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AN EFFECTIVE LAGRANGIAN FOR THE PURE N = 1 SUPERSYMMETRIC YANG-MILLS THEORY

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ABSTRACT

An effective Lagrangian for the pure, N = 1, supersymmetric Yang-Mills theory is proposed by suitably modifying that of QCD. The quantum breaking of scale and chiral invariance by the corresponding anomalies generates a massive Wess-Zumino supermultiplet while preserving supersymmetry. The large N_c limit is discussed for an $SU(N_c)$ gauge group.

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The mechanism by which supersymmetry (SUSY) can possibly be broken dynamically is still not clearly understood[1]. Yet such an understanding is of doubtless importance for building realistic models based on supersymmetry.

In order to expose some of the distinctive properties of SUSY, while making as much use as possible of the recently acquired understanding of non-Abelian gauge theories without fundamental scalars, we have looked at the pure $N = 1$ supersymmetric Yang-Mills theory[2] with an $SU(N_c)$ gauge group. This theory is thus a one flavour ($N_f = 1$) gauge theory with Majorana (Weyl) fermions in the adjoint representation (rather than Dirac fermions in the fundamental representation as would be the case for QCD). The Lagrangian is

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{i}{2} \bar{\lambda}^a \not{D}_{ab} \lambda^b + \text{gauge fixing + ghost terms} + \text{auxiliary fields} \qquad (1)$$

$(a = 1,2,\ldots,N_c^2 - 1)$

with repeated indices summed over. The metric used is $\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$. In Eq. (1) λ^a is the spinor field and $F^a_{\mu\nu}$, D_μ are the usual Yang-Mills field strength and covariant derivative respectively. This theory is supersymmetric for any N_c, hence it enjoys a SUSY $1/N_c$ expansion. The topological classification of the diagrams contributing to order $(1/N_c)^n$ is the same as that of QCD diagrams in the limit $N_c \to \infty$ with N_f/N_c fixed[3], i.e., boson and fermion loops count alike (supersymmetry) and a graph with genus h (h "handles") is of order $(1/N_c)^{2h}$ relative to the leading ($h = 0$) diagrams. In spite of this, assuming colour confinement, the leading term of the expansion is closer to that of QCD in the limit $N_c \to \infty$, $N_f/N_c \to 0$[4], in that it describes colourless narrow bound states whose residual n body interactions go to zero at N_c^{2-n}. These "hadrons" are coherent states made out of an indefinite number of constituent "quarks" and "gluons". "Mesons" and "baryons" contain an even or odd number of quarks respectively and coexist as $N_c \to \infty$, unlike in QCD where only the mesons survive in the limit. Finally, this theory is asymptotically free and its coupling g get transmuted into the usual scale parameter Λ in terms of which "hadronic" masses are all of $O(1)$.

At the classical level this theory is scale and chiral invariant. These two symmetries are broken at the quantum level by the anomalies of the energy momentum tensor and of the axial current respectively

$$\partial_\mu (x_\nu \Theta_{\mu\nu}) = \Theta_{\mu\mu} = \frac{\beta(g)}{2g} F^a_{\mu\nu} F^a_{\mu\nu}$$
$$\partial_\mu J_{\mu 5} = -\frac{\beta(g)}{2g} F^a_{\mu\nu} \tilde{F}^a_{\mu\nu} \quad ; \quad \tilde{F}^a_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\sigma\tau} F^a_{\sigma\tau} \qquad (2)$$

where $\beta(g)$ is the usual renormalization group β function. Similar equations (but with different coefficients) hold for QCD. In particular note that the axial current defined here does not satisfy the Adler-Bardeen no-renormalization theorem[5]. What distinguishes this theory from QCD is the existence of a conserved SUSY current S_μ^α which has a γ-trace anomaly:

$$\partial_\mu (x_\lambda \gamma^\lambda) S_\mu = \gamma^\mu S_\mu = 2 \frac{\beta(g)}{2g} F_{\mu\nu}^a \sigma_{\mu\nu} \lambda^a \qquad (3)$$

It has been shown[6] that both the current $J_{\mu s}$, $\theta_{\mu\nu}$ and S_μ and their anomalies $\partial_\mu J^5_\mu$, $\theta_{\mu\mu}$, $\gamma_\mu S_\mu$ belong to supermultiplet structures. Recent explicit calculations by Jones and Leveille[7] appear to indicate that this is indeed the case up to fourth order in the gauge coupling. The supermultiplet structure determines the coefficients in Eqs. (2) and (3) uniquely. It is clear therefore that the axial current which resides in the same supermultiplet with the energy momentum tensor cannot satisfy the Adler-Bardeen theorem. In the QCD case, it has been possible to construct large N_c effective Lagrangians[8] which incorporate properly the axial anomaly and the effect of topological charge, and offer a simple picture for the resolution of the U(1) problem. The multiplet structure of the anomalies will be our basis for extending that construction to the SUSY case. Before going into that we would like, however, to make some general considerations.

There are two interesting lowest dimension gauge invariant order parameters in this theory, $F^a_{\mu\nu}F^a_{\mu\nu}$ and $\bar\lambda^a\lambda^a$. If the first gets a vacuum expectation value SUSY is broken. This comes from the fact that[9]

$$\frac{1}{8} Tr \{\gamma^\mu S^\alpha_\mu, \bar Q^\alpha\} = \theta_{\mu\mu} = \frac{\beta(g)}{2g} F^a_{\mu\nu} F^a_{\mu\nu} \qquad (4)$$

An expectation value of the right-hand side of Eq. (4) would imply that Q_α does not annihilate the vacuum (alternatively, the vacuum energy $\langle 0|\theta^\mu_\mu|0\rangle$ is non-zero). It has been suggested[10] that the expectation value of the renormalized operator in the SUSY theory possesses positivity properties such that, for example

$$\langle F^2_{01}\rangle_0 \,, \langle F^2_{23}\rangle_0 \geq 0 \qquad (5)$$

Combining this equation with Lorentz invariance of the vacuum:

$$\langle F^a_{\mu\nu} F^a_{\rho\sigma} \rangle_0 = (\eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\mu\sigma}\eta_{\nu\rho}) \frac{\langle F^2 \rangle_0}{12} \tag{6}$$

one concludes that $\langle F^2 \rangle_0 = 0$. This conclusion is, in our opinion, unwarranted. Indeed, by explicit calculation of the anticommutator in Eq. (4) one finds:

$$\Theta_{\mu\mu} \underset{N\to 4}{=} (4-N)\left(-\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{3i}{8} \bar{\lambda}^a \not{D} \lambda^a\right) \tag{7}$$

The first term in Eq. (4) is the conventional one while the second, also present in non-SUSY theories[11] with a different numerical coefficient, is a rather trivial operator because of the equation of motion. The coefficient 3/8 appearing in Eq. (7) is just what is needed in order to achieve the exact cancellation of gluonic and fermionic loops (at the one loop level) needed to leave the vacuum energy at zero. This is consistent with the fact that in deriving Eq. (7) we have eliminated the auxiliary field D through its equation of motion $D = 0$. In any case, the above argument shows that the operator appearing on the left-hand side of Eq. (2) is not just F^2, a fermionic free term having been subtracted. This spoils the positivity arguments given above and leaves the question of whether $\langle F^2 \rangle_0 = 0$ or not, a dynamical one.

Let us now discuss the expectation value of $\langle \bar{\lambda}\lambda \rangle$. The question of whether $\langle \bar{\lambda}\lambda \rangle \neq 0$ breaks SUSY is a more subtle one since we have to use the anticommutator of Q_α with a gauge dependent quantity in order to obtain $\langle \bar{\lambda}\lambda \rangle$, i.e.,

$$\langle \bar{\lambda}\lambda \rangle = \frac{1}{4} \langle \{ Q_\alpha, (\bar{\lambda} \not{A})_\alpha \} \rangle \tag{8}$$

[In Eq. (8) we have used the fact that Lorentz symmetry is not broken, hence only scalar operators can have a non-zero vacuum expectation value.] We shall see explicitly how SUSY is preserved even if $\langle \lambda\lambda \rangle \neq 0$.

On the other hand, the question of whether $\langle \bar{\lambda}\lambda \rangle \neq 0$ represents in some sense a spontaneous symmetry breaking deserves more discussion, due to the explicit breaking by the anomaly. In QCD the anomaly can be switched off by going to the limit $N_f/N_c \to 0$ and in this case the possibility of a spontaneous breaking of the $U(1)$ symmetry is a meaningful one. Unfortunately, in the present SUSY case, no parameter can be adjusted to switch off the anomaly while preserving

SUSY [in the present SUSY theory $N_f = 1$ and the anomaly is larger than in QCD by a factor $O(N_c)$]. Yet the diagrams contributing to the large N_c limit of the three axial-current amplitude can be classified according to whether the three currents are attached to the same fermion loop or not. It is the first type of diagrams which in the QCD case survives in the large N_c limit and which is responsible for spontaneous chiral symmetry breaking while the other ones, being related to the anomaly, provide a mass for the would-be Goldstone boson (the η'). In the SUSY case both sets of diagrams should be considered simultaneously. However, by analogy, we still expect the first type to induce a spontaneous breaking with a Goldstone boson while the others provide the mass shift through explicit breaking. To substantiate the same conclusion one can regard the supersymmetric case as the $N_f \to 1$ limit of a more general theory with N_f flavours of Majorana fermions in the adjoint representation. This theory is not supersymmetric (if $N_f \neq 1$) but has instead a $U(N_f)$ chiral symmetry broken to $SU(N_f)$ by the strong anomaly. In this case, taking also $N_c \to \infty$, we can easily show that a Coleman-Witten[12] type of result follows, using 't Hooft anomaly equations[13]. One expects $SU(N_f)$ to be broken to $O(N_f)$ with

$$\langle \bar{\lambda}_i^a \lambda_j^a \rangle \sim \delta_{ij} \qquad i,j = 1,2 \cdots N_f \tag{9}$$

Notice that only the diagrams of the first type discussed above contribute to the $SU(N_f)$ anomalies and are responsible for the vacuum expectation value of Eq. (9). Unless something discontinuous happens as we come down to $N_f = 1$, we expect the same diagrams to give $\langle \bar{\lambda}^a \lambda^a \rangle \neq 0$ in the SUSY case as well.

Another argument for spontaneous breaking of chiral symmetry uses the fermion loop expansion (a non-supersymmetric expansion in powers of N_f). Assuming, as in QCD, that the quarkless theory has non-trivial θ dependence, one finds[14] that this can be cancelled in the full theory with massless fermions only if a Goldstone pole appears at the one-fermion loop level [which gets a mass $O(\sqrt{N_f})$ after resumming all fermion loops]. The above arguments make it plausible that one of our low-lying degrees of freedom should be the would-be Goldstone boson of chiral symmetry.

We are now ready to turn to the effective Lagrangian approach which has proved to be so useful in QCD. The two bosonic gauge invariant composite fields used there were $F\tilde{F}$ and $\bar{\lambda}\lambda$. It is very natural here to add to them the other members of the anomaly supermultiplet[15]. The resulting effective theory will then be of the Wess-Zumino type with the components of the supermultiplet given by the composite fields

$$\phi = C\,\bar{\lambda}_R \lambda_L \quad ; \quad \phi^* = C\,\bar{\lambda}_L \lambda_R$$

$$\chi = \frac{i}{2} C\, F_{\mu\nu}\, \sigma_{\mu\nu}\, \lambda_L \tag{10}$$

$$M = -\frac{C}{2}\left(F^2 + i F\tilde{F}\right) \quad ; \quad M^+ = -\frac{C}{2}\left(F^2 - i F\tilde{F}\right)$$

where terms that vanish upon use of the equations of motion have been omitted and

$$C = \frac{\beta(g)}{2g} = -\frac{3 g^2 N_c}{32\pi^2}$$

$$\lambda_L = \tfrac{1}{2}(1 - i\gamma_5)\lambda \quad ; \quad \lambda_R = \tfrac{1}{2}(1 + i\gamma_5)\lambda$$

in our $SU(N_c)$ theory. Alternatively, one can use the definite parity combinations $a = c\bar{\lambda}\lambda$, $b = ci\bar{\lambda}\gamma_5$, χ, cF^2, $-cF\tilde{F}$. The fields in Eq. (10) can be combined to form the chiral multiplet (in two component notation).

$$S(x,\theta) = \phi + 2\theta\chi - \theta^2 M - i(\theta\sigma_\mu\bar{\theta})\partial_\mu\phi$$
$$- i\theta^2(\bar{\theta}\bar{\partial}\chi) - \tfrac{1}{4}\theta^2\bar{\theta}^2\Box\phi \tag{11}$$

In the QCD effective Lagrangian there were three crucial terms; the kinetic term for ϕ, a quadratic term in $F\tilde{F}$ and a coupling of $F\tilde{F}$ to $\log\phi$ to give the correct anomaly. The standard kinetic term one uses in supersymmetric theories, i.e., $(S^*S)_D$ (in standard notation) would give a kinetic term for both ϕ and χ as well as a quadratic term in the auxiliary field $F\tilde{F}$. However, because in our case ϕ is a composite field, it does not carry the canonical dimension of a fundamental scalar field (the same applies also to χ). To ensure scale invariance of the kinetic terms one modifies the kinetic Lagrangian by taking

$$\mathcal{L}_{kin} = \frac{g}{\alpha}\,(S^*S)_D^{1/3} \tag{12}$$

with α a dimensionless constant. Writing this down in components one can see that L_{kin} contains automatically a term

$$\frac{1}{\alpha}(\phi^*\phi)^{-2/3} M M^+ = \frac{1}{\alpha}(\phi^*\phi)^{-2/3}\frac{1}{4}\left[(cF^2)^2 + (cF\tilde{F})^2\right] \tag{13}$$

which hence effectively includes the needed quadratic term in $(F\tilde{F})$. Notice that in QCD we could regard $(F\tilde{F})$ as an auxiliary field only in the low energy, $N_c \to \infty$ limit in which the Lagrangian is at most quadratic in $F\tilde{F}$ and derivative could be neglected. In this SUSY theory $F\tilde{F}$ and F^2, being auxiliary fields, can appear at most quadratically and have no kinetic terms, so they can always be eliminated. We have now to add a term which produces the three anomalies of Eqs. (2) and (3). Mimicking the QCD case we can do that by adding to L_{kin} an F term

$$\mathcal{L}_{anom.} = \frac{1}{3}\left(S \log S/\mu^3 - S\right)_F + h.c. \tag{14}$$

This is the only possible structure that is scale invariant up to an anomaly. Indeed, under a chiral transformation

$$S(x,\theta) \longrightarrow e^{3i\beta} S(x, \theta e^{-i\frac{3}{2}\beta}) \tag{15}$$

(recalling that λ has chirality 3/2). Hence

$$(S)_F \equiv \int d^4x\, d^2\theta\, S(x,\theta) \longrightarrow \int d^4x\, d^2\theta'\, S(x,\theta') \qquad \theta' = \theta e^{-i3/2}$$

$$\delta\left(S \log S/\mu^3\right)_F + h.c. = 3i\beta \int d^4x\, d^2\theta'\, S(x,\theta') + h.c. \tag{16}$$

$$= 3i\beta \int d^4x\,(-M+M^+) = -3\beta C F\tilde{F}$$

Under a scale transformation

$$S(x,\theta) \longrightarrow e^{3\gamma} S(xe^{\gamma}, \theta e^{\gamma/2}) \tag{17}$$

so that

$$(S)_F \equiv \int d^4x\, d^2\theta\, S(x,\theta) \longrightarrow \int d^4x' d^2\theta'\, S(x',\theta') \qquad \theta' = \theta e^{\gamma/2},\ x' = xe^{\gamma}$$

$$\delta\left(S \log S/\mu^3\right)_F + h.c. = -3\gamma \int d^4x\, d^2\theta'\,(S(x,\theta') + h.c) \tag{18}$$

$$= -3\gamma \int d^4x\,(M+M^+) = 3\gamma C F^2$$

Finally we have checked that the $\gamma \cdot S$ anomaly is also fulfilled by performing the explicit infinitesimal superconformal transformation[9),15)]

$$\delta(S)_F = 0$$

$$\delta(S\log S/\mu^3)_F + h.c = 6c\,\bar{\xi}\chi \tag{19}$$

These are indeed the correct transformation properties following Eqs. (2) and (3). Having checked that the effective Lagrangian

$$\mathcal{L}_{eff.} = \frac{9}{\alpha}(S^*S)_D^{1/3} + \frac{1}{3}\left[(S\log S/\mu^3 - S)_F + h.c\right] \tag{20}$$

has the correct transformation properties, we can turn to the analysis of its consequences. This is done most easily by writing down L_{eff} in Eq. (20) in components. The auxiliary fields turn out to be given by:

$$(M+M^\dagger) = -cF^2 = -\frac{1}{3}\left\{2\left(\frac{\bar{\chi}_R\chi_L}{\phi} + \frac{\bar{\chi}_L\chi_R}{\phi^*}\right) - \alpha(\phi^*\phi)^{2/3}\log\left(\frac{\phi^*\phi}{\mu^6}\right)\right\}$$

$$-i(M-M^\dagger) = -cF\tilde{F} = \frac{i}{3}\left\{2\left(\frac{\bar{\chi}_R\chi_L}{\phi} - \frac{\bar{\chi}_L\chi_R}{\phi^*}\right) - \alpha(\phi^*\phi)^{2/3}\log\left(\phi^*/\phi\right)\right\} \tag{21}$$

In terms of ϕ and χ alone the effective Lagrangian Eq. (20) takes the form

$$\begin{aligned}\mathcal{L}_{eff.} = {} & \frac{1}{\alpha}(\phi^*\phi)^{-2/3}\left(\partial_\mu\phi^*\partial_\mu\phi + i\bar{\chi}\partial\!\!\!/\chi\right) \\ & - \frac{1}{3}\left(\frac{\bar{\chi}_L\chi_R}{\phi^*} + \frac{\bar{\chi}_R\chi_L}{\phi}\right) \\ & - \frac{\alpha}{9}(\phi^*\phi)^{2/3}\log\phi/\mu^3\log\phi^*/\mu^3 \\ & + \frac{2}{9}\left(\frac{\bar{\chi}_L\chi_R}{\phi^*}\log\phi^*/\mu^3 + \frac{\bar{\chi}_R\chi_L}{\phi}\log\phi/\mu^3\right) \\ & + \frac{2}{3\alpha}\frac{(\bar{\chi}\gamma_\mu\gamma_5\chi)(\phi^*\overleftrightarrow{\partial_\mu}\phi)}{(\phi^*\phi)^{5/3}}\end{aligned} \tag{22}$$

Equation (22) is quadratic in the fermionic fields. This arises from our choice of a factorized D term, Eq. (12), while a four fermi interaction $\chi_L^2\chi_R^2$ will be at most present in the general case[16].

Notice that the non-polynomial form of Eq. (14) reflects itself in Eq. (22) as non-polynomial terms in ϕ, ϕ^* but not in X, $\bar{X}$. In particular, the scalar potential takes the form

$$V = \frac{\alpha}{3} (\phi^* \phi)^{2/3} \log \phi/\mu^3 \log \phi^*/\mu^3 \tag{23}$$

with minima at $\langle\phi\rangle = \langle\phi^*\rangle = 0$ or $\langle\phi\rangle = \langle\phi^*\rangle = \mu^3$. By rescaling the ϕ and X fields in Eq. (22) ($\hat{\phi} = (3/\sqrt{\alpha})\phi^{1/3}$, $\hat{X} = (\sqrt{2}/\alpha)\phi^{-2/3}X$) so that the kinetic terms take the conventional form, one can see that only the $\langle\phi\rangle = \mu^3$ solution makes sense in agreement with our expectation. In particular, the scale of the "non-renormalizable" terms is given by μ.

Expanding the effective Lagrangian around its minimum we find that ϕ describes a scalar and a pseudoscalar massive boson and X a massive (Majorana) fermion. As expected from the fact that $V = 0$ at the minimum, these particles have the same mass:

$$m_F = m_B = \frac{1}{3}\alpha\mu \tag{24}$$

and form a Wess-Zumino scalar multiplet. Hence, in spite of $\langle\lambda\bar{\lambda}\rangle \neq 0$, SUSY is not broken (in particular, $\langle F^2\rangle = 0$). The would-be pseudoscalar Goldstone has received a mass from the anomaly (as the η' in QCD*)) but, because of SUSY, has dragged along a scalar and a fermion.

The fact that $\langle\phi\rangle \neq 0$ does not imply supersymmetry breaking is intimately related to the fact that $\bar{\lambda}\lambda$ is the lowest component of the chiral multiplet S [Eq. (11)] whose components are gauge invariant composite operators. The lowest component can never be obtained as a commutator of the supersymmetry charge with some other components. Thus the usual Goldstone type argument cannot be applied here. Only when the F term of the supermultiplet (the auxiliary field) gets non-zero vacuum expectation value, can one conclude that supersymmetry is broken. At first sight it seems that by adding linear F terms we can achieve a situation where the auxiliary field gets non-zero vacuum expectation value and supersymmetry is broken. This, however, is not the case, since in our effective Lagrangian, Eq. (14), such an F term can be swallowed by a redefinition of μ. This is just a specific manifestation of the well-known fact that the most general Wess-Zumino type model does not include a linear term in the chiral multiplet. Such a term can always be eliminated by a supersymmetry transformation which shifts the chiral multiplet by an appropriate constant[15].

*) Indeed one can also show here that a Witten type formula[14] holds, relating the mass of the pseudoscalar boson to the coefficient of $(F\tilde{F})^2$ and to the expectation value F_π of the properly normalized scalar field.

One could be worried about the consistency of our result with the large N_c behaviour of masses and couplings. This is easily fixed by noticing that with "quarks" in the adjoint representation one should have

$$\mu^3 = \langle \phi \rangle \sim \langle \bar{\lambda}\lambda \rangle \sim N_c^2 \tag{25}$$

and that the correct N_c dependence of the ϕ, χ kinetic terms demands $\alpha \sim N^{-2/3}$. Writing then $\mu^3 \equiv N_c^2\Lambda^3$ and $\alpha = \gamma N^{-2/3}$ one finds $m_B = m_F = \gamma\Lambda/3$, $\langle\hat{\phi}\rangle \sim N\Lambda$ and the couplings of the rescaled fields $g_{\chi\chi\phi} \approx 1/N$, $g_{\chi\chi\chi\phi^*\phi} \approx 1/N^2\Lambda^2$, etc., in complete agreement with large N_c counting.

Finally, let us remark that, while the structure of the F term in our effective Lagrangian Eq. (20) is uniquely determined, this is not the case for the kinetic D term. We can write other scale invariant terms. Such terms will be of the form $(S^*S)^p(S^q + S^{*q})$ with $6p + 3q = 2$. We have looked at these terms and they do not seem to qualitatively change our conclusions.

To summarize, we have argued that the pure $N = 1$ supersymmetric Yang-Mills theory leads to a spectrum of massive multiplets of composite hadrons (of which ours is expected to be the lowest one for small values of α) which become weakly interacting in the large N_c limit. Perhaps disappointingly, SUSY is not broken*) since only the lowest member of a scalar multiplet has developed a vacuum expectation value $(\bar{\lambda}\lambda)$. A similar result was obtained in the two-dimensional SUSY, CP^n model at large n[17]. A similar scenario was advocated for some four-dimensional models including matter multiplets[18]. What remains to be seen is whether other SUSY gauge theories (such as adding matter multiplets to the $N = 1$ theory or going to extended SUSYs) can lead to a different and phenomenologically more interesting behaviour.

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*) This result is in agreement with those following from the index analysis of Witten [Ref. 1)].

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