Ref.TH.2802-CERN

GAUGE AND RENORMALIZATION SCHEME DEPENDENCE IN GUTS

P. Binétruy
CERN - Geneva

and

T. Schücker*)
Université de Genève

ABSTRACT

We use a variant of the minimal subtraction scheme to calculate, in a transparent way, relevant physical quantities in GUTS and study the dependence on various parameters: gauge, Λ, top quark mass, etc.

---

*) Partially supported by the Swiss National Science fundation.

Ref.TH.2802-CERN
21 December 1979

It turns out that this quantity is finite and is expressed in terms of renormalized quantities

$$m_{eff}(p^2) = m\,\frac{B(p^2)}{A(p^2)} \tag{3}$$

where m is the renormalized mass: $m = m_0 - \delta m$, etc. Since $m_{eff}(p^2)$ is expressed in terms of bare quantities [Eq. (2)], it is independent of the renormalization procedure: renormalization scheme and renormalization point (scale). We shall then compute it in the minimal subtraction scheme[10] where we obtain :

$$\begin{aligned} m_{eff}(p^2) = m \Big\{ 1 + \frac{\alpha_s}{3\pi} \Big[& -3(\gamma - \ln 4\pi) - 3 \ln \frac{m^2-p^2}{\mu^2} \\ & + (3+a)\frac{m^2}{p^2} \ln \frac{m^2-p^2}{m^2} + 4 + a - a\frac{m^4}{p^4} \ln\frac{m^2-p^2}{m^2} - a\frac{m^2}{p^2} \Big] \Big\} \end{aligned} \tag{4}$$

where α_s is the renormalized QCD coupling constant, a the renormalized gauge parameter, γ the Euler constant and μ the scale. One important feature of Eq. (4) is the gauge dependence that we shall now discuss. Since, thanks to Eq. (2), m_{eff} is scale independent, one can write a renormalization group equation for $m_{eff}(p^2)$ whose solution is:

$$\begin{aligned} m_{eff}(p^2) = \bar{m}(-p^2) \Big\{ 1 + & \frac{\bar{\alpha}_s(-p^2)}{3\pi} \Big[-3(\gamma - \ln 4\pi) \\ & - 3 \ln \frac{\bar{m}(-p^2)^2 - p^2}{(-p^2)} - (3+\bar{a}(-p^2))\frac{\bar{m}(-p^2)^2}{(-p^2)} \ln \frac{\bar{m}(-p^2)^2 - p^2}{\bar{m}(-p^2)^2} + 4 + \bar{a}(-p^2) \\ & - \bar{a}(-p^2)\frac{\bar{m}(-p^2)^4}{p^4} \ln \frac{\bar{m}(-p^2)^2-p^2}{\bar{m}(-p^2)^2} + \bar{a}(-p^2)\frac{\bar{m}(-p^2)^2}{(-p^2)} \Big] \Big\} \end{aligned} \tag{5}$$

where $\bar{m}(p^2)$, $\bar{\alpha}_s(p^2)$, $\bar{a}(p^2)$ are the running mass, coupling and gauge parameter, that is the solution of the renormalization group equations:

$$\begin{aligned} \frac{M}{\bar{m}(M)} \frac{d\bar{m}(M)}{dM} &= \gamma_m(\alpha_s) = \gamma_m^{(0)} \frac{\bar{\alpha}_s(M)}{4\pi} + \gamma_m^{(1)} \frac{\bar{\alpha}_s(M)^2}{16\pi^2} + \cdots \\ \frac{M}{\bar{\alpha}_s(M)} \frac{d\bar{\alpha}_s(M)}{dM} &= \beta(\alpha_s) = -\beta_0 \frac{\bar{\alpha}_s(M)}{2\pi} - \beta_1 \frac{\bar{\alpha}_s(M)^2}{8\pi^2} + \cdots \\ \frac{M}{\bar{a}(M)} \frac{d\bar{a}(M)}{dM} &= \beta_a(\alpha_s) = \cdots \end{aligned} \tag{6}$$

1. INTRODUCTION

A large amount of the recent literature on unified gauge theories[1)-3)] has been devoted to the problems of quark masses and proton decay. The first one is of importance because constraints on the number of generations is based on the determination of the mass of the bottom quark[3),4)]. Besides, there exists a controversy on the computation of the strange quark mass[5)]. On the other hand, the utmost importance of proton decay need not be emphasized. We therefore have to be very cautious when computing contributions to proton lifetime or quark masses. In this paper, we stress the relevance of gauge and renormalization scheme dependence on such computations.

In Section 2 we calculate renormalized masses and show that, when one includes mass effects, the constituent mass used to give numerical predictions becomes gauge dependent. Instead, as one expects, the true pole of the propagator is gauge independent and therefore allows safe (if not checkable) predictions. We then review, in Section 3, different renormalization schemes and stress the fact that one should stick to a given renormalization procedure and a given gauge throughout the calculations. This is especially true for proton decay computations where one has to take into account numerous contributions calculated by various authors[3),6)-8)]. We define a variant of the minimal subtraction scheme of renormalization which incorporates the decoupling of superheavy fields. This allows us to compute the superheavy gauge boson mass in a clean and transparent way. Technical details are given in an appendix. Finally in Section 4, we give some numerical results and our conclusions.

2. QUARK MASSES

We first start by reminding the different masses available on the QCD market. If one computes a fermion propagator with bare quantities, one obtains

$$S^{-1}(p) = A^0(p^2)\,\not{p} - m^0\,B^0(p^2) \tag{1}$$

where the superscript ^{0} denotes bare quantities. We define the effective mass by[9)]

$$m_{eff}(p^2) = m^0\,\frac{B^0(p^2)}{A^0(p^2)} \tag{2}$$

$$m_{eff}\,(p^2 = m_p^2) = m_p \tag{9}$$

that is

$$m_p = \bar{m}(-m_p^2)\left\{1 + \frac{\bar{\alpha}_s(-m_p^2)}{3\pi}\left[4 - 3(\gamma - \ln 4\pi) - 3i\pi\right]\right\} \tag{10}$$

This expression looks complex. However, if $\pi \ll \ln(p^2/\Lambda^2)$, then $\bar{m}(-p^2) = \bar{m}(p^2)\left[1 + i\,(\gamma_m^{(0)}/8)\,\bar{\alpha}_s(p^2)\right]$ and to the first order

$$m_p = \bar{m}(m_p^2)\left\{1 + \frac{\bar{\alpha}_s(m_p^2)}{3\pi}\left[4 - 3(\gamma - \ln 4\pi)\right]\right\} \tag{11}$$

The reason is clear: if $\pi \ll \ln(p^2/\Lambda^2)$, it is sufficient to sum the leading logs $\ln(p^2/\Lambda^2)$ and use therefore $\bar{\alpha}_s(p^2)$. However, if $\pi \sim \ln(p^2/\Lambda^2)$, one has to use a continuity procedure.

The $\gamma - \ln 4\pi$ factor is the usual factor that appears whenever one uses the minimal subtraction scheme.

In summary, m_p given by Eq. (11) is the only definition of quark masses which is gauge independent and renormalization scheme independent. To compare with the constituent mass given by (8), one has to express $\bar{m}(m^2)$ in terms of $\bar{m}(4m^2)$ which gives, to the first order

$$m_p = \bar{m}(4m_p^2)\left\{1 + \frac{\bar{\alpha}_s(4m_p^2)}{3\pi}\left[4 - 3(\gamma - \ln 4\pi) + 6\ln 2\right]\right\} \tag{12}$$

The coefficient of $\bar{\alpha}_s(4m_p^2)/3\pi$ is numerically about 14. The pole is therefore higher than the constituent mass (evaluated for a not too pathological gauge) and the difference is substantial for the strange quark mass. We do not claim that this is the right definition of mass but this only shows that the accuracy to which one can predict the strange quark mass is certainly bad.

One should note that in the minimal subtraction scheme, as well as in any mass independent renormalization scheme, these equations are decoupled and it is therefore straightforward to solve them.

Now the constituent mass m_c is defined by[11)]

$$m_{eff}\left(p^2 = -4m_c^2\right) = m_c \tag{7}$$

which gives

$$m_c = \bar{m}(4m_c^2)\left\{1 + \frac{\bar{\alpha}_s(4m_c^2)}{3\pi}\left[4 - 3(\gamma - \ln 4\pi) + 6\ln 2 - \frac{15}{4}\ln 5 + \frac{5}{4}\,\bar{a}(4m_c^2)\left(1 - \frac{1}{4}\ln 5\right)\right]\right\} \tag{8}$$

This shows that the constituent mass is explicitly gauge dependent. For example, in the Landau gauge, the coefficient of $\bar{\alpha}_s(4m_c^2)/3\pi$ is approximately 8; in the Feynman gauge $(\bar{a}(4m_c^2) = 1)$, it is of the order of 7.2. Therefore the effect is certainly negligible for the bottom quark mass $(\bar{\alpha}_s(4m_b^2) << 1)$ but starts being sensitive at the strange quark mass $(\bar{\alpha}_s(4m_s^2) \sim 1)$ in the minimal subtraction scheme.

One should note that authors of Ref. 3) define the constituent mass in the timelike sector, namely the one available in e^+e^- annihilation

$$m_{eff}\left(p^2 = 4m_c^2\right) = m_c \tag{7'}$$

which gives

$$m_c = \bar{m}(-4m_c^2)\left\{1 + \frac{\bar{\alpha}_s(-4m_c^2)}{3\pi}\left[4 - 3(\gamma - \ln 4\pi) + 6\ln 2 - \frac{9}{4}\ln 3 + \frac{3}{4}\,\bar{a}(-4m_c^2)\left(1 + \frac{1}{4}\ln 3\right) + \frac{3i\pi}{4}\left(1 + \frac{1}{4}\,\bar{a}(-4m_c^2)\right)\right]\right\} \tag{8'}$$

There is, in this expression, an explicit and implicit $(\bar{m}(-4m_c^2)\ \ldots)$ imaginary contribution. One should therefore use a procedure for continuing to the space-like region[3)-12)]. Anyway, both the difference of the real parts between (8) and (8') and the uncertainty in the choice of procedure increase the uncertainty in the estimate of the constituent mass.

On the other hand, it is clear from Eq. (5) that the only gauge invariant definition of mass (to this order) is the perturbative pole of the fermion propagator m_p which satisfies

3. RENORMALIZATION SCHEME DEPENDENCE

The minimal subtraction, which allowed us to give analytical expressions because the equations (6) were decoupled, incorporates two unwanted features. The first one is that the decoupling theorem of Appelquist and Carazzone[14)] does not hold: if one takes, for example, a fermion (boson) loop in a Green's function, this loop will give a $1/(n-4)$ (n is the dimension) pole and therefore the minimal subtraction will take this loop into account, whatever the mass of the fermion (boson) is. This situation seems dramatic since this does not seem to allow any breaking of symmetry: if one has to keep the leptoquark contribution at low energy, each coupling is the original SU(5) coupling and for example, formula (14) does not hold. The other feature is that large logarithms are creeping into the calculation: the large $\ln(m/\mu)$ where m is a large fermion (boson) mass and μ the dimensional regularization scale (of the order of the external momentum) are not subtracted (whereas they are in a subtraction at $p^2 = -M^2$) and spoil the perturbation approach. This is the main reason why people did not use the minimal subtraction sheme for mass calculations[11)], despite its simplicity. The answer to these two problems is clear: the large $\ln(m/\mu)$ are just the one that spoil the decoupling theorem. Resumming them should therefore give variables that agree with this theorem. We have checked explicitly that this is the case for fermion masses. Actually the only diagrams that give a leading $\ln(m/\mu)$ contribution to the gauge coupling are (see Appendix):

and their summation turns the N (total number of fermion) dependence of the running gauge coupling into an effective N. We believe that the same summation holds for boson masses (actually, the renormalization at $p^2 = -M^2$ is nothing else than an example of such a summation) and therefore the theory decouples and is free of large logarithms. Obviously, such a decoupling property is vital to any GUT consideration: the fact that we want to describe low energy phenomena via three distinct interactions requires that the superheavy gauge bosons (X,Y) must decouple. From now on, we will therefore refer to the minimal subtraction scheme in the following sense: we renormalize a Green function of typical external

After this rather long discussion within QCD, we can now add the well-known GUT ingredients to Eq. (11) and find the bottom mass (defined as the pole) versus the τ mass for example:

$$m_b = m_\tau \left(\frac{\bar{\alpha}_s(m_b)}{\bar{\alpha}_s(M_u)}\right)^{-\frac{\gamma_m^{(0)}}{2\beta_0}} \left(\frac{\bar{\alpha}_2(m_b)}{\bar{\alpha}_2(M_u)}\right)^{-\frac{\gamma_{m,2}^{(0)b}}{2\beta_{0,2}}} \left(\frac{\bar{\alpha}_1(m_b)}{\bar{\alpha}_1(M_u)}\right)^{-\frac{\gamma_{m,1}^{(0)b}}{2\beta_{0,1}}}$$
$$\left(\frac{\bar{\alpha}_2(m_\tau)}{\bar{\alpha}_2(M_u)}\right)^{\frac{\gamma_{m,2}^{(0)\tau}}{2\beta_{0,2}}} \left(\frac{\bar{\alpha}_1(m_\tau)}{\bar{\alpha}_1(M_u)}\right)^{-\frac{\gamma_{m,1}^{(0)\tau}}{2\beta_{0,1}}} \left[1 - \frac{\gamma_m^{(0)}}{8\beta_0}\left(\frac{\gamma_m^{(1)}}{\gamma_m^{(0)}} - \frac{\beta_1}{\beta_0}\right)\frac{\bar{\alpha}_s(m_b) - \bar{\alpha}_s(M_u)}{\pi} + \frac{\bar{\alpha}_s(m_b)}{3\pi}\left(4 - 3(\gamma - \ln 4\pi)\right)\right] \tag{13}$$

where β_0, $\gamma_m^{(0)}$, $\gamma_m^{(1)}$ are defined in Eq. (6);

$\beta_{0,i}$, $\gamma_{m,i}^{(0)b}$, $\gamma_{m,i}^{(0)\tau}$ are the same quantities for the groups SU(2) and U(1) (the anomalous dimensions referring to b and τ respectively);

M_u is the grand unification mass*) in the minimal subtraction scheme (see below); in particular $\bar{m}_b(M_u) = \bar{m}_\tau(M_u)$.

We have introduced in Eq. (13) only the first order QCD corrections. As can be seen, we had to take into account the α_s^2 term in the β function which gives a contribution to this order (but in our approximation we have neglected $\alpha_s\alpha_1$ or $\alpha_s\alpha_2$ terms).

As pointed out elsewhere[13)], the anomalous dimensions $\gamma_{m,i}^{(0)b,\tau}$ $i = 1,2$ are gauge dependent. However, this gauge dependence disappears in the formula (13) because $\gamma_{m,i}^{(0)b} - \gamma_{m,i}^{(0)\tau}$ is gauge independent [the tiny gauge dependence, coming from the fact that we make a ratio of two quantities at different points $\bar{\alpha}_i(m_b)/$ $/\bar{\alpha}_i(m_\tau)$, is of order $\bar{\alpha}_i(m_{b,\tau})$ and should disappear, if we take these terms into account in (13) because each is gauge independent as a pole] and we obtain ($\gamma_{m,2}^{(0)b} = \gamma_{m,2}^{(0)\tau}$)

$$m_b = m_\tau \left(\frac{\bar{\alpha}_s(m_b)}{\bar{\alpha}_s(M_u)}\right)^{-\frac{\gamma_m^{(0)}}{2\beta_0}} \left(\frac{\bar{\alpha}_1(m_b)}{\bar{\alpha}_1(M_u)}\right)^{-\frac{\gamma_{m,1}^{(0)b} - \gamma_{m,1}^{(0)\tau}}{2\beta_{0,1}}}$$
$$\left[1 - \frac{\gamma_m^{(0)}}{8\beta_0}\left(\frac{\gamma_m^{(1)}}{\gamma_m^{(0)}} - \frac{\beta_1}{\beta_0}\right)\frac{\bar{\alpha}_s(m_b)}{\pi} + \frac{\bar{\alpha}_s(m_b)}{3\pi}\left(4 - 3(\gamma - \ln 4\pi)\right)\right] \tag{14}$$

Before giving any number, we need to precise our scheme of renormalization.

---

*) Throughout the paper, we define the grand unification mass as the mass scale where the three couplings are equal to the SU(5) coupling or (in some schemes) so near to it as to be effectively unified.

As we shall now see, the simplicity of our calculation is not shared by every renormalization scheme. Let us take, for example, the renormalization at $p^2 = -\mu^2$. In such a scheme, Eqs (6) form a system of coupled differential equations because β_0, $\gamma_m^{(0)} \ldots$ are functions of $\bar{m}(M)$, $\bar{a}$ (M). However, we can use what we learned above by parametrizing the solution in terms of the solution of the decoupled equations, namely in terms of the minimal subtraction quantity. For instance, if we renormalize the QCD coupling constant via the three gluon couplings at the symmetrical point $(p^2 = q^2 = r^2 = \mu^2)$, we have

$$\bar{m}'(\mu^2) = \bar{m}(\mu^2)\left\{1 + \frac{\bar{\alpha}_s(\mu^2)}{\pi} A(\mu)\right\} \tag{16a}$$

$$\bar{\alpha}_s'(\mu^2) = \bar{\alpha}_s(\mu^2)\left\{1 + \frac{\bar{\alpha}_s(\mu^2)}{\pi} B(\mu)\right\} \tag{17a}$$

where $\bar{m}'(\mu)$, $\bar{\alpha}_s'(\mu)$ are the running quantities in the renormalization at $p^2 = -\mu^2$, and $\bar{m}(\mu)$, $\bar{\alpha}_s(\mu)$ the running quantities in the minimal subtraction scheme.

It is straightforward to obtain $A(\mu)$

$$A(\mu) = -(\gamma - \ln 4\pi) - \ln\frac{\bar{m}(\mu^2)+\mu^2}{\mu^2} - \left(1 + \frac{\bar{a}(\mu^2)}{3}\right)\frac{\bar{m}(\mu^2)^2}{\mu^2}\ln\frac{\bar{m}(\mu^2)^2+\mu^2}{\bar{m}(\mu^2)^2} + \frac{4}{3} + \frac{1}{3}\bar{a}(\mu^2) - \frac{1}{3}\bar{a}(\mu^2)\frac{\bar{m}(\mu^2)^4}{\mu^4}\ln\frac{\bar{m}(\mu^2)^2+\mu^2}{\bar{m}(\mu^2)^2} + \frac{1}{3}\bar{a}(\mu^2)\frac{\bar{m}(\mu^2)^2}{\mu^2} \tag{16b}$$

As for $B(\mu)$ [16),18)]

$$\begin{aligned} B(\mu) = {} & -\beta_0\left[\frac{\gamma - \ln 4\pi}{4} + \frac{1}{4}\ln\frac{M^2}{\mu^2} - \frac{1}{2}\right] + C_2(G)\frac{1}{2}\left[\frac{3}{8}\bar{a} - \frac{1}{4}\bar{a}^2 + \frac{1}{24}\bar{a}^3 + \frac{I}{12}\left(\frac{23}{6} - \frac{9}{2}\bar{a} + \bar{a}^2\right)\right] - \frac{4}{9}T(R)\,I\,N_f + \frac{1}{2}T(R)\sum_{j=1}^{N_f}\Bigg[\\ & \frac{4}{3}\int_0^{1/3} dy\, p(y)\left(-2\ln\frac{M^2y+m_j^2}{M^2y} + \frac{3m_j^2}{M^2y+m_j^2}\right) - 2\left(\frac{2m_j^2}{M^2} - 1\right)\left(\frac{4m_j^2}{M^2}+1\right)^{1/2} \times \\ & \times \ln\frac{\left(\frac{4m_j^2}{M^2}+1\right)^{1/2}+1}{\left(\frac{4m_j^2}{M^2}+1\right)^{1/2}-1} + 8\frac{m_j^2}{M^2} + 2\ln\frac{m_j^2}{M^2}\Bigg] \end{aligned} \tag{17b}$$

where $\beta_0 = 11/3\; C_2(G) - 4/3\; T(R)N_f$

$C_2(G)$, $T(R)$ are defined as usual (respectively 3 and ½ for SU(3))

$I = 2.344$ [see Ref. 16)]

$p(y) = \iiint dx_1 dx_2 dx_3\; \delta(1-x_1-x_2-x_3)\; \delta(y-x_1x_2-x_2x_3-x_1x_3)$ [see Ref. 18)].

momentum M by subtracting the pole $1/(n-4)$ (pure minimal subtraction) when $M^2 \geq M_X^2$(or M_W^2) and by subtracting the pole and the appropriate $\ell n(M_X^2/\mu^2)$ terms when $M^2 \leq M_X^2$. "Appropriate" means the $\ell n(M_X^2/\mu^2)$ terms that one would find far from the threshold: $M^2 \ll M_X^2$. This gives us, as a final result, β,γ functions that behave as step functions at the X (or W) threshold, that is nothing other than the β,γ functions that most people use. The fact that one crosses thresholds via step functions does not seem quite physical but neither is the running coupling constant (even if the threshold was crossed aesthetically). We follow the same procedure for fermion thresholds.

In such a scheme, the grand unification mass coincides with the mass of the superheavy gauge bosons. We therefore first test the method by computing it and comparing our result with the value given by Goldman and Ross[7]. These authors use the value $\Lambda^2 = 0.1$ GeV^2 for six flavours. On the other hand, they are in the momentum subtraction scheme since they use threshold effects computed in Ref. 6). To compare, we must therefore start, in our minimal subtraction scheme, with $\Lambda^2 = 0.1/(5.5)^2$ GeV^2 = 0.33×10^{-2} GeV^2 [16]. It is then straightforward to obtain M_X [see, for example, Ref. 15)].

$$\ell n \frac{M_X}{M_W} = -\frac{1}{11}\left[\frac{14}{3}\ell n\frac{M_W^2}{\Lambda^2} - \pi\,\alpha_{e.m.}^{-1}(M_W)\right] \tag{15}$$

In our scheme where β functions behave as step functions at the pole, we find*) $\alpha_{e.m.}^{-1}(M_W) = 128$ and obtain $M_X = 1.35 \times 10^{15}$ GeV. This should be compared with the value $M_X = 1.8 \times 10^{15}$ GeV obtained in Ref. 7), before higher order and Higgs contributions are taken into account. We interpret such an agreement as a success of our method. The reason is very transparent: Λ_{mom} is approximately six times Λ_{MS} [16]; this, more or less, translates the grand unification masses of the same amount, that is $(M_u)_{mom} = 6(M_u)_{MS} = 6M_X$. This factor 6 is just the one advocated by Ross[6] for threshold effects. Actually, it turns out not to be exactly 6 (5.5) and not to translate by exactly the same amount the unification masses [see Eq. (15)] and this gives the above value . If we now take into account higher orders and Higgs contributions with a factor 20/3 reduction as in Ref. 7), the final value is $M_X = 2 \times 10^{14}$ GeV. Actually the value of Λ was taken above mainly for comparison purposes and is somewhat low compared with experiments[17]. We give other values in Table 1.

---

*) Of course, Goldman and Ross find a different value for $\alpha_{e.m.}$ namely $\alpha_{e.m.}(2M_W) = 130.4$, because they use a momentum subtraction scheme where they have to include threshold effects.

4. RESULTS AND CONCLUSION

After this rather long discussion on renormalization scheme dependence, we can now give a numerical result to Eq. (14). Using Refs 4),13),19)

$$\gamma_m^{(0)} = -8 \qquad \gamma_{m,1}^{(0)b} = \frac{1}{5} + \frac{3}{20}a \qquad \gamma_{m,1}^{(0)\tau} = -\frac{9}{5} + \frac{3}{20}a$$

$$\beta_0 = 11 - \frac{2}{3}N_f \qquad \beta_{0,1} = -\frac{2}{3}N_f$$

$$\gamma_m^{(1)} = -\frac{324}{3} + \frac{40}{9}N_f \qquad \beta_1 = 102 - \frac{38}{3}N_f$$

we obtain, for six flavours

$$m_b = m_\tau \left(\frac{\bar{\alpha}_s(m_b)}{\bar{\alpha}_s(M_u)}\right)^{\frac{4}{7}} \left(\frac{\bar{\alpha}_1(m_b)}{\bar{\alpha}_1(M_u)}\right)^{\frac{1}{4}} \left[1 + 0.92\,\frac{\alpha_s(m_b)}{\pi} + 3.29\,\frac{\alpha_s(m_b)}{\pi}\right] \tag{20}$$

Our numerical results are given in Table 1 for different values of Λ. One should stress that in Eq. (20) we have neglected the top threshold. However, it turns out that taking it into account, within a large range of $m_t (15\ \text{GeV} \leq m_t \leq \leq M_W)$, changes only the result by a few (2 to 3) per cent. Results given in Table 1 are high. The mass defined as the pole is too high. As for the constituent mass, it is certainly high if one looks at the values corresponding to the experimentally favoured Λ [17]: $\Lambda_{\overline{MS}} \simeq 0.19$ GeV. The situation could be saved if Λ turned out to be lower and if one stuck to usual gauges (but why should we?). If not, the model is in trouble. There are several ways to get out of it. One is to search for another definition of quark masses (we feel free to do so, since confinement may spoil any of our definitions). We tend to favour a definition that would be based on physical processes involving Higgs particles (the fermion-Higgs coupling being proportional to the mass): for instance, the S matrix element describing the process $e^+e^- \to \text{Higgs} \to q\bar{q}$ should provide us with a gauge independent $m_{eff}(p^2)$ and therefore a gauge independent definition of the mass. The strange quark mass problem is worse. It turns out [see Table 1 note (e)] that the term of order α_s in Eq. (14) (adapted to the strange quark) gives rise to a correction that ranges from 30% to 100%, in most cases. The high values given in Table 1 should therefore not be trusted. Anyway, this suggests either a change of the relations $m_q(M_u) = m_\ell(M_u)$ [5] or a search for a new perturbative definition of masses.

Of course, B would be different, had we chosen to renormalize the gauge coupling through the quark-quark-gluon interaction. In the momentum subtraction scheme, we have

$$m_{eff}(p^2) = \bar{m}'(-p^2) \tag{18}$$

and therefore as expected, we recover the expression given in (5). If one thinks that the perturbative expansion converges more rapidly in this scheme than in the minimal subtraction one, one can express $m_{eff}(p^2)$ in terms of $\bar{\alpha}'_s(p^2)$ through Eqs (5) and (17). The same treatment can be applied to Eq. (14) which gives

$$m_b = m_\tau \left(\frac{\bar{\alpha}'_s(m_b)}{\bar{\alpha}'_s(M'_u)}\right)^{-\frac{\gamma_m^{(0)}}{2\beta_0}} \left(\frac{\bar{\alpha}'_1(m_b)}{\bar{\alpha}'_1(M'_u)}\right)^{-\frac{\gamma_{m,1}^{(0)b} - \gamma_{m,1}^{(0)\tau}}{2\beta_{0,1}}} \times$$

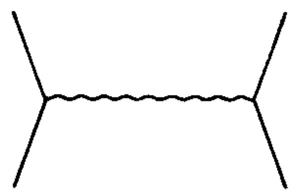
$$\times\left[1 - \frac{\gamma_m^{(0)}}{8\beta_0}\left(\frac{\gamma_m^{(1)}}{\gamma_m^{(0)}} - \frac{\beta_1}{\beta_0}\right)\frac{\bar{\alpha}'_s(m_b)}{\pi} + \frac{\bar{\alpha}'_s(m_b)}{3\pi}\left(4 - 3(\gamma - \ln 4\pi) + \frac{3}{2}\frac{\gamma_m^{(0)}}{\beta_0}B(m_b)\right)\right] \tag{19}$$

One can check that the $(\gamma - \ell n 4\pi)$ term disappears $(\gamma_m^{(0)} = -8)$ in this expression. M'_u is the grand unification mass in the scheme considered. Of course, as is obvious from Eq. (17), $M_u \neq M'_u$ that is the grand unification mass depends on the scheme used to renormalize (whereas the physical quantity M_X is independent; this was the basis of the above calculation). Actually things turn out to be even more complicated as we shall now see.

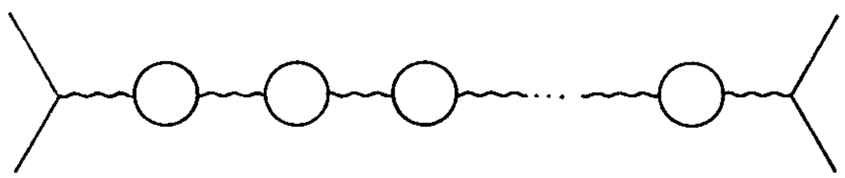
In the minimal subtraction scheme, the three coupling constants intersect at M_X. If we now go to the momentum subtraction this will no longer be the case. Not only they will intersect at another point but the three of them will no longer intersect at the same point. As is clear from Eq. (17b), B depends in a complicated way on the gauge group through $C_2(G)$ and $T(R)$. Therefore if $\bar{\alpha}_s(M_X) = \bar{\alpha}_2(M_X)$ then the point M'_u where $\bar{\alpha}'_s(M'_u) = \bar{\alpha}'_2(M'_u)$ will depend on the gauge groups SU(3) and SU(2), and so forth for other pairs of gauge couplings. So they would only intesect two by two. Similarly, running masses would intersect at still another point. The answer to this puzzle is clear: as stressed by Ross[6)], threshold effects play an important role and force the couplings to join a unique value beyond M_X. But the moral of it is clear too: threshold effects and grand unification point $(M_u \gtrsim M_X)$ strongly depend on the renormalization procedure : scheme, point chosen to renormalize (quark-quark-gluon; three gluon...) and also gauge (B is gauge dependent). It is therefore of great importance, especially for proton decay where one has to add so many ingredients[7)-8)], to compute every contribution in the same scheme, with the same gauge. It is not clear in the literature that this is the case.

APPENDIX

Consider two interacting (light) quarks described to lowest order by the following graph

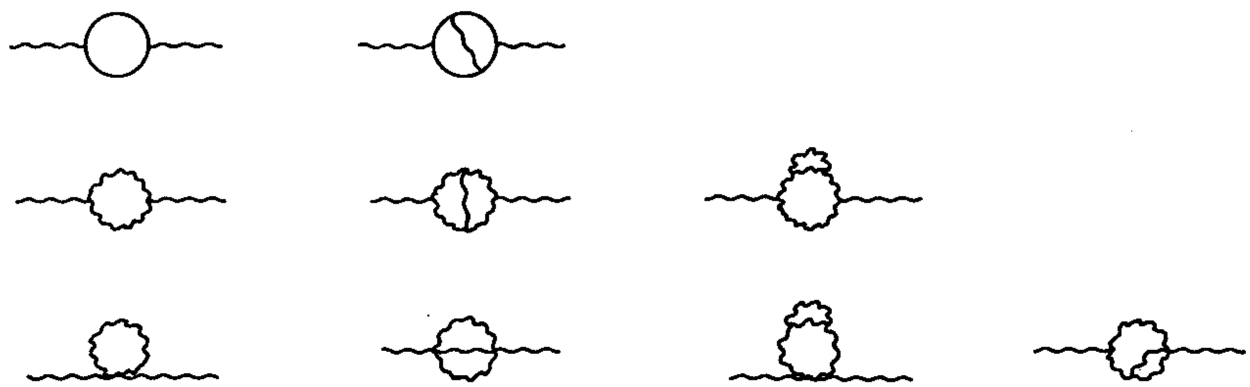


Let us add higher order corrections involving heavy quarks of mass M. We show that in the minimal subtraction scheme the only graphs contributing terms proportional to $\alpha(\alpha\ \ell n(M/\mu))^n$ (leading logs) are repeated insertions of simple heavy quark loops into the gluon propogator:



First we remark that a graph containing a converging loop does not contribute a $\alpha(\alpha\ \ell n(M/\mu))^n$ term. This is the case whenever a heavy quark loop is attached to a light quark line. Therefore it is sufficient to consider only higher order corrections to the gluon propagator which contain no convergent loop.

The following is a complete list of irreducible gluon propagators of any order not containing any convergent loop:



It remains to show that the second graph has a zero $(\alpha\ \ell n(M/\mu))^n$ term.

Finally, about the number of fermions N_f, as already noted by Nanopoulos and Ross[4),*)], it is of vital importance for the model that $N_f = 6$.

As a conclusion, we think that the variant of the minimal subtraction scheme that we use provides us with a very simple way to calculate physical quantities, especially the mass of the superheavy gauge bosons. The bottom and strange quark mass turn out to be high, even with six flavours and whatever definition (pole or constituent) that one uses. This raises the question of a suitable perturbative definition of quark masses. We have not yet succeeded in answering this question.

After this manuscript was completed, we learnt from S. Weinberg that he had developed a very similar prescription for calculating renormalization in unified theories.

ACKNOWLEDGEMENTS

We wish to thank M.K. Gaillard and J. Ellis for active support and friendly encouragement. We also acknowledge helpful discussions with R. Coquereaux, D. Nanopoulos and E. de Rafael.

*) One should also note that, within our scheme, we recover the original results of these authors for the b and s constituent mass at ordinary gauges ($a = 0$ or 1).

Λ_4 GeV	Λ_6 GeV	M_X simple minded GeV	M_X sophisticated GeV	m_b pole GeV	m_b constituent mass GeV			m_s pole GeV	m_s constituent mass GeV		
					a = 0	a = 1	a = 10		a = 0	a = 1	a = 10
(a)	(b)	(c)	(d)	(e)	(e)	(e)	(e)	(e)	(e)	(e)	(e)
0.05	0.017 ± 0.002	$(4.8 \pm 0.4) \times 10^{14}$	$(7.2 \pm 0.6) \times 10^{13}$	5.75	5.1	5.15	5.6	*0.54*	0.42	0.43	*0.49*
0.12	0.049 ± 0.004	$(1.2 \pm 0.1) \times 10^{15}$	$(1.8 \pm 0.1) \times 10^{14}$	6.7	5.8	5.85	*6.45*	~~0.82~~	*0.55*	*0.56*	~~0.66~~
0.19	0.085 ± 0.007	$(1.9 \pm 0.1) \times 10^{15}$	$(2.8 \pm 0.2) \times 10^{14}$	*7.4*	6.25	6.3	*7.0*	~~1.21~~	*0.67*	~~0.68~~	~~0.83~~

Table 1

Superheavy gauge boson and quark mass results

(a) value of Λ in the minimal subtraction scheme for four flavours

(b) idem for six flavours: the uncertainty comes from the top quark mass ($15\ \text{GeV} \leq m_t \leq 100\ \text{GeV}$)

(c) value of M_X as computed in the text, that is without higher orders or Higgs contribution

(d) final value of M_X if we assume that the remaining corrections give a factor 0.15 reduction as in Ref. 7).

(e) the masses are written normally when the correcting term of order α_s [see Eq. (14)] is less than 30%. They are written in italic when the term is between 30% and 50% and they are barred when it is higher than 50%.

We have checked it explicitly but the reasons of this behaviour is quite general: any diagram that contains a loop with a degree of divergence $D = 0$ has no $(\alpha\, \ln(M/\mu))^n$ term. Actually it is easy to show that the divergent part corresponding to such a loop is just $\Gamma(\varepsilon)$ with no dependence on external parameters as M. Therefore the subtraction corresponding to such a loop provides no $\ln(M/\mu)$. On the other hand, in case of overlapping divergences, after having done the lower order subtractions for the whole diagram, one cannot end with a $(1/\varepsilon^n)\, M^\varepsilon$ term since that would allow a $(1/\varepsilon^{n-1})\ln M$ counterterm which is not local. We thus obtain no $(\alpha\, \ln(M/\mu))^n$ term.

As a conclusion, the only diagrams that contribute to the leading log level are repeated insertions of heavy quark loops as indicated above and the summation is obvious. The effective low energy coupling constant α_{eff} is then expressed in terms of the original one α by:

$$\alpha_{eff}(\mu) = \frac{\alpha(\mu)}{1 - \frac{2}{3}\sum_F T(R) \ln\frac{M_F}{\mu}\, \frac{\alpha(\mu)}{\pi}} \tag{A.1}$$

where the sum runs over heavy fermions and $T(R) = \frac{1}{2}$ for the fundamental representation of $SU(N)$.

A summation of the leading $\ln(M_X/\mu)$ with M_X a superheavy gauge boson mass would follow the same lines. The effective low energy coupling constant $\alpha^{(i)}_{eff}$ [$i = 1,2,3$ for $U(1)$, $SU(2)$ and $SU(3)$ respectively] is expressed in terms of the unified $(SU(5))$ coupling constant α by:

$$\alpha^{(i)}_{eff}(\mu) = \frac{\alpha(\mu)}{1 + \frac{11}{6}\left(C_2(G) - C_2(G_i)\right) \ln\frac{M_X}{\mu}\, \frac{\alpha(\mu)}{\pi}} \tag{A.2}$$

where G is the unification group $(SU(5))$ and G_i $i = 1,2,3$ the low energy groups [$U(1)$, $SU(2)$ and $SU(3)$ respectively]. The result is easily extended to more complicate groups or breakings of symmetry and to Higgs contributions.

Finally we think that the summation of next to leading terms of order $\alpha(\alpha\, \ln(M/\mu))^n$, for example, will make the second order term in the β function (β_1) decouple and so forth to all orders.

REFERENCES

1) H. Georgi and S.L. Glashow, Phys. Rev. Letters 32 (1974) 438.

2) H. Georgi, H.R. Quinn and S. Weinberg, Phys. Rev. Letters 33 (1974) 451.

3) A.J. Buras, J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Nucl. Phys. B135 (1978) 66.

4) D.V. Nanopoulos and D.A. Ross, Nucl. Phys. B157 (1979) 273.

5) H. Georgi and C. Jarlskog, Harvard University Preprint HUTP-79/A026 (1979).

6) D.A. Ross, Nucl. Phys. B140 (1978) 1.

7) T. Goldman and D.A. Ross, Phys. Lett. 84B (1979) 208.

8) W.J. Marciano, Phys. Rev. D20 (1979) 274.

9) H.D. Politzer, Nucl. Phys. B117 (1976) 397.

10) G 't Hooft and M. Veltman, Nucl. Phys. B44 (1972) 189.

11) H. Georgi and H.D. Politzer, Phys. Rev. D14 (1976) 1829.

12) R.G. Moorhouse, M.R. Pennington and G.G. Ross, Nucl. Phys. B124 (1977) 285.

13) V. Elias, ICTP Preprint IC/79/52 (May 1979).

14) T. Appelquist and J. Carazzone, Phys. Rev. D11 (1975) 2856.

15) M.K. Gaillard and L. Maiani, Lectures in Cargèse Preprint LAPP-TH-09 (1979).

16) W. Celmaster and R.J. Gonsalves, University of California, San Diego Preprint UCSD-10P10-202 (1979).

17) J. Ellis' talk at the 1979 Symposium on Lepton and Hadron Interactions FNAL, Preprint CERN-TH.2744 (1979).

18) O. Nachtman and W. Wetzel, Nucl. Phys. B146 (1978) 273.

19) W. Caswell, Phys. Rev. Letters 33 (1974) 244;
D.R.T. Jones, Nucl. Phys. B75 (1974) 531.