

INFRA-RED LIMIT OF THE AXIAL GAUGE GLUON PROPAGATOR
AT HIGH TEMPERATURE

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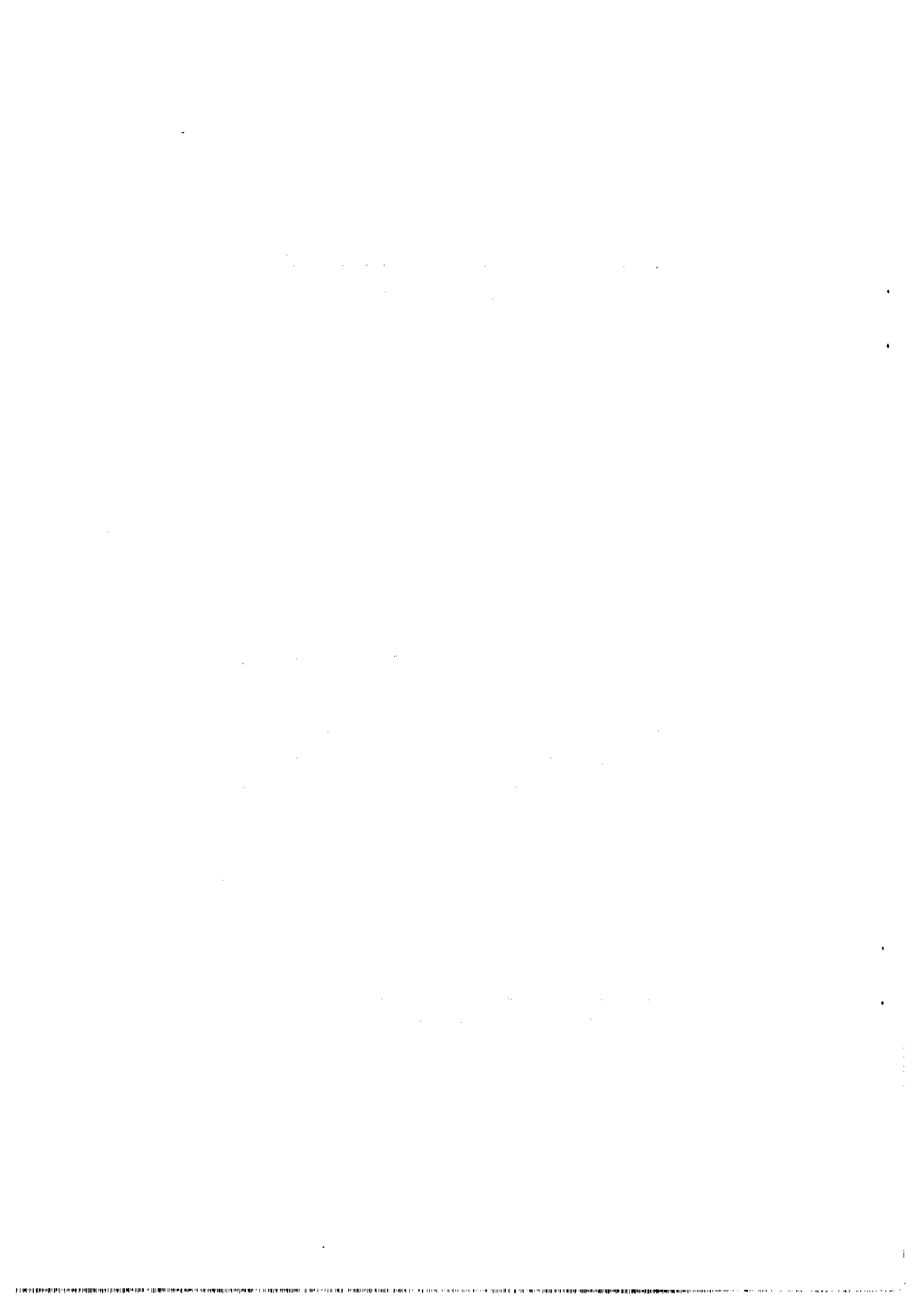
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A B S T R A C T

Based on a self-consistent solution of the Dyson equation at high temperature, we show that gluons acquire a magnetic mass-squared of order $g^3 T^2$. This implies that colour magnetic fields are screened, and that a semi-perturbative calculation of the thermodynamic potential to all orders in g is possible.

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In recent papers¹⁾⁻³⁾ it was pointed out that the thermodynamic potential of QCD at high temperature may not be perturbatively calculable beyond the fifth power of the coupling constant. This situation is almost without precedent in quantum field theory. The origin of this problem is the fact that gluons acquire an electric mass-squared of order $g^2 T^2$ whereas the magnetic mass-squared is zero to this order. If the magnetic mass-squared was strictly zero then the resulting infra-red divergence would cause the coefficient of the $g^6 T^4$ term in the thermodynamic potential to be infinite, unless unexpected cancellations occur. On the other hand if the magnetic mass-squared was of order $g^4 T^2$ then the coefficient would be finite but uncalculable since an infinite number of diagrams would contribute²⁾. In this letter we will show that this infra-red problem cures itself by generating a non-perturbative magnetic mass-squared of order $g^3 T^2$.

We study a pure SU(N) gauge theory since even massless quarks should not influence the infra-red behaviour of the gluon propagator. We work in a physical gauge, the temporal axial gauge with $A_a^0 = 0$. The most general form of the gluon propagator is the same as in the vacuum, namely

$$\mathcal{D}_{00} = \mathcal{D}_{0i} = \mathcal{D}_{i0} = 0, \quad (1)$$

$$\mathcal{D}_{ij} = \frac{1}{k^2 + F} \left(\delta_{ij} - \frac{k_i k_j}{\vec{k}^2} \right) + \frac{1}{k^2 + G} \frac{k^2}{k_0^2} \frac{k_i k_j}{\vec{k}^2}. \quad (2)$$

The metric is Euclidean. The functions to be determined are

$$\begin{aligned} F(k_0, |\vec{k}|) &= \frac{k^2}{\vec{k}^2} \Pi_{00}(k_0, |\vec{k}|), \\ G(k_0, |\vec{k}|) &= \frac{1}{2} \Pi_{\mu\mu}(k_0, |\vec{k}|) - \frac{k^2}{\vec{k}^2} \Pi_{00}(k_0, |\vec{k}|), \end{aligned} \quad (3)$$

where $\pi_{\mu\nu}$ is the gluon self-energy. The advantage of working in this gauge is that there are only two independent functions of two variables each. This contrasts with the Landau gauge at finite temperature which has two functions for the gluon propagator and another for the ghost. A general covariant gauge, such as the Feynman gauge, has an additional third function for the gluon propagator³⁾. Furthermore our gauge does not have any of the complications displayed by the Coulomb gauge^{4),5)}.

At first sight the singularity in Eq. (2) at $k_0 = 0$ would seem to be disastrous since $k_0 = 2\pi nT$ is discrete. However, the same principal value description⁶⁾ which is necessary and sufficient at $T = 0$ is valid also at $T \neq 0$ so that no new temperature-induced divergences appear. This can be seen by expressing the frequency sum in terms of contour integrals. In Minkowski space, we have⁷⁾

$$\begin{aligned} \text{P.V. } T \sum_{n=-\infty}^{\infty} \frac{1}{k_0^2} f(k_0 = 2\pi nT i) &= \text{P.V. } \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} \frac{dk_0}{k_0^2} f(k_0) \\ &+ \frac{1}{2\pi i} \int_{-i\infty+\epsilon}^{i\infty+\epsilon} \frac{dk_0}{k_0^2} \frac{f(k_0) + f(-k_0)}{\exp(k_0/T) - 1} . \end{aligned} \quad (4)$$

Finite temperature effects arise from the second integral, which is normally evaluated by encircling the poles of the integrand in the right half-plane and by calculating the residues. The pole at $k_0 = 0$ is outside the contour of integration.

We are interested in the static infra-red limit of F and G , $k_0 = 0$ and $|\vec{k}| \rightarrow 0$ ¹⁾⁻³⁾. It can be shown that the static limits of F and G are directly associated with the electric and magnetic fields, respectively. To calculate the limits of these two functions we set up the one-loop self-consistent approximation to the full Dyson equation shown in the Figure. Clearly a self-consistent equation is necessary to determine the non-perturbative effects we are looking for. The one-loop approximation should be sufficient because, at high temperature, the effective coupling constant $g(T)$ is approaching zero. We also note that, although the gluon propagator is in general gauge-variant, we do not violate gauge invariance because we are doing a self-consistent loop expansion with a fixed number of loops⁸⁾.

After some algebra, the diagrams of the Figure give us

$$\begin{aligned} m_{el}^2 \equiv F(0,0) &= g^2 N T \sum_n \int \frac{d^3k}{(2\pi)^3} \left[\frac{k^2}{k_0^2 (k^2 + F)} \right. \\ &+ \frac{2}{k^2 + G} - \frac{4k_0^2}{(k^2 + G)^2} - \left. \frac{2}{k_0^2} \left(\frac{k^2}{k^2 + F} \right)^2 \right] , \end{aligned}$$

$$m_{mag}^2 \equiv G(0,0) = g^2 N T \sum_n \int \frac{d^3 k}{(2\pi)^3} \left[\frac{k^2}{k_0^2 (k^2 + F)} + \frac{2}{k^2 + G} - \frac{2 \vec{k}^2}{(k^2 + G)^2} - \frac{1}{k_0^2} \frac{k^2}{k^2 + F} \frac{\vec{k}^2}{k^2 + G} \right]. \quad (6)$$

In the above integrands, $F = F(k_0, |\vec{k}|)$ and $G = G(k_0, |\vec{k}|)$. At zero temperature we must have $m_{el}^2 = m_{mag}^2 = 0$ as required by gauge invariance. We can perform a lowest order perturbative analysis of these equations by setting $F = G = 0$ in the integrand. The result is the same as in the covariant gauges,

$$m_{el}^2 = \frac{1}{3} g^2 N T^2, \quad (7)$$

$$m_{mag}^2 = 0. \quad (8)$$

One would at first think that corrections to these values of m_{el}^2 and m_{mag}^2 would be of order $g^4 T^2$ and would be obtained by expanding the integrands in Eqs. (5) and (6) to first order in F and G . This is not the case, however, because of the resulting divergence of the integrand at $k = 0$. To alleviate this problem and to pick out the dominant non-perturbative effect as $g \rightarrow 0$, we write:

$$\frac{1}{k^2 + F(k_0, |\vec{k}|)} = \frac{1}{k^2 + F(0,0)} + \left[\frac{1}{k^2 + F(k_0, |\vec{k}|)} - \frac{1}{k^2 + F(0,0)} \right]. \quad (9)$$

The first term on the right-hand side picks out the dominant infra-red behaviour. The second term contributes to higher order. Using this expansion and the lowest order results for $F(0,0)$ and $G(0,0)$ in the right-hand side of Eq. (5), we obtain

$$m_{el}^2 = \frac{1}{3} g^2 N T^2 - \frac{15}{16\pi} \left(\frac{1}{3} g^2 N \right)^{3/2} T^2. \quad (10)$$

The analysis of Eq. (6) is a little more subtle, however, because of the last term in the integrand. This term comes from the first diagram of the Figure, and corresponds to the coupling between transverse and longitudinal

oscillations in the plasma. For this term one cannot neglect the momentum dependence of F and G because they are different. A careful, but rather lengthy, analysis shows that to order g inclusive this term behaves as $-\vec{k}^2/k_0^2 k^2$ and not as $-\vec{k}^2/k_0^2(k^2+F(0,0))$. We then find that

$$m_{mag}^2 = \frac{3}{8\pi} \left(\frac{1}{3} g^2 N \right)^{3/2} T^2. \quad (11)$$

This is the main result of our letter.

Recently there have appeared two complementary papers^{9),10)} which calculate the magnetic mass at high temperature on the lattice using Monte Carlo techniques. The first paper finds a result consistent with $0.24 g^2 T$, although with Monte Carlo techniques it would be difficult to distinguish between that and $c g^{\frac{3}{2}} T$. The second paper makes no assumption concerning the g dependence, but only calculates at one temperature^{*}). They find $m_{mag} = (0.79 \pm 0.08) T$, whereas for $SU(2)$, Eq. (11) predicts $m_{mag} = 0.42 T$ at the appropriate value of g . This factor of two difference is not surprising because $g^2 = 2.959$ may not be small enough to neglect higher order corrections to Eq. (11).

Further details of our calculations, as well as an analysis of the beta function at finite temperature, will be presented in a larger publication.

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^{*}) The Monte Carlo results are consistent with each other in their domain of overlap.

REFERENCES

- 1) A.D. Linde - Rep.Progr.Phys. 42 (1979) 389.
- 2) A.D. Linde - Phys.Letters 96B (1980) 289.
- 3) D. Gross, R. Pisarski and L. Yaffe - Revs.Modern Phys. 53 (1981) 43.
- 4) V.N. Gribov - Lecture at the 12th Winter School, Leningrad (1977), unpublished.
- 5) N.H. Christ and T.D. Lee - Phys.Rev. D22 (1980) 939.
- 6) W. Kummer - Acta Phys.Austr. 41 (1975) 315.
- 7) J.I. Kapusta - Nucl.Phys. B148 (1979) 461.
- 8) S. Coleman and E. Weinberg - Phys.Rev. D7 (1973) 1888.
- 9) A. Billoire, G. Lazarides and Q. Shafi - Phys.Letters 103B (1981) 450.
- 10) T.A. DeGrand and D. Toussaint - UCSB TH-29 (1981).

FIGURE CAPTION

The one-loop self-consistent expression for the gluon self-energy in temporal axial gauge.

$$\Pi = -\frac{1}{2} \text{---} \text{---} \text{---} \text{---} \frac{1}{2} \text{---} \text{---} \text{---} \text{---}$$