Ref.TH.3156-CERN

ASYMPTOTIC LEVEL DENSITY OF CONSTRAINED AND INTERACTING FIELDS

J. Kapusta *)
CERN, Geneva **)

and

Physics Department
McGill University
Montreal, Canada

ABSTRACT

An expression for the asymptotic level density of interacting quantum fields is formulated in terms of the functional integral. The energy, the momentum, the angular momentum, and any conserved internal quantum numbers, may be given definitely prescribed values. Of particular interest is the asymptotic mass spectrum of the bag model. It is shown that interactions, treated perturbatively, do not change the form of the asymptotic mass spectrum. Specifying the values of internal quantum numbers, such as baryon number, leads to results which are consistent with the statistical bootstrap model.

---

*) NATO - NSF Postdoctoral Fellow.

**) Present address.

Ref.TH.3156-CERN
14 September 1981

1. INTRODUCTION

In the original paper on the MIT bag model[1] the remarkable observation was made that the asymptotic mass spectrum grows exponentially with the mass of the hadronic bag. This is a consequence of a particular feature of the model, that the mass of a bag is proportional to its time-averaged volume. When energy is pumped into the bag, the bag boils away surrounding vacuum, making new partons rather than raising the average kinetic energy of the partons in the bag. This exponential rise and the associated limiting temperature of partons in the bag are reminiscent of the statistical bootstrap model of hadrons[2]. Indeed, the inverse slope of the exponential in the bag model, T_0, is of the order of $B^{1/4}$, B being the bag constant. With $B \sim (150\text{ MeV})^4$, T_0 comes out very close in value to Hagedorn's limiting temperature of 160 MeV. Subsequently it was argued that this is not a limiting temperature after all, rather it is the temperature of a phase transition[3], specifically from a gas of hadronic resonances to a gas of free quarks and gluons[4]. Recently the pre-exponential factor was calculated for the bag model with the result[5]

$$\rho(m) \sim c\, m^{-3} \exp(m/T_0). \tag{1.1}$$

However, it was proven, at least in the context of the bag model, that T_0 cannot be the temperature of a phase transition from a gas of resonances to a gas of free partons; that must happen at a higher temperature, $T_c > T_0$.

The result (1.1) was derived in the context of the micro-canonical ensemble (with strict momentum conservation) for massless, non-interacting partons with arbitrary spin and statistics. It is just a sum over n-body phase space, from $n = 2$ to ∞, with specified total energy and momentum[6]. The effect of the latter constraints is crucial; without them one would obtain a different (incorrect) pre-exponential factor.

The question naturally arises as to how to include interactions among the partons. In fact, it is easy to write some formulae. The level density is defined as

$$\rho(E) = \sum_{\nu}{}' \delta(E - E_\nu). \tag{1.2}$$

The prime on the sum indicates that the sum is restricted to those states which, for example, have zero total momentum and Q units of electric charge. The restriction depends upon the system whose level density one is interested in. The restriction on the sum may be removed if Kronecker deltas are used,

$$\rho(E) = \sum_{\nu} \delta(E - E_\nu)\, \delta_{\vec{p}_\nu, \vec{0}}\, \delta_{Q_\nu, Q}\,. \tag{1.3}$$

In this paper we shall be interested in placing fields with energy E in a volume V, and calculating the level density in the limit $E \to \infty$, $V \to \infty$, E/V finite. In this case (1.3) can be written as

$$\rho(E, V) = \frac{(2\pi)^3}{V}\, \mathrm{Tr}\ \delta(H - E)\, \delta(\vec{P})\, \delta_{\hat{Q}, Q}\,. \tag{1.4}$$

Here H is the Hamiltonian operator, $\vec{P}$ is the momentum operator, and $\hat{Q}$ is the charge operator. In general, we may have one Kronecker delta for each of the conserved, mutually commuting, and additive charge operators of the system under consideration. Angular momentum and non-Abelian symmetries require a special discussion but, as shown later, they can be handled with ease with the methods used in this paper.

The purpose of this paper is to demonstrate that (1.4) can be evaluated using the formalism and approximation techniques of relativistic quantum field theory. We shall mostly be interested in the applications to hadronic physics and the bag model. However, it should be of wider application to classical or quantum fields when the effects of constraints are important.

The plan of the paper is as follows. In Section 2 we consider a self-interacting neutral scalar field. Using field theory techniques we write the level density as the Laplace transform of a functional integral over the scalar field. A perturbative evaluation leads to a bag model mass spectrum of the form of (1.1) where now c and T_0 depend on the quartic coupling constant. In Section 3 we consider the effects of constraints on the internal symmetry quantum numbers of massless non-interacting fermions. We show that the bag model asymptotic mass spectrum, with fixed baryon number, has precisely the form predicted by the statistical bootstrap model. We also consider the effect of introducing an internal $SU(2)$ symmetry and restricting the states to $SU(2)$ singlets. We close with some comments in Section 4.

2. INTERACTIONS

For simplicity, consider a theory with no conserved charges, or where we sum over states of all charge. Then (1.4) can be written as a Laplace transform of a partition function Z,

$$\rho(E) = \frac{1}{(2\pi i)^4} \int_{-i\infty+\epsilon}^{i\infty+\epsilon} d\beta \, d^3\lambda \, \exp(\beta E) \, Z(\beta, \vec{\lambda}),$$

$$Z(\beta, \vec{\lambda}) = \mathrm{Tr} \, \exp(-\beta H + \vec{\lambda} \cdot \vec{P}). \tag{2.1}$$

For example, with a Lagrangian density of

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - U(\phi), \tag{2.2}$$

we know that the Hamiltonian density and momentum density are given by

$$\mathcal{H} = \frac{1}{2} \pi^2 + \frac{1}{2} (\vec{\nabla}\phi)^2 + U(\phi),$$

$$\vec{p} = \pi \vec{\nabla} \phi. \tag{2.3}$$

Here π is the field momentum conjugate to ϕ. First we make the convenient change of variable $\vec{\lambda} \to \beta\vec{\lambda}$. Then we write Z in path integral form[7)-9)]

$$Z = N \int_{\text{periodic}} [d\phi] \int [d\pi] \, \exp\left\{ \int_0^\beta d\tau \int_V d^3x \, \cdot \right.$$

$$\left. \cdot \left[\pi \left(i \frac{\partial \phi}{\partial \tau} + \vec{\lambda} \cdot \vec{\nabla} \phi \right) - \frac{1}{2} \pi^2 - \frac{1}{2} (\vec{\nabla}\phi)^2 - U(\phi) \right] \right\}. \tag{2.4}$$

N is an (infinite) normalization constant, the integration in ϕ is restricted to paths periodic in τ of period β, and the integration in π is unrestricted.

If we were calculating the grand canonical partition function $\vec{\lambda}$ would be set to zero and β would be the inverse temperature. The fact that we will eventually do a contour integral over (possibly) imaginary values of β is no problem; we merely analytically continue. Since the π integration in (2.4) is Gaussian it can be performed directly to yield[9]

$$Z = N'(\beta) \int_{\text{periodic}} [d\phi] \exp\left\{ \int_0^\beta d\tau \int_V d^3x \left[-\frac{1}{2}\left(\frac{\partial\phi}{\partial\tau} - i\vec{\lambda}\cdot\vec{\nabla}\phi \right)^2 - \frac{1}{2}(\vec{\nabla}\phi)^2 - U(\phi) \right] \right\}. \tag{2.5}$$

Here $N'(\beta)$ is a new, β dependent, normalization whose β dependence has been computed[9].

The functional integral can be done exactly only when $U(\phi)$ is no more than quadratic in ϕ, i.e., for free fields. Therefore, one can develop the usual perturbation expansion in powers of the interaction.

$$\begin{aligned} \ln Z &= \ln Z_0 + \ln Z_I, \\ \ln Z_0 &= \ln N'(\beta) + \ln\left\{ \int [d\phi] \exp(S_0) \right\}, \\ \ln Z_I &= \ln\left\{ 1 + \frac{\sum_{n=1}^{\infty} \frac{1}{n!} \int [d\phi] \exp(S_0)\, S_I^n}{\int [d\phi] \exp(S_0)} \right\}. \end{aligned} \tag{2.6}$$

Here S_0 is that part of the action which is quadratic in the fields, S_I is the rest. Since we are dealing with relativistic interacting quantum fields we must also deal with renormalization. As in the case of field theories at finite temperature[10),11)] we do not expect to find divergences in the system not already present in the vacuum ($\beta \to \infty$, $\vec{\lambda} \to 0$). Therefore, the usual vacuum renormalization schemes should be sufficient to regulate the theory. We have not attempted to prove this in general. We have done a two loop calculation, presented below, in which no β or $\vec{\lambda}$ dependent infinities arose. We can also present the following naive argument. Finite β is like finite temperature. Finite temperature does not affect ultra-violet divergences, and often softens infra-red divergences due to the screening effect. Finite $\vec{\lambda}$ is due to the momentum constraint. Constraining the total momentum of the state should not introduce any new infinities; at best it may even serve to soften them.

As an example, consider the potential $U(\phi) = g^2\phi^4$. The $\ln Z_0$ can be evaluated in the usual way[9),12)] ($\omega_n = 2\pi n/\beta$).

$$\begin{aligned}
\ln Z_0 &= \ln N'(\beta) - \tfrac{1}{2} \ln \det\left[(\omega_n - i\vec{\lambda}\cdot\vec{k})^2 + \vec{k}^2\right] \\
&= \ln N'(\beta) - \tfrac{1}{2} \operatorname{Tr} \ln\left[(\omega_n - i\vec{\lambda}\cdot\vec{k})^2 + \vec{k}^2\right] \\
&= -\tfrac{1}{2} V \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} \left\{ \ln \beta^2 + \ln\left[(\omega_n - i\vec{\lambda}\cdot\vec{k})^2 + \vec{k}^2\right]\right\} \qquad (2.7) \\
&= \frac{\pi^2 V}{90\,\beta^3 (1-\vec{\lambda}^2)^2} .
\end{aligned}$$

In (2.7) we have thrown away an infinite constant which is independent of β and $\vec{\lambda}$, as well as the infinite zero point energy of the field. If we set $\vec{\lambda} = 0$ we recognize $(\ln Z_0)/\beta V$ as the pressure of a non-interacting gas of massless bosons. Notice that (2.7) was calculated assuming that $V \to \infty$. Surface and higher order corrections have not been included.

The order g^2 contribution to $\ln Z_I$ is the figure-eight diagram shown in Fig. 1. There are the usual finite temperature Feynman rules with the only change being a $\vec{\lambda}$ dependent propagator.

$$\ln Z_2 = -3g^2 \beta V \left[\frac{1}{\beta}\sum_n \int \frac{d^3k}{(2\pi)^3} \frac{1}{(\omega_n - i\vec{\lambda}\cdot\vec{k})^2 + \vec{k}^2}\right]^2 + \text{subtractions.} \qquad (2.8)$$

The subtractions [see, for example, Ref. 11)] just cancel the infinities in the quantity in brackets so that

$$\ln Z_2 = -\frac{g^2 V}{48\,\beta^3 (1-\vec{\lambda}^2)^2} . \tag{2.9}$$

Hence,

$$\rho(E) = \frac{(2\pi)^3}{V} \frac{1}{(2\pi i)^4} \int_{-i\infty+\epsilon}^{i\infty+\epsilon} d\beta\, d^3\lambda \, \exp(f), \tag{2.10}$$

where we have scaled $\vec{\lambda} \to \vec{\lambda}/\beta$ and

$$f = \beta E + \ln Z \approx \beta E + \frac{\beta V}{(\beta^2 - \vec{\lambda}^2)^2} \left(\frac{\pi^2}{90} - \frac{g^2}{48}\right). \tag{2.11}$$

With $g^2 = 0$ we recognize (2.10) and (2.11) as the same expression derived on the basis of single particle phase space[6),5)].

To evaluate (2.10) asymptotically, i.e., as $V \to \infty$, $E \to \infty$, E/V fixed, we use the saddle point, or stationary phase, approximation. Then

$$\rho(E) \sim \frac{(2\pi)^3}{V} \frac{1}{(2\pi)^2} \frac{\exp(f)}{|\det|^{1/2}} , \tag{2.12}$$

which is evaluated at the saddle point determined by the equations

$$\frac{\partial f}{\partial \beta} = 0, \qquad \vec{\nabla}_\lambda f = 0 . \tag{2.13}$$

These give, as the saddle point,

$$\vec{\lambda} = 0,$$
$$\beta^4 = \frac{3V}{E}\left(\frac{\pi^2}{90} - \frac{g^2}{48}\right). \qquad (2.14)$$

The det is the 4×4 determinant of the second order derivatives of f with respect to β and λ_i. It factorizes into $(\partial^2 f/\partial\beta^2)(\partial^2 f/\partial\lambda_3^2)^3$ because of rotational invariance. Putting it all together results in

$$\rho(E) \sim \frac{\pi}{8\cdot 3^{1/2}} \frac{1}{\left(\frac{\pi^2}{90} - \frac{g^2}{48}\right)^2} \frac{\beta^{10}}{V^3} \exp\left(\tfrac{4}{3}\beta E\right). \qquad (2.15)$$

where β is given by (2.14).

In the bag model the total energy is not E but is $m = E + BV$, and the equilibrium value of the volume is $V = m/4B$ [1)]. Using these relationships among E, m and V in (2.15) gives us the asymptotic level density for hadrons in the bag model.

$$\rho(m) \underset{m\to\infty}{\sim} \frac{4\pi^3}{45\cdot 3^{1/2}}\left(1 - \frac{15}{8}\frac{g^2}{\pi^2}\right)\beta^{-2} m^{-3} \exp(\beta m),$$
$$\beta^4 = \frac{1}{B}\left(\frac{\pi^2}{90} - \frac{g^2}{48}\right). \qquad (2.16)$$

Notice that this has the same form as in the case of no interactions, namely $cm^{-3}\exp(\beta m)$. The difference now is that c and β depend on g^2. This result is quite clear. If we compute a level density, and then turn on the interactions perturbatively, the positioning of the energy levels will change a bit but the total number of levels will not. We expect this to remain true to all orders of perturbation theory, although we have not proven it. We suspect that, if $P_{th}(\beta)$ is the thermal pressure of the bosons calculated in the grand canonical ensemble, then β will be determined by the balance of pressure[1),5)]

$$B = P_{th}(\beta),$$

and

$$\rho(m) \sim \frac{4\pi^3}{45 \cdot 3^{1/2}} \left[\frac{90}{\pi^2} \beta^4 P_{th}(\beta) \right] \beta^{-2} m^{-3} \exp(\beta m).$$

If we choose $U(\phi) = \frac{1}{2} M^2 \phi^2$ then we have a non-interacting but massive boson. The Z can be evaluated exactly. The result for the bag model is

$$\rho(m) \sim 128\pi \left[\frac{P^4}{\frac{\partial^2}{\partial\beta^2}(\beta P) \left(\frac{\partial P}{\partial \beta}\right)^3} \right]^{1/2} P\, m^{-3} \exp(\beta m), \tag{2.17}$$

where

$$P = \int \frac{d^3k}{(2\pi)^3} \frac{k^2}{3\omega} \frac{1}{\exp(\beta\omega) - 1},$$

$$\omega = (k^2 + M^2)^{1/2}. \tag{2.18}$$

The saddle point is found by setting the thermal pressure of bosons equal to the bag pressure, $P(\beta,M) = B$. Thus, the introduction of an intrinsic energy scale, namely the boson mass M, does not change the asymptotic form of the mass spectrum.

3. INTERNAL SYMMETRIES

Having established that perturbative interactions do not change the analytic form of the asymptotic level density, let us consider what happens when we constrain the internal quantum numbers of the states. For simplicity, we shall consider only massless, non-interacting fermion fields. We could work directly in terms of the single particle phase space[6] for these examples. However, the derivations and calculations are simpler in terms of the functional integral method, especially if we were interested in states of fixed angular momentum. First we will consider a global $U(1)$ "baryon" symmetry and then a global $SU(2)$ "colour" symmetry.

With a Lagrangian density of

$$\mathcal{L} = \overline{\Psi}\, i\, \not{\partial}\, \Psi, \tag{3.1}$$

and a conserved baryon current of

$$J^{\mu} = \overline{\Psi}\, \gamma^{\mu}\, \Psi, \tag{3.2}$$

we have the level density given by (1.4), where the charge operator is

$$\hat{Q} = \hat{b} = \int_V d^3x\, \overline{\Psi}\, \gamma^0\, \Psi, \tag{3.3}$$

and where the total baryon number of the state is $Q = b$. Writing the energy, momentum and baryon number constraints in integral representation, and writing the trace as a path integral over anti-periodic Dirac fields, we arrive at

$$\rho(E, b) = \frac{(2\pi)^3}{V} \frac{1}{(2\pi i)^4} \frac{1}{2\pi} \int_{-i\infty+\epsilon}^{i\infty+\epsilon} d\beta\, d^3\lambda\, \beta^4 \exp(\beta E) \cdot$$

$$\cdot \int_{-\pi/\beta}^{\pi/\beta} d\mu\, \exp(-i\beta\mu b)\, Z,$$

$$Z = N \int [d\overline{\Psi}][d\Psi] \exp\left\{ \int_0^{\beta} d\tau \int_V d^3x\, \overline{\Psi} \left[i\, \not{\partial} + \right.\right. \tag{3.4}$$

$$\left.\left. + i \gamma^0 \vec{\lambda}\cdot\vec{\nabla} + i \mu \gamma^0 \right] \Psi \right\}.$$

Notice that μ is like an imaginary chemical potential which ranges from $-\pi/\beta$ to π/β because of the discreteness of baryon number. Note also we have scaled $\vec{\lambda} \to \beta\vec{\lambda}$ and $\mu \to \beta\mu$ for convenience.

The partition function, being quadratic in the fields, can now be evaluated in the usual way

$$\ln Z = -4 \ln N'(\beta) + \ln \det\left[\gamma^0\left(i\partial_0 + i\vec{\lambda}\cdot\vec{\nabla} + i\mu\right) + i\vec{\gamma}\cdot\vec{\nabla}\right]$$

$$= -4 \ln N'(\beta) + \mathrm{Tr} \ln\left[\gamma^0\left(-iE_n - \vec{\lambda}\cdot\vec{k} + i\mu\right) - \vec{\gamma}\cdot\vec{k}\right]. \tag{3.5}$$

Here $E_n = (2n + 1)\pi/\beta$. The -4 appears above because for fermions there are four independent degrees of freedom and a minus sign for anti-periodicity. Carrying out the trace and dropping the zero point energy and an infinite constant, we find

$$\ln Z = \frac{V}{12\pi^2} \frac{1}{\beta^3(1-\vec{\lambda}^2)^2}\left[\beta^4\mu^4 - 2\pi^2\beta^2\mu^2 + \frac{7}{15}\pi^4\right]. \tag{3.6}$$

If we set $\vec{\lambda} = 0$ and let $\mu \to i\mu$ we would recognize $(\ln Z)/\beta V$ as the pressure of a gas of massless non-interacting fermions. Making the change of variables $\vec{\lambda} \to \vec{\lambda}/\beta$ and $\mu \to -i\mu/\beta$ we have

$$\rho(E, b) = \frac{(2\pi)^3}{V} \frac{1}{(2\pi i)^5} \int_{-i\infty+\epsilon}^{i\infty+\epsilon} d\beta\, d^3\lambda \int_{-i\pi}^{i\pi} d\mu \; \exp(f),$$

$$f = \beta E - \mu b + \frac{V}{12\pi^2} \frac{\beta}{(\beta^2-\vec{\lambda}^2)^2}\left[\mu^4 + 2\pi^2\mu^2 + \frac{7}{15}\pi^4\right]. \tag{3.7}$$

To evaluate (3.7) asymptotically we use the saddle point approximation. The equations which determine the saddle point are

$$\vec{\lambda} = 0,$$

$$\frac{E}{V} = \frac{1}{4\pi^2\beta^4}\left[\mu^4 + 2\pi^2\mu^2 + \frac{7}{15}\pi^4\right], \tag{3.8}$$

$$\frac{b}{V} = \frac{1}{3\pi^2\beta^3}\left[\mu^2 + \pi^2\right]\mu.$$

In fact, (3.8) is just the usual expression for the energy density and baryon density in terms of the "temperature" β^{-1} and "chemical potential" $\beta\mu$. Unfortunately, we cannot solve for $\mu(E/V, b/V)$ and $\beta(E/V, b/V)$ in closed form. Therefore, we restrict our attention to small values of b/V and determine the saddle point approximately as

$$\beta^4 \approx \frac{7\pi^2}{60}\frac{E}{V},$$

$$\mu \approx 3\beta^3\frac{b}{V}. \tag{3.9}$$

Evaluating (3.7) in the context of the bag model $(m = 4BV = E + BV)$ gives

$$\rho(m, b) \sim \frac{4\pi^2}{27}\left(\frac{7\pi}{10}\right)^{3/2}\beta^{-5/2}m^{-7/2}\exp\left(\beta m - \frac{7\pi^2}{30}\frac{b^2}{\beta m}\right). \tag{3.10}$$

We see that the mass spectrum is cut off with a Gaussian in the baryon number. Amazingly this has precisely the same analytic form as found in the statistical bootstrap model[13)]! Summing over all baryon numbers gives back the pre-exponential power of m^{-3}, as well as the correct absolute normalization[5)].

Next, let us consider what happens when our massless quarks obey a global SU(2) "colour" symmetry. We will require that the hadronic states all be colour singlets. How is that accomplished? The conserved current is

$$J^\mu_a = \bar{\Psi}\gamma^\mu \tfrac{1}{2}\tau_a \Psi. \tag{3.11}$$

We cannot specify the values of all three charges simultaneously since they do not commute. We can specify Q^2 and Q_3. However, we cannot follow the usual statistical mechanical procedure, i.e., introduce a chemical potential for $\hat{Q}^2$ and then take the Laplace transform, since $\hat{Q}^2$ is not an additive observable.

It was noticed, in the context of the angular momentum of nuclear states[14], that

$$\rho(m, Q) = \rho(m, Q_3 = Q) - \rho(m, Q_3 = Q+1). \tag{3.12}$$

That is, the number of states with total charge Q is equal to the difference between the number of states with third component of charge Q and $Q + 1$. Therefore, if we wish to compute the level density for colour singlet states of SU(2) it suffices to calculate the level density for fixed Q_3 equal to zero and one. This evaluation is nearly identical to the case of fixed baryon number. It leads to

$$\ln Z = \frac{V}{6\pi^2} \frac{\beta}{(\beta^2 - \vec{\lambda}^2)^2} \left[\left(\frac{\mu}{2}\right)^4 + 2\pi^2 \left(\frac{\mu}{2}\right)^2 + \frac{7}{15}\pi^4 \right], \tag{3.13}$$

and to

$$\rho(m, Q_3) \sim 2\pi^{1/2} \left(\frac{14\pi^2}{45}\right)^{3/2} \beta^{-5/2} m^{-7/2} \exp\left(\beta m - \frac{14\pi^2}{15} \frac{Q_3^2}{\beta m}\right), \tag{3.14}$$

where the saddle point value of β is

$$\beta^4 = \frac{7\pi^2}{90 B}. \tag{3.15}$$

Applying this to (3.12) gives the singlet level density

$$\rho_s(m) = \rho(m, Q = 0) \sim 6\pi^{1/2} \left(\frac{14\pi^2}{45}\right)^{5/2} \beta^{-7/2} m^{-9/2} \exp(\beta m). \tag{3.16}$$

Notice that the power of m is reduced by 3/2 units, whereas the power was reduced by 1/2 unit in the case of fixed baryon number. This is in agreement with observations made in the statistical bootstrap model[13)] that restricting each symmetry reduces the power by 1/2 unit. This is in spite of the fact that the three generators of SU(2) do not commute.

The group theoretical techniques to generalize (3.12) to any internal symmetry group have been given[15),16)]. We refer the interested reader to those papers.

4. CONCLUSIONS

We have derived an expression for the asymptotic level density of interacting relativistic quantum fields, obeying constraints in a very large volume. This expression may be evaluated using the functional integral and the associated approximations and techniques. In the context of the bag model we showed that perturbative interactions do not change the analytic structure of the asymptotic mass spectrum. Restricting the baryon number to a specified value leads to results identical in form to the results of the statistical bootstrap model. The restriction of states to SU(2) singlets was also carried out. Restricting the states to a fixed value of the total angular momentum, with or without interactions among the partons, should be a straightforward exercise.

In this paper we have imposed periodic boundary conditions on the bosonic fields and anti-periodic boundary conditions on the fermionic fields. These are not the correct boundary conditions for the bag model. However, different boundary conditions usually lead to the same bulk properties of the system, only the surface terms are different. Therefore, we should expect our asymptotic estimates of the level density of the bag model to remain valid.

This statement needs a little discussion for the case of a non-Abelian gauge symmetry. The bag model boundary conditions require that all states be colour singlets. If we impose (anti)periodic boundary conditions and do coupling constant perturbation theory non-singlet states will contribute to the level density. If we could evaluate Z exactly then we would expect the non-singlet states to have infinite energy and so not contribute to the level density. This is the expected colour confinement. As a practical matter we could pick out by hand (with Kronecker deltas) only the colour singlet states and do perturbation theory about them.

ACKNOWLEDGEMENTS

I have enjoyed discussions with S. Das Gupta, R. Hagedorn, L. Van Hove, B.K. Jennings, H. Pirner and J. Rafelski.

REFERENCES

1) A. Chodos, R.L. Jaffe, K. Johnson and V.F. Weisskopf, Phys. Rev. D9 (1974) 3471.

2) R. Hagedorn, in "Cargèse Lectures in Physics" ed. E. Schatzmann (Gordon and Breach, New York, 1973), vol. 6.

3) N. Cabibbo and G. Parisi, Phys. Lett. 59B (1975) 67.

4) J.I. Kapusta, Nucl. Phys. B148 (1979) 461.

5) J.I. Kapusta, Phys. Rev. D23 (1981) 2444.

6) V.B. Magalinskii and Ia.P. Terletskii, Zh. Eksp. Teor. Fiz. 32 (1957) 584 (Sov. Phys. JETP 5 (1957) 483).

7) R.P. Feynman, Phys. Rev. 91 (1953) 1291.

8) L.D. Faddeev, Teor. Mat. Fiz. 1 (1969) 3, (Theor. Math. Phys. 1 (1969) 1).

9) C. Bernard, Phys. Rev. D9 (1974) 3312.

10) R.E. Norton and J.M. Cornwall, Ann. of Phys. 91 (1975) 106.

11) B.A. Freedman and L.D. McLerran, Phys. Rev. D16 (1977) 1130.

12) J.I. Kapusta, Phys. Rev. D24 (1981) 426.

13) C.J. Hamer and S.C. Frautschi, Phys. Rev. D4 (1971) 2125.

14) H.A. Bethe, Phys. Rev. 50 (1936) 332.

15) K. Redlich and L. Turko, Z. Phys. C5 (1980) 201.

16) L. Turko, Phys. Lett. 104B (1981) 153.

FIGURE CAPTION

The order g^2 contribution to $\ln Z_I$.

-3