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NEUTRINO MIXING AND THE m_W / m_Z MASS RATIO *)

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A B S T R A C T

A comparison of neutral and charged current data according to the standard model provides bounds on neutrino mixing parameters, independently of the number of fermions. This mixing may also affect the determination of $\sin^2\theta$ and m_W/m_Z.

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The usual analysis of lepton mixing relies on charged-current data alone. These essentially provide upper bounds for the off-diagonal elements of the mixing matrix. Limits on the diagonal elements are then obtained indirectly, by assuming that all leptons have actually been observed, and making use of the unitarity of the matrix[1,2].

In this note we want to point out that the comparison of charged- and neutral-current data provides some more direct information on those diagonal elements, in a way now independent of the number of existing fermions.

Since neutrino mixing affects charged currents and leaves neutral currents untouched, its effects are in many ways similar to a change in the m_W/m_Z mass ratio. The standard prediction of the one-scalar boson doublet SU(2) × U(1) model is $\rho = m_W^2/m_Z^2 \cos^2\theta = 1$. Some time ago, Veltman[3] showed that the presence of heavy fermions would modify this relation: the vacuum polarization loop involving any fermionic doublet of masses (m_1, m_2) would indeed increase ρ by an amount:

$$\rho - 1 = \frac{G_F}{8\sqrt{2}\pi^2}\left(\frac{2 m_1^2 m_2^2}{m_1^2 - m_2^2} \cdot \ln\frac{m_2^2}{m_1^2} + m_1^2 + m_2^2\right) \qquad (1)$$

(It is an easy matter to check that this quantity is positive, and vanishes only if the doublet is mass degenerate.) This observation is quite exciting: for instance, an observed value of $\rho = 1.01$ would point to the presence of fermions with a mass splitting of O(400 GeV), otherwise completely inaccessible to experiments!

The importance of this possible conclusion however justifies some care. Veltman and Green[4] have recently shown that other fourth-order processes would, independently of any heavy fermions, increase the apparent value of ρ by 0.5%.

We will check that even in a three-neutrino world, an extra 0.5% correction may arise from neutrino mixing alone.

For reasons of simplicity, we will first assume that only two neutrinos are "light". The ν_τ, and any other neutrinos belonging to hypothetical extra "generations", will be assumed to have masses ≥ 105 MeV. We will relax this hypothesis at the end of the paper.

Anyway, we limit ourselves to the case where the oscillation length between "heavy" and "light" neutrinos is small enough not to be detected. A mass difference of a fraction of a MeV is sufficient to ensure this.

Since the possibility of lepton mixing forbids the determination of M_W from low-energy data, we choose to express the quantities considered below in terms of M_{Z^0} and $\sin^2 \theta_W$. [These can, at least in principle, be determined at present energies, e.g. in the electron-deuterium scattering[5)]]. Owing to the high neutrino masses assumed, we choose to work in terms of mass eigenstates, and introduce the matrix A relating these to the "current" eigenstates:

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \cdot \\ \cdot \\ \cdot \\ \nu_n \end{bmatrix}_{\text{current}} = \begin{bmatrix} a_{11} & a_{12} & \cdots \\ a_{21} & a_{22} & \\ \vdots & & \\ & & \end{bmatrix} \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \cdot \\ \cdot \\ \cdot \\ \nu_n \end{bmatrix}_{\text{mass}} \tag{2}$$

Before turning to neutrino scattering experiments, we first reconsider the limits arising from e-μ universality, and μ decay.

If neutrinos ℓ to n are heavy enough not to be produced in π or K decays, the ratios $R_\pi = \sigma(\pi^- \to e^-\bar{\nu})/\sigma(\pi^- \to \mu^-\bar{\nu})$ and $R_K = \sigma(K^- \to e^-\bar{\nu})/\sigma(K^- \to \mu^-\bar{\nu})$ will be affected by a corrective coefficient:

$$R_{e/\mu} = \frac{1 - \sum_{i=\ell}^{n} |a_{1i}|^2}{1 - \sum_{i=\ell}^{n} |a_{2i}|^2} \tag{3}$$

Experiment tells us that at the one-standard deviation level, $R_{e\mu}$ is constrained to [6)]:

$$R_{e/\mu} = 1.033 \pm 0.02 \tag{4}$$

(Note that this differs by 1 st. dev. from the older value quoted in Ref. 1.)

In the case where some neutrino masses lie in the range 10 MeV $< m_{\nu_i} < m_K$, their incidence on R_π and R_K could be much more dramatic than is suggested by Eq. (3) (see Ref. 2), since the suppression of e decay with respect to μ decay which usually results from the chiral structure of the current, no longer takes place. Any mixing of ν_e with such neutrinos is thus constrained to be extremely small ($|a_{1i}| < 0.01$).

Considering now the muon decay rate, we note that, because of the assumption on masses 3 to n, we must include the suppression factors A_1 and A_2. Rewriting this rate in terms of M_Z and ρ, we have:

$$\frac{1}{\tau_\mu} \sim \left(\frac{g^2\cos^2\theta}{\sin^2\theta}\right)^2 \frac{A_1^2 A_2^2}{M_Z^4 \rho^2}$$

$$A_1^2 = \left(1 - \sum_{i=3}^{n} |a_{1i}|^2\right) \tag{5}$$

$$A_2^2 = \left(1 - \sum_{i=3}^{n} |a_{2i}|^2\right)$$

We believe the argument (see Ref. 2) -- that such an effect could be detected because it would imperil Cabibbo universality -- to be somewhat misleading. There is *a priori* no reason[1] for the elements b_{11} and b_{12} in the matrix B describing the mixing of quarks to satisfy $b_{11}^2 + b_{12}^2 = 1$. As a matter of fact, such an occurrence would be highly undesirable in a six quarks world, since it would forbid the Kobayashi-Maskawa matrix[7] to account for CP violation.

We consider instead that the presence of A_1 and A_2 in Eq. (5) should be investigated by making a comparison with neutral-current processes. For this reason we now envisage neutrino scattering processes.

We consider a pure ν_μ mass-eigenstate beam, interacting with hadrons. The behaviour of the charged cross-section now depends on the mass and decay properties of the *charged* leptons belonging to generations 3 to n. Of course if this mass exceeds the available centre-of-mass energy (currently < 30 GeV) for i = k, ..., n, the corresponding charged-current interaction will be suppressed:

$$\sigma^{cc} \sim \left(\frac{\cos^2\vartheta}{\sin^2\theta}\frac{g^2}{M_Z^2}\right)^2 \frac{1}{\rho^2}\left[1-\sum_{i=k}^{n} |a_{i2}|^2\right] \tag{6}$$

(We assume that the usual corrections resulting from quark mixings have been performed, and their uncertainties taken into account.)*).

The case of lighter leptons (such as the τ) is more intricate. They would be produced, but their short decay time would prevent their positive identification. Let us consider the example of the τ. In 20% of the cases this decays into $\mu\bar{\nu}$, and is thus correctly interpreted as a charged-current event. However, 60% of the decays produce only hadrons, and could thus be mistaken for neutral-current events. The fate of e decays depends upon the type of detector used. If we assume that they are not identified, and limit ourselves to three families, we get:

$$\sigma_{cc}^{\text{observed}}(\nu_\mu \cdot \text{hadr}) = \sigma_{cc}^{\text{unmixed}}(1-|a_{32}|^2+0.2|a_{32}|^2)$$
$$\sigma_{N.C}^{\text{observed}}(\nu_\mu \cdot \text{had.}) = \sigma_{NC} + \sigma_{cc}^{\text{unmixed}} \times 0.8 \times |a_{32}|^2 \tag{7}$$

We thus see that intermediate-mass leptons would have a more complex but *stronger* impact on $R_\nu = \sigma^\nu_{NC}/\sigma^\nu_{CC}$ or $R_{\bar{\nu}}$ than would the extremely heavy ones. Apart from this last experimental difficulty, the neutral-current cross-section is of course not affected by lepton mixing.

Finally, we consider ν_μ-electron scattering. As long as the heavy neutrino masses are not bigger than the centre-of-mass energy (300 MeV for 200 GeV neutrinos), the treatment of charged currents is analogous to the case of hadronic cross-sections, since no suppression occurs at the electronic vertex. However, if such a suppression arises for neutrinos ℓ to n, we have to insert an extra factor A into Eq. (6):

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*) While the value of these mixing parameters b_{ij} would be affected by the consideration of lepton mixing (see Ref. 1), the value of the product $g \cdot b_{ij}$ stays unchanged: no more corrections should thus be performed at the hadronic vertex.

$$A = \left[1 - \sum_{i=\ell}^{n} |a_{1i}|^2\right] \tag{8}$$

We also note that owing to (ep) mixing, some slight contribution to apparent neutral-current events may take place, as shown in Fig. 1. Both these effects would thus lead to an apparent increase of the ratios $R_\nu^{(e)}$ and $R_{\bar{\nu}}^{(e)}$.

Having listed the above effects of neutrino mixing, we now use them to get some limitations on the matrix A. We perform some more simplifications, trying essentially to obtain a lower bound on the element $|a_{22}|$. First, we consider the simple Eq. (6), keeping σ_{NC} unchanged, rather than its more complicated counterpart Eq. (7). This underestimates the effect of ν_μ-ν_i mixings for $i \geq 3$, and suits our purpose. Two different suppression factors are related to a_{22}, namely:

$$\begin{aligned} |A_2|^2 &= 1 - \sum_{i\geq 3}^{n} |a_{2i}|^2 = |a_{22}|^2 + |a_{12}|^2 \\ |A_2'|^2 &= 1 - \sum_{i\geq 3}^{n} |a_{i2}|^2 = |a_{22}|^2 + |a_{21}|^2 \end{aligned} \tag{9}$$

In the case of massive ν_τ, which interests us, Kolb and Goldman[2)] have obtained the limitation: $|a_{12}|^2$ and $|a_{21}|^2 < 0.64 \times 10^{-2}$ from neutrino oscillations alone. (They improved this limit to 0.16×10^{-2} by an iteration procedure.) In the case of n generations, their limit now reads:

$$|a_{12}|^2, \quad |a_{21}|^2 < 0.64 \, \frac{n}{3} \cdot 10^{-2} \tag{10}$$

The difference between $|A_2|^2$ and $|A_2'|^2$ being at most $0.64\ (n/3) \times 10^{-2}$, we tentatively neglect it.

Finally, still in order to find a lower bound for A_2, we maximize the value of $|A_1|$ in Eq. (5), by taking it equal to 1.

After those drastic simplifications, we remark that the effect of neutrino mixing not only goes in the same direction as an *increase* of ρ, as was already apparent, *but also exactly mimics it:* we may thus simply replace ρ everywhere by an effective parameter $\rho' = \rho/|A_2|$ and make use of the habitual fits[8], which now gives us (at 1 st. dev.)

$$\rho' = \rho/|A_2| = 1.025 \begin{smallmatrix} +.38 \\ -.40 \end{smallmatrix} \tag{11}$$

Since we know that $\rho \geq 1$ in the standard model, and that $|A_2| \leq 1$, we obtain simultaneous limits,

$$\left.\begin{array}{l} \rho \leqslant 1.063 \\ |A_2| \geqslant 0.938 \end{array}\right\} \text{1 st. dev.} \tag{12}$$

This lower bound on A_2 is very conservative, for two reasons. First, we have neglected the radiative corrections of Ref. 4 which would improve Eq. (12). Furthermore, going back to Eq. (4) we see that the lowest value for A_2 in Eq. (12) would violate the present bound on e-μ universality if A_1 is kept equal to 1. An improved bound on A_2 could thus be expected from a simultaneous fit to both parameters A_1 and A_2.

This fit, however, would prove more difficult (as would a fit to A_1 with A_2 set equal to 1) since the effect of neutrino mixing can no longer be rephrased in terms of the effective ρ' parameter.

While the limit obtained in Eq. (12) is clearly weaker than the one presented in Ref. 2 for the special case of six leptons, we would like to stress again that Eq. (12) presents the important feature of being independent of the number of existing fermions (we have only assumed that the extra neutrinos had masses > 100 MeV).

Our study is even not superfluous in the case of three neutrinos, since we can now as announced, reverse the argument and make use of the limits provided by Kolb and Goldman to estimate the possible impact of lepton mixing on the determination of ρ.

From the mixing matrix in Ref. 2, we see that we are precisely in the situation where only A_2 may differ appreciably from unity: $A_2 > 99.5$ implies that a correction of -0.5% can result when extracting the true value of ρ from the effective parameter ρ'. This contribution could thus be of the same order as the one calculated in Ref. 4.

The hypothesis on the mass of the neutrinos ($m_i > 100$ MeV, $i \geq 3$) was introduced essentially to allow the introduction of the effective parameter ρ' in Eq. (11). If we want to relax this restriction, the suppression factor disappears from μ decay [Eq. (5)]. Only in this case could we determine the mass of $W^{\pm}$ from μ decay. Also, the value of A_2 can be reached directly from Eq. (7). However, some extra care needs to be taken, since the hypothesis of a pure ν_μ beam is no longer reasonable and should be reconsidered according to each experiment. In general, the admixture of other neutrinos in the beam would further reduce σ_{CC} with respect to σ_{NC}, and this reinforces the effect of the suppression factor in Eq. (6). Also note that the existing limitations on mixing angles are very different in this context[9].

Finally, we want to envisage the effect of lepton mixing on the experimental values of m_W and m_Z. In order to be specific, we return to the assumption of $m_{\nu_i} > 105$ MeV, $i \geq 3$, and neglect A_1 to simply replace ρ by ρ'. Liede and Roos[8] have shown that the values of $\sin^2 \theta_W$ and ρ (or, in the present case ρ') are strongly correlated.

Let us, for instance, assume that ρ' departs by 2.5% from its standard values 1 [see Eq. (11)]. This would, according to Ref. 8, be accompanied by an increase of $\sim 5\%$ for $\sin^2 \theta$. Through Eq. (5), we see that this in turn implies a decrease of m_Z by about 2%. If the effect on ρ' is attributed only to neutrino mixing, the relation $(m_W/m_Z) \cos \theta = 1$ then implies a similar decrease of m_W. This effect might partly compensate the shift in the location of the pole of the W and Z propagators recently calculated by Veltman[10] and by Antonelli et al.[10].

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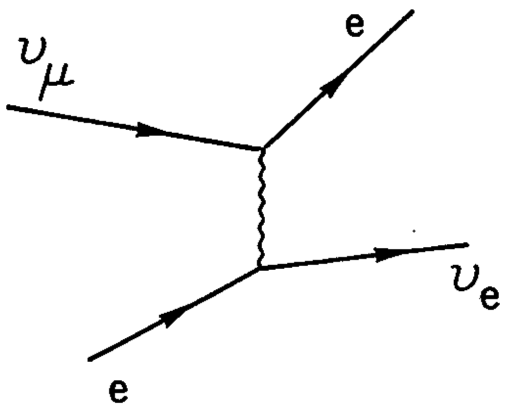

Fig. 1