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FLAVOUR-CHANGING NEUTRAL INTERACTIONS IN BROKEN SUPERSYMMETRIC THEORIES

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ABSTRACT

We point out that in order to ensure an efficient "super-GIM" suppression of flavour-changing neutral interactions, the supersymmetric partners of conventional fermions (squarks and sleptons) must be almost degenerate in mass. The strongest constraints on squark mass differences of $\Delta m^2_{sq}/m^2_{sq} < O(10^{-3})$ come from the K_1-K_2 mass matrix, while the non-observation of $\mu\to e\gamma$ imposes $\Delta m^2_{s\ell}/m^2_{s\ell} < O(10^{-3})$ if the supersymmetric partners of the SU(2) and SU(1) bosons have masses O(100) GeV. These results help motivate a susy gauge theory with an extra $\tilde{U}(1)$ symmetry spontaneously broken at low energy, perhaps of a non-minimal type.

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Recently there has been renewed interest[1] in simply supersymmetric (susy) gauge theories, largely because of the promise they hold for solving the hierarchy problem encountered in grand unified theories (GUTs). Unfortunately no wholly satisfactory susy GUT has yet been constructed, and one stumbling block is a satisfactory mechanism for breaking susy. Fundamental fermions and their bosonic superpartners would have equal masses if susy were manifest, so one should break the symmetry either spontaneously or explicitly. Spontaneous susy breaking requires introducing a new $\tilde{U}(1)$ gauge symmetry valid down to an energy scale of order 100 GeV and yields characteristic mass formulae[2]

$$m^2_{sf} - m^2_f = c\,\tilde{Y}_f \tag{1}$$

where $\tilde{Y}_f$ is the common $\tilde{U}(1)$ charge of a chiral fermion and its spin-zero superpartner sf, and c is a universal constant. Inventing a $\tilde{U}(1)$ symmetry which gives phenomenologically acceptable masses for the unseen spin-zero particles has proved rather tricky[3], and embedding it in an acceptable susy GUT looks even hairier. Therefore some authors[1] have resorted to explicit soft susy breaking as an unaesthetic and hopefully temporary expedient which introduces large numbers of new arbitrary parameters into the theory. It is clear from Eq. (1) that for particles in a spontaneously broken susy theory with the same $\tilde{U}(1)$ charge, such presumably as particles with identical $SU(3)\times SU(2)\times U(1)$ quantum numbers which belong to different generations,

$$m^2_{sf_i} - m^2_{sf_j} = m^2_{f_i} - m^2_{f_j} \tag{2}$$

and Cabibbo mixing in the sf sector is intimately related to that in the f sector. However, these statements do not apply to softly broken susy theories. At present few constraints on the parameters of these theories are known[3], apart from the transparent necessity that all the sf masses be larger than $O(15)$ GeV, since none of them has been seen at PETRA or PEP[4].

In this paper we compute the constraints on broken susy gauge theories imposed by the phenomenology of flavour-changing neutral interactions (FCNI). Their natural suppression[5] in the standard minimal $SU(3)\times SU(2)\times U(1)$ model is not automatically maintained in more complicated theories[6], which has sometimes proved their Nemesis[7]. FCNI are naturally suppressed in susy theories by a simple super-GIM mechanism. However, the extent of this suppression depends on the differences Δm^2_{sf} in $(\text{mass})^2$ between spin-zero superpartners (squarks and sleptons) of the quarks and leptons in different generations. We find that $\Delta m^2_{sf}/m^2_{sf}$ must be much smaller than unity both for squarks and for sleptons.

Our results are set out in the Table, where it is seen that the best constraints on the first two super-generations come from

$$\left.\begin{array}{l} \mathrm{Im}\,(K_1 - K_2) \;\Rightarrow\; \dfrac{\Delta m^2_{sq}}{m^2_{sq}} < O(10^{-3}) \\[2ex] \mu \to e\gamma \;\Rightarrow\; \dfrac{\Delta m^2_{sl}}{m^2_{sl}} < O(10^{-3}) \end{array}\right\} \qquad (3)$$

if the susy partners $\tilde{W}$, $\tilde{B}$ of the SU(2) and U(1) gauge bosons have masses O(100) GeV and the super-Cabibbo angles are not much smaller than the familiar Cabibbo angle. These results imply that the thresholds in e^+e^- annihilation for the pair production of different flavours of squark or slepton with identical charges must be almost degenerate. In particular, the su and sc (or sd and ss) thresholds may be closer together than the e^+e^- beam energy spread! The constraints (3) are suggestive that something approaching the mass relations (2) must be true even in a softly broken susy theory. Clearly a spontaneously broken susy theory (1), if it exists, would be compatible with our constraints (3) as long as the squark masses are larger than about 40 GeV. This, however, is also an upper limit on the squark masses in the simplest model[3] of spontaneous susy breaking.

Before analyzing the different FCNI constraints, we first recall a few salient features of couplings in susy gauge theories[3]. The conventional $SU(3)\times SU(2)\times U(1)$ gauge bosons $(W^{\pm}_{0}, B, g)$ are joined by Majorana spin $\frac{1}{2}$ fermions $(\tilde{W}^{\pm}_{0}, \tilde{B}, \tilde{g})$ which couple conventional quarks and leptons to their squark and slepton partners of the same handedness [e.g., left-handed doublets and right-handed singlets for the SU(2) $\tilde{W}^{\pm}_{0}$]. For simplicity, we neglect $\tilde{W}^0$-B mixing because this should not change the essential results. The trilinear superfield couplings containing the Yukawa interactions responsible for the quark and lepton masses also contribute to the squark and slepton mass matrices. However, in broken susy theories one can also have other contributions to the squark and slepton mass matrices, coming for example from soft quadratic terms in the scalar fields such as $|\phi|^2$. Therefore the diagonalizations of the mass matrices of quarks and leptons and of their scalar partners will in general be different, with the gauge fermion couplings characterized by novel super-Cabibbo-Kobayashi-Maskawa[8] unitary matrices. We will assume without justification approximately diagonal forms for these super-CKM matrices:

$$V \approx \begin{pmatrix} 1 & O(\epsilon) & O(\epsilon^2) \\ O(\epsilon) & 1 & O(\epsilon) \\ O(\epsilon^2) & O(\epsilon) & 1 \end{pmatrix} : \begin{array}{l} \epsilon = O(\sin\theta_c) \\ |\epsilon|^2 \approx O\left(\frac{1}{20}\right) \end{array} \qquad (4)$$

Since gauge couplings are generally much larger than Higgs couplings, in this first paper on the FCNI problem in broken susy theories we will not investigate effects due to the exchanges of shiggses (supersymmetric partners of the Higgses).

The first FCNI we study is the $\Delta S = 2$ term in the K_1-K_2 mass matrix, for which the most important diagrams are shown in Fig. 1. We consider first the $\tilde{W}^{\pm}$ exchange diagram and assume that the Majorana mass $m_{\tilde{W}} >> m_{sq}$. We therefore remove factors of $1/m_{\tilde{W}}$ for each fermion propagator and notice that there are super-GIM cancellations on each of the squark lines. We therefore find that the coefficient of the $\Delta S = 2$ operator to be compared with the experimental value of the real part of the K_1-K_2 mass matrix is of order

$$\frac{1}{m^2_{\tilde{W}}} \; \frac{g_2^4}{192\pi^2}\left(\frac{\Delta m^2_{sq}}{m^2_{sq}}\right)^2 \epsilon^2 < \frac{1}{2} \times 10^{-12}\ \mathrm{GeV}^{-2} \tag{5}$$

Using the super-CKM assumption (4) in Eq. (5) we find

$$\frac{1}{m^2_{\tilde{W}}} \left(\frac{\Delta m^2_{sq}}{m^2_{sq}}\right)^2 < O(10^{-7})\,\mathrm{GeV}^{-2} \tag{6}$$

implying if $m_{\tilde{W}} \approx 100$ GeV that

$$\mathrm{Re}\,(K_1 - K_2) \Rightarrow \frac{\Delta m^2_{sq}}{m^2_{sq}} < O\left(\frac{1}{30}\right) \tag{7}$$

justifying *a posteriori* the assumption $\Delta m^2_{sq}/m^2_{sq} << 1$. If one further constrains the $\Delta S = 2$ operator by the imaginary part of the K_1-K_2 mass matrix - and there is no reason why it should be real relative to the presumably dominant u and c quark box diagrams - one can strengthen the inequality (6) by three orders of magnitude and replace Eq. (7) by

$$\mathrm{Im}\,(K_1 - K_2) \Rightarrow \frac{\Delta m^2_{sq}}{m^2_{sq}} < O(10^{-3}) \tag{8}$$

The constraints (6) and (7) apply to the u and c squarks, but we could obtain similar constraints for the d and s squarks by considering the $\tilde{W}^0$, $\tilde{B}^0$ and g exchange diagrams in Fig. 1. It may be that the $\tilde{B}^0$ or $\tilde{g}$ has a mass $<< m_{sq}$: in this case one can derive an inequality like Eq. (6) with

$$\frac{1}{m^2_{\tilde{W}}} \rightarrow \frac{1}{m^2_{sq}} \tag{9}$$

Since the $\tilde{g}$ diagram has strong gauge couplings, and m_{sq} may well be considerably less than $m_{\tilde{W}}$, the replacement (9) promises to provide a tighter constraint than (7) or (8), but we will be conservative and stick to these already interesting bounds. Henceforward we will mainly consider diagrams in which the Majorana gauge fermion masses are larger than the squark and slepton masses. Consideration of the $\Delta C = 2$ operator and the upper bound[9] on D^0-$\bar{D}^0$ mixing would give a bound less stringent than (7) by a factor of 2 or 3.

Next we consider the leptonic decay $K^0_L \to \mu^+\mu^-$ and the diagrams of Fig. 2. The $\tilde{W}^\pm$ box diagram of Fig. 2a is similar to that of Fig. 1 except that now we can only expect one super-GIM cancellation, on the squark line. Assuming that $m_{s\ell} \ll m_{sq}$ in order to compare the $K^0_L \to \mu^+\mu^-$ rate given by this diagram to the conventional $K^+ \to \mu^+\nu_\mu$ decay rate, we get

$$\frac{\Gamma(K^0_L \to \mu^+\mu^-)}{\Gamma(K^+ \to \mu^+\nu_\mu)} = 2\left[\frac{g_2^2}{64\pi^2}\,\frac{m_W^2}{m_{\tilde{W}}^2}\,\frac{\Delta m_{sq}^2}{m_{sq}^2}\right]^2 \frac{|\epsilon|^2}{\sin^2\theta_c} \tag{10}$$

Demanding that this be less than the experimental ratio[10] we obtain

$$\frac{1}{m_{\tilde{W}}^2}\left(\frac{\Delta m_{sq}^2}{m_{sq}^2}\right) < O(10^{-5})\,\mathrm{GeV}^{-2} \tag{11}$$

which is rather less stringent than (7) or (8) if we assume $m_{\tilde{W}} \approx 100$ GeV. Consideration of the vertex correction diagram in Fig. 2b would give us an analogous bound on the combination

$$\frac{1}{m_{\tilde{W}}^2}\left(\frac{\Delta m_{sq}^2}{m_Z^2}\right) \tag{12}$$

In the hope that $m_{sq} < O(m_Z)$, we fancy (11) to be the more stringent bound, which is in any case not competitive with (7) or (8) for likely values of $m_{\tilde{W}}$.

The leptonic decay $K^0_L \to \mu e$ does not give us any useful new information. The only diagrams contributing are those of Fig. 2a, but there will be super-GIM cancellations on both the squark and slepton legs. Since the experimental upper limit[11] on the $K^0_L \to \mu e$ decay rate is comparable to the observed $K^0_L \to \mu^+\mu^-$ decay rate, one can only get a bound on the product

$$\frac{1}{m_{\tilde{W}}^2}\left(\frac{\Delta m_{sq}^2}{m_{sq}^2}\right)\left(\frac{\Delta m_{s\ell}^2}{m_{s\ell}^2}\right) \times \epsilon^2 \tag{13}$$

which is comparable to the inequality (11) and hence gives us neither a useful constraint on $\Delta m^2_{s\ell}$ nor useful new information on Δm^2_{sq}.

We now turn to FCNI which do provide useful constraints on $\Delta m^2_{s\ell}$, starting with anomalous muon conversion $\mu N \to eN$. As shown in Fig. 3a, there are box diagram contributions to this process which suffer a super-GIM cancellation on the slepton line. They give a $(\bar{\mu}_L \gamma_\mu e_L)(\bar{p}\gamma_\mu p, \bar{n}\gamma_\mu n)$ interaction with coefficient of order of magnitude

$$\frac{1}{m^2_{\tilde{W}}} \left(\frac{\Delta m^2_{s\ell}}{m^2_{s\ell}}\right) \frac{g_2^4}{256\pi^2} \times \epsilon^2 \tag{14}$$

Using the standard calculations[12] of $\mu N \to eN$ capture and the experimental upper limit of 1.5×10^{-10} relative to the dominant $\mu N \to \nu_\mu N$, we deduce

$$\frac{1}{m^2_{\tilde{W}}} \left(\frac{\Delta m^2_{s\ell}}{m^2_{s\ell}}\right) < O(10^{-5})\,\mathrm{GeV}^{-2} \tag{15}$$

There is also a contribution to $\mu N \to eN$ coming from the helicity-conserving $\bar{\mu}e\gamma$ vertex shown in Fig. 3b. This contribution is comparable in magnitude to the box diagram, and so does not affect the order of magnitude of the deduced bound (15).

Coming finally to purely leptonic processes, we find that the decay $\mu \to e\bar{e}e$ can proceed via a box diagram like those discussed previously for other processes. However, the experimental constraint[13] $\Gamma(\mu\to e\bar{e}e)/\Gamma(\mu\to e\bar{\nu}\nu) < 1.9\times10^{-9}$ gives us a bound on $\Delta m^2_{s\ell}$ which is somewhat less stringent than (15):

$$\frac{1}{m^2_{\tilde{W}}} \left(\frac{\Delta m^2_{s\ell}}{m^2_{s\ell}}\right) < O(10^{-4})\,\mathrm{GeV}^{-2} \tag{16}$$

which is not very useful if $m_{\tilde{W}} \gtrsim O(100)$ GeV as we suspect.

As a final example of a FCNI we consider the classic muon-number violating decay $\mu \to e\gamma$. Since this is mediated by a helicity-flip operator, the $\tilde{W}^{\pm,0}$ cannot contribute at the one-loop level as they only couple to left-handed fermions (see Fig. 3b). Diagrams involving the $\tilde{B}^0$ have an internal charged slepton line to which the photon can couple, but the photon cannot of course couple to the $\tilde{B}^0$. The remaining diagram is very analogous to that discussed by Fayet[3] in his analysis of the anomalous magnetic moment of the muon. We conclude from the experimental upper limit of 1.9×10^{-10} on $B(\mu\to e\gamma)$ that if $m_{\tilde{B}^0} \ll m_{s\ell}$

$$\frac{1}{m_{s\ell}^2}\left(\frac{\Delta m_{s\ell}^2}{m_{s\ell}^2}\right) < O(10^{-7})\ \mathrm{GeV}^{-2} \tag{17}$$

while a similar limit with one power of $m_{s\ell}^{-2}$ replaced by $m_{\tilde{B}^0}^{-2}$ applies if $m_{\tilde{B}^0} >> m_{s\ell}$.

It is apparent from the above analysis that our star constraint on $\Delta m_{sq}^2/m_{sq}^2$ comes from the imaginary part of the K_1-K_2 mass matrix, while the best constraint on $\Delta m_{s\ell}^2/m_{s\ell}^2$ comes from the non-observation of $\mu \to e\gamma$. Since leptons have a knack of acquiring masses considerably less than their quark partners, one might speculate that the same could be true of sleptons: $m_{s\ell}^2 << m_{sq}^2$? In this case there is a possibility that $\Delta m_{s\ell}^2/m_{s\ell}^2$ might be larger than $\Delta m_{sq}^2/m_{sq}^2$, and it seems interesting to push the experimental search for $\mu \to e\gamma$ as far as possible.

We have various constraints on both the $I = \pm\frac{1}{2}$ members of the first two generations of squark and slepton doublets. However, we do not have any very useful constraint on the mass differences with third generation squarks and sleptons, nor can we prove that there should be approximate degeneracy between, e.g., $I = \pm\frac{1}{2}$ squarks. (Should $m_{su} \approx m_{sd}$?) However, the squark constraint (8) suggests that the thresholds for producing different charge $\frac{2}{3}$ squarks (or different charge $-\frac{1}{3}$ squarks) in e^+e^- annihilation should be almost degenerate, which may well have interesting phenomenological consequences[14].

The proliferation of mass and mixing parameters in softly broken susy gauge theories is not very attractive. This and the degeneracy between different squarks enforced by FCNI impel us in the direction of spontaneously broken susy[2),3)] with its mass relations (2). However, in the simplest versions[3)] of such a theory there is an upper limit on the masses of the supersymmetric partners of the conventional fermions of order 40 GeV. It may be difficult to reconcile this constraint with the second line of the Table: either some squarks have masses very close to this limit or else a more flexible method of spontaneous susy breaking must be used.

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Flavour-changing transition	Constraint	
	for arbitrary $m_{\tilde{W}}$	if $M_{\tilde{W}}$ or $m_{\tilde{B}}$ = 100 GeV
$Re(K_1-K_2)$	$\frac{1}{m^2_{\tilde{W}}}\left(\frac{\Delta m^2_{sq}}{m^2_{sq}}\right)^2 < O(10^{-7})\ \mathrm{GeV}^{-2}$	$\left(\frac{\Delta m^2_{sq}}{m^2_{sq}}\right) < O(\frac{1}{30})$
$Im(K_1-K_2)$	$\frac{1}{m^2_{\tilde{W}}}\left(\frac{\Delta m^2_{sq}}{m^2_{sq}}\right)^2 < O(10^{-10})\ \mathrm{GeV}^{-2}$	$\left(\frac{\Delta m^2_{sq}}{m^2_{sq}}\right) < O(10^{-3})$
(D_1-D_2)	$\frac{1}{m^2_{\tilde{W}}}\left(\frac{\Delta m^2_{sq}}{m^2_{sq}}\right)^2 < O(10^{-6})\ \mathrm{GeV}^{-2}$	$\left(\frac{\Delta m^2_{sq}}{m^2_{sq}}\right) < O(\frac{1}{10})$
$K^0_L \to \mu\mu$	$\frac{1}{m^2_{\tilde{W}}}\left(\frac{\Delta m^2_{sq}}{m^2_{sq}}\right) < O(10^{-5})\ \mathrm{GeV}^{-2}$	$\left(\frac{\Delta m^2_{sq}}{m^2_{sq}}\right) < O(\frac{1}{10})$
$K^0_L \to \mu e$	no useful constraint	
$\mu N \to eN$	$\frac{1}{m^2_{\tilde{W}}}\left(\frac{\Delta m^2_{s\ell}}{m^2_{s\ell}}\right) < O(10^{-5})\ \mathrm{GeV}^{-2}$	$\left(\frac{\Delta m^2_{s\ell}}{m^2_{s\ell}}\right) < O(\frac{1}{10})$
$\mu \to e\bar{e}e$	$\frac{1}{m^2_{\tilde{W}}}\left(\frac{\Delta m^2_{s\ell}}{m^2_{s\ell}}\right) < O(10^{-4})\ \mathrm{GeV}^{-2}$	$\left(\frac{\Delta m^2_{s\ell}}{m^2_{s\ell}}\right) < O(1)$
$\mu \to e\gamma$	$\frac{1}{m^2_{\tilde{B}}}\left(\frac{\Delta m^2_{s\ell}}{m^2_{s\ell}}\right) < O(10^{-7})\ \mathrm{GeV}^{-2}$	$\left(\frac{\Delta m^2_{s\ell}}{m^2_{s\ell}}\right) < O(10^{-3})$

<u>TABLE</u> - FCNI constraints on broken susy theories.

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FIGURE CAPTIONS

Figure 1 Box diagrams contributing to K_1-K_2 mixing in broken susy gauge theories. The symbols $\otimes$ denote Majorana fermion mass terms.

Figure 2 Diagrams contributing to $K_L^0 \to \mu\mu$ and μe in broken susy gauge theories.

Figure 3 (a) A box diagram contribution to the $(\bar{\mu}e)(\bar{q}q)$ vertex, and
(b) a contribution to the $\mu e\gamma$ vertex.

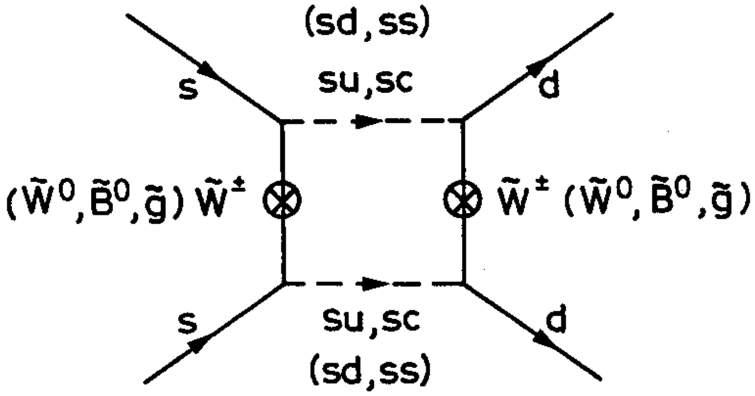

FIG. 1

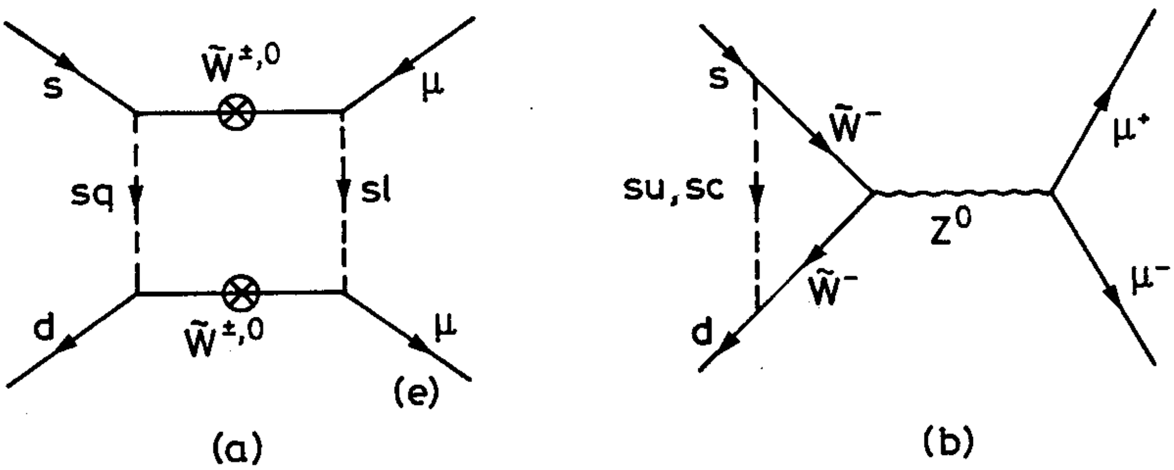

FIG. 2

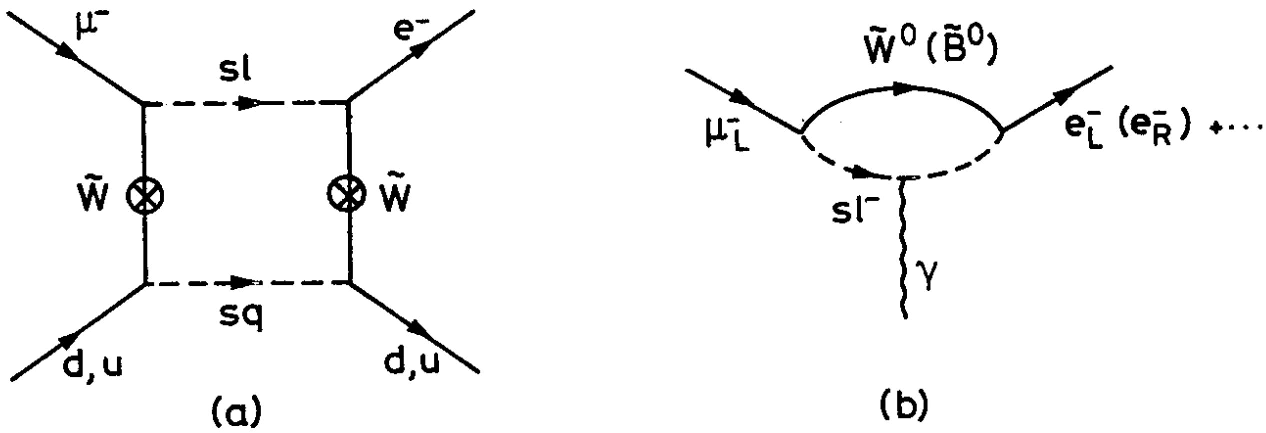

FIG. 3