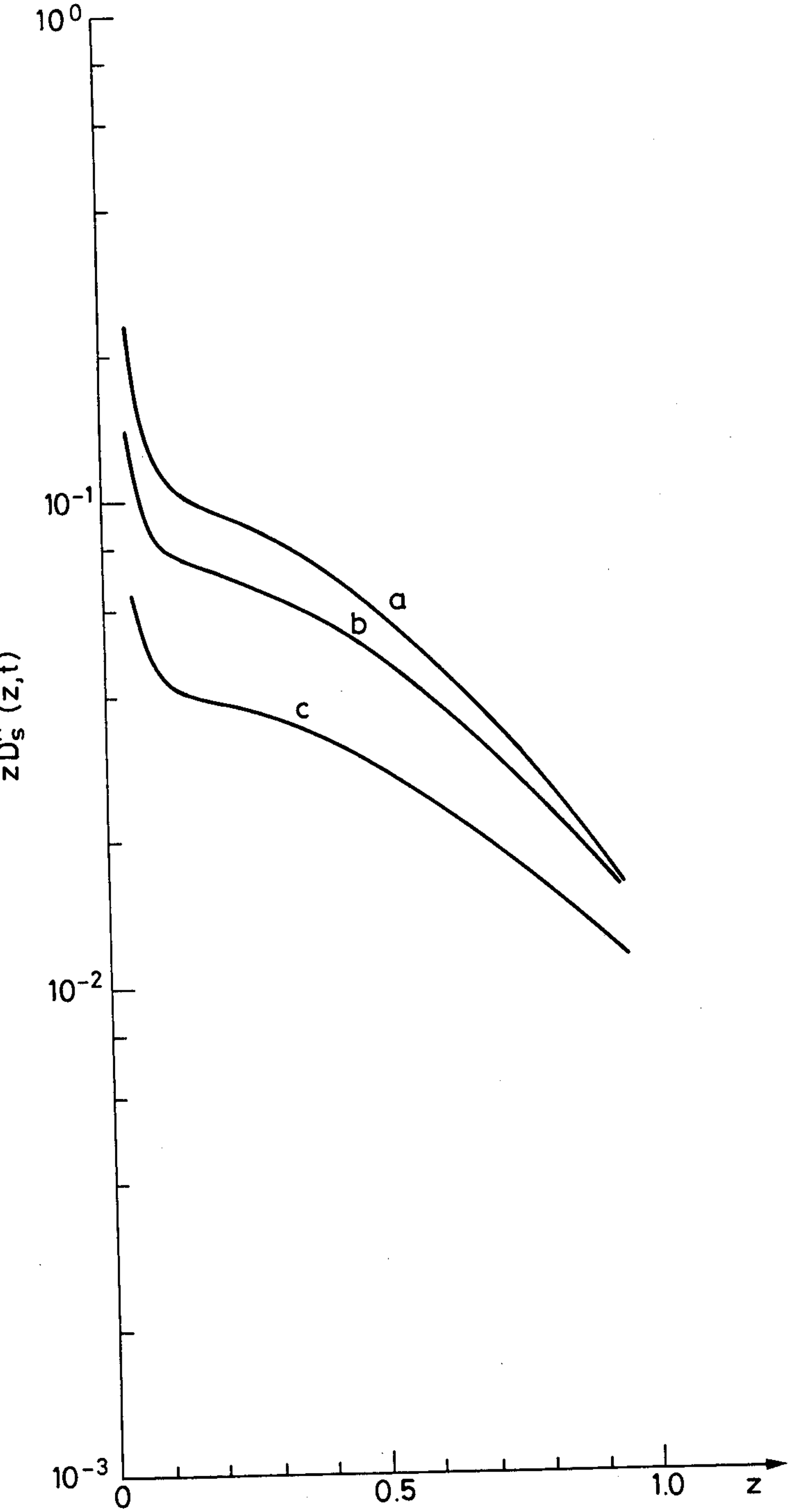

FIG.5

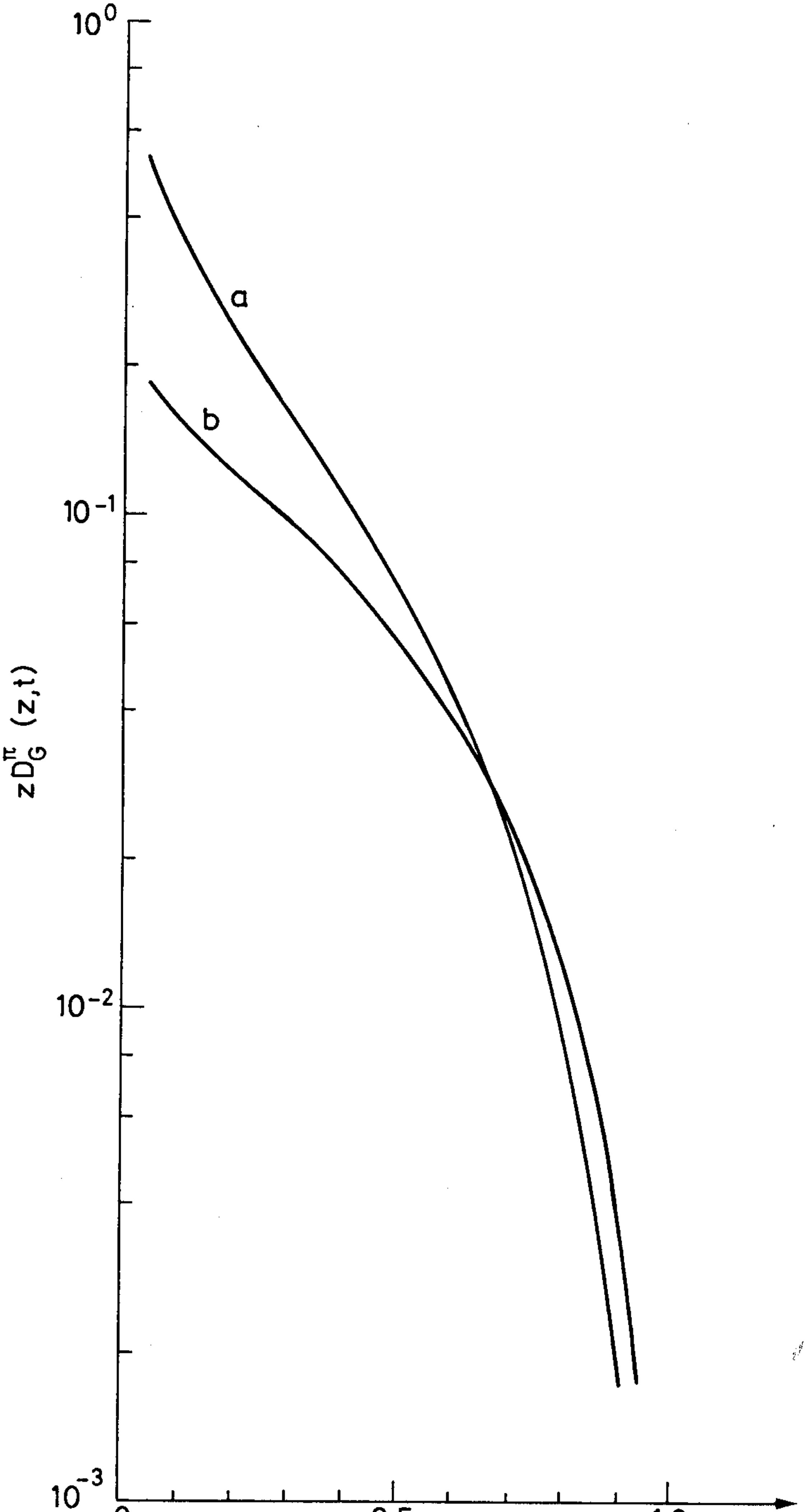

FIG.6

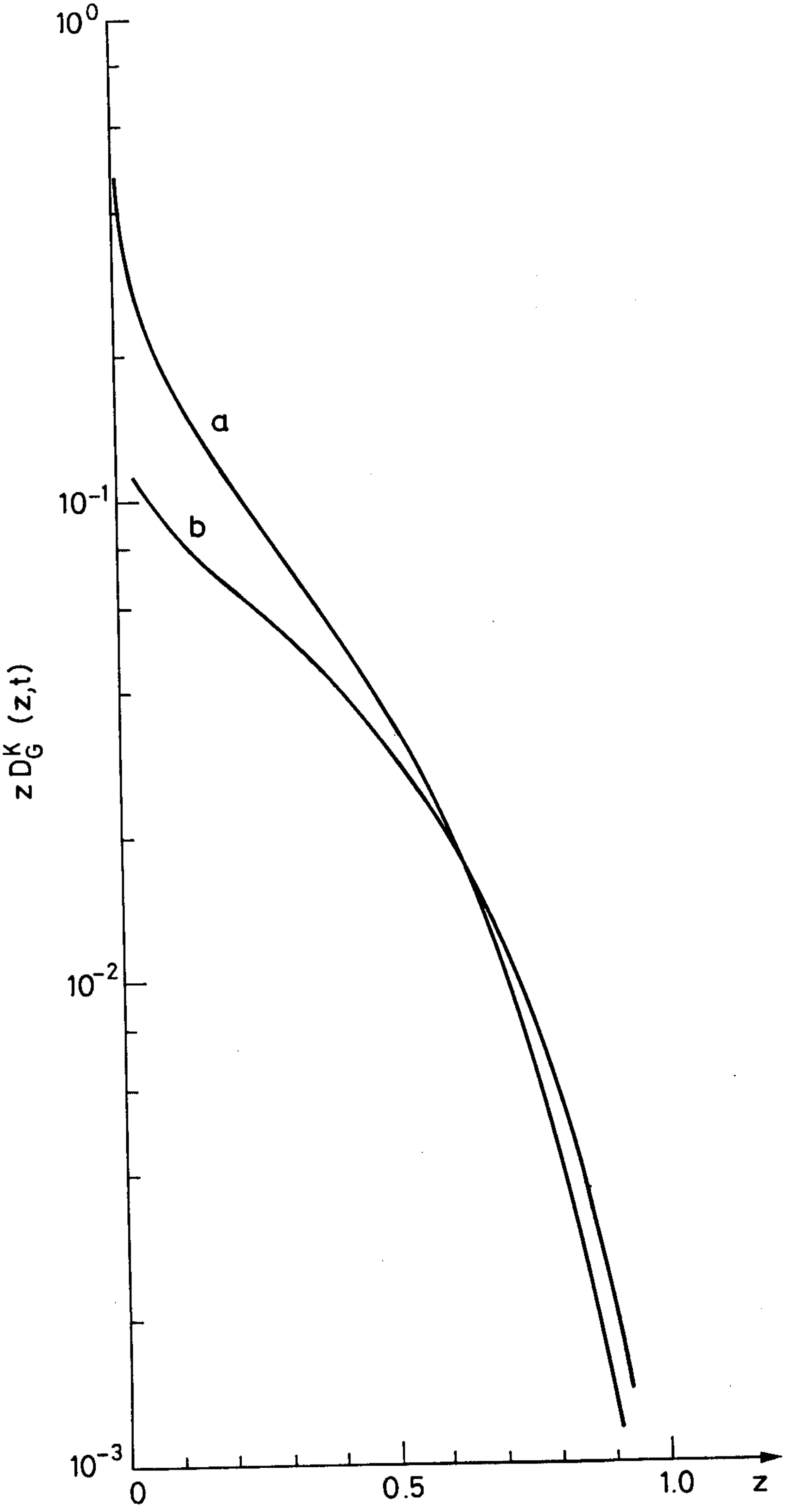

FIG.7