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SOME INSTANTON EFFECTS AT LARGE MOMENTA

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ABSTRACT

We briefly review results of some recent attempts to estimate the influence of non-perturbative effects associated with the existence of a non-trivial vacuum structure in QCD on processes at large momenta.

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In this talk I will discuss briefly the appearance of non-trivial Euclidean solutions to the classical equations of an SU(2) (SU(N)) gauge theory, the so-called instantons[1], and the modifications they cause in defining vacuum expectation values in the theory. I will present the main results of some attempts[2)-5)] to understand some phenomenological and some not so phenomenological consequences of these modifications. Further details can be found in one of several reviews that exist in print or in Refs 2)-5).

The main starting point is the observation made by Callan, Dashen and Gross[6] that in a theory based on a Lagrangian density:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^{a} F_{a}^{\mu\nu} , \quad F_{\mu\nu}^{a} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{abc} A_\mu^b A_\nu^c \tag{1}$$

where f_{abc} are SU(N) structure constants, the vacuum is more complicated than $A_\mu = 0$ or its gauge transform, due to the existence of an infinite number of vacuum-like states $|n\rangle$, $(n=-\infty,\ldots 0,\ldots\infty)$ separated by finite barriers. Thus instead of considering $\langle 0|\exp(-Ht)|0\rangle$, which for $t\to\infty$ gives the functional integral

$$\langle 0|\exp(-Ht)|0\rangle \xrightarrow[t\to\infty]{} \int \mathcal{D}(A_\mu) \exp\left(-\int d^4x\, \mathcal{L}(A_\mu)\right) \tag{2}$$

we have to consider

$$\langle n|\exp(-Ht) m\rangle \xrightarrow[t\to\infty]{} \int \mathcal{D}(A_\mu)_{n-m} \exp\left(-\int d^4x\, \mathcal{L}(A_\mu)\right) \tag{3}$$

where the functional integral has to be performed over all A_μ of homotopy class $\nu = n-m$. For $\nu = 0$ the minimal action vanishes ($A_\mu = 0$) and $\langle n|\exp(-Ht)|n\rangle \sim O(1)$. For $\nu \neq 1$ the minimal action is bounded from below. For example, for $\nu = 1$ the minimal action $= 8\pi^2/g^2$ and the corresponding field configuration

$$A_\mu = \frac{2}{g} \frac{\sigma_{\mu\nu}(x-x_0)^\nu}{(x-x_0)^2+\rho^2} \tag{4}$$

is the so-called one instanton solution. Here $\sigma_{\mu\nu} = i\eta_{\mu\nu}^a \sigma^a$ where $\eta_{\mu\nu}^a$ is the 't Hooft tensor[7], and x_o^μ and ρ are usually referred to as the position and the size of the instanton[1]. Thus

$$\langle n|\exp(-Ht)|n+1\rangle \sim \langle 1|\exp(-Ht)|0\rangle \sim O\left(\exp\left(-\frac{8\pi^2}{g^2}\right)\right) \tag{5}$$

which behaves like a typical tunnelling amplitude (vanishing exponentially as $g \to 0$), and as such is unseen by the usual pertubation theory. The real vacuum of the theory is given by

$$|\theta\rangle = \sum_n e^{in\theta} |n\rangle \tag{6}$$

One can show[8] that each $|\theta\rangle$ vacuum is the ground state of a physically inequivalent world and that different θ worlds do not communicate with each other. However when $\theta \neq 0$ the theory exhibits a spontaneous breakdown of $P + T$ invariance. As the theory we are considering is thought of as a model for the theory of strong interactions, clearly $\theta \sim 0$. The question why physics chooses this value has not as yet been satisfactorily answered[9].

In his pioneering work[7] 't Hooft, and later also others[10] managed to calculate $\langle 1 |\exp(-Ht)| 0\rangle_{t \to \infty}$ in a one-loop approximation. It was found that

$$\begin{aligned} \langle 1|\exp(-Ht)|0\rangle_{t\to\infty} &\xrightarrow[t\to\infty]{} c \int d^4x_0 \int \frac{d\rho}{\rho^5} \left[\frac{8\pi^2}{\bar{g}^2(\frac{1}{\mu\rho})}\right]^{2N} \exp\left(-\frac{8\pi^2}{\bar{g}^2(\frac{1}{\mu\rho})}\right) \\ &\equiv c \int d^4x_0 \int d\rho \, d(\rho) \end{aligned} \tag{7}$$

where $\bar{g}$ is the familiar running coupling constant, μ is the renormalization scale parameter and $d(\rho)$, instanton density function, is defined by the above expression. For SU(3) (without fermions)

$$\frac{8\pi^2}{\bar{g}^2(\frac{1}{\rho\mu})} \xrightarrow[\rho\to 0]{} 11 \ln\frac{1}{\rho\mu} \quad \Rightarrow \quad d(\rho) \sim \rho^6 \tag{8}$$

and we see that the contribution of small size instantons is suppressed. However, the integration extends to arbitrarily large ρ , for which the running coupling constant is not calculable and so for complete understanding we need more information about the infra-red limit of the theory or we have to choose a process in which, for some reason, large size instantons are also suppressed. This is the case in our applications.

When we calculate the vacuum expectation value of an operator A we have

$$\langle vac|A|vac\rangle = \frac{\sum_{n=-\infty}^{\infty} \int \mathcal{D}(A_\mu)_n \, e^{-\int d^4x \, \mathcal{L}(A_\mu)} A[A_\mu]}{\sum_{n=-\infty}^{\infty} \int \mathcal{D}(A_\mu)_n \, e^{-\int d^4x \, \mathcal{L}(A_\mu)}} \tag{9}$$

where the sum runs over the discrete sectors of function space corresponding to different homotopy classes and where we have set $\theta = 0$. The conventional perturbation approach takes only $n = 0$ terms.

Three obvious questions pose themselves at this point:

1) Is this correct ? i.e. are we sure that quantum corrections do not change the theory so much that the discussion based on homotopy classes is misleading ?

2) How to calculate functional integrals ?

3) How to perform the sum ?

The answers to these questions are as follows : 1) this answer is unclear at this point. Although Witten[11)] has raised some objections based on the analogy to a two-dimensional theory it is not clear to what extent his arguments apply to this four-dimensional theory. 2)We only know how to perform Gaussian integration and so we have to resort to perturbation theory in each sector[12)]. 3) Not clear at all. As conventional perturbation theory works so well we expand[13)] in the number of instantons (and anti-instantons)

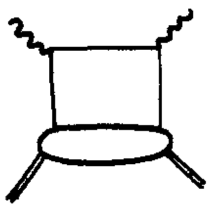
$$\langle vac|A|vac\rangle = \langle A\rangle_0 + \sum_{\pm} \int d^4x_0^{\pm}\, d\rho^{\pm}\, d(\rho^{\pm}) \left\{ \langle A\rangle_{A_\mu} - \langle A\rangle_0 \right\} \tag{10}$$

$+$ two instanton contribution +

It is not clear, at this stage, whether such an expansion makes sense. Recent work of Fateev et. al[14)], based however on a two-dimensional model, suggests caution when drawing conclusions from such expansions. However, until the theory is better understood or a different calculational scheme found we shall use the expansion given above and interpret our results with caution.

Our procedure will be to choose processes for which we can isolate dominant perturbative contributions and then calculate one instanton (and one anti-instanton) corrections to these contributions.

The "cleanest" application is $\sigma(e^+e^- \to \text{anything})$ (at large Q^2), where  dominates. We shall also consider deep inelastic scattering and the Drell-Yan process which are asymptotically described by the diagrams:



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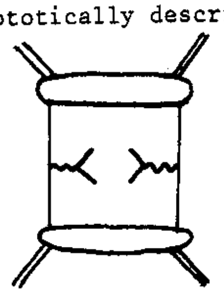
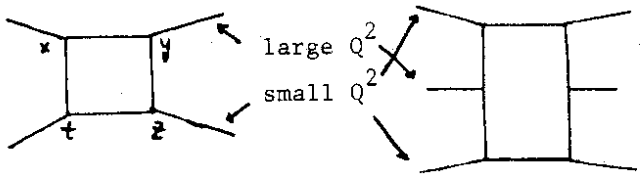


respectively.

As the structure of the proton cannot be determined pertubatively we shall use a toy model in which protons are represented by "scalar photons" with $q^2 = 0$ and for simplicity we also replace other photons by "scalar" photons with Q^2 large. Thus we study the effects of instantons on diagrams:



hoping that the observed effects persist in the complete theory.

To perform calculations we use propagators of scalar and spinor particles in an instanton background field, obtained by Brown et. al[15]. In this way, when we consider $\sigma(e^+e^- \to \text{anything})$, integrate over instanton position and size (with a cut-off) and then having continued the expressions to Minkowski space take the appropriate imaginary part we find that the result is independent (for $Q^2 \to \infty$) of the instanton-size cut-off. To understand this point we observe that there is only one scale in this problem $(1/Q)$ and that the dominant contribution comes from the instanton sizes of this scale. In this way we find the asymptotic behaviour of the instanton correction to R

$$\Delta R \underset{Q\to\infty}{\sim} Q^{-11-N_f/3} \left[\ln Q^2\right]^{6\cdot\frac{33-4N_f}{33-2N_f}} \tag{11}$$

Putting all factors in for the calculated fermion loop we obtain our phenomenological estimates:

$$N_f = 3 \qquad \frac{\Delta R}{R} \approx \left(\frac{Q}{1.4\ \text{GeV}}\right)^{-12} \left(\ln Q^2\right)^{3.67} \tag{12}$$

$$N_f = 4 \qquad \frac{\Delta R}{R} \approx \left(\frac{Q}{1.1\ \text{GeV}}\right)^{-12\frac{1}{3}} \left(\ln Q^2\right)^{4.08}$$

This result holds in the limit of vanishing fermion mass. It can be shown that the lowest order mass corrections[5] do not alter this conclusion. It is of the greatest importance to determine whether this result is altered by the introduction of higher mass insertions or consideration of multi-instanton contributions. Unfortunately, at the moment, technical problems prevent us from being able to determine these contributions.

The same problem was also studied by Andrei and Gross[13], who reached a different conclusion. This difference stems from the fact that they did not isolate the required imaginary part; their contribution is entirely real. It may be that the inclusion of the higher mass insertions, or higher order terms in pertubation theory, brings together the behaviour of the real and the imaginary part of the amplitude, as they expect, but to check this, further work is required. At the level of the calculations performed so far the instanton corrections are given by the expressions above.

When we consider a model for deep inelastic scattering, based on the "box diagram" mentioned before we can show that as $Q^2 \to \infty$ $(x-y)^2 \to 0$, while z and t are arbitrary. Thus in general the leading short distance singularity $(x-y)^2 \to \infty$ of the instanton correction is the same as in conventional pertubation theory. The details depend on the details of the z and t behaviour and thus are unknown. However, one can show that the instanton contribution effectively renormalizes perturbative effects[4),12)], that it can be incorporated in the definition of the quark distributions and that in general it does not go away as $Q^2 \to \infty$.

Conventional perturbative calculations show that the Drell-Yan cross section factorizes into a product of quark and anti-quark distributions as measured in deep inelastic scattering. The question then arises whether this is true also for the distributions which include the effects of instanton corrections. When we calculate the Drell-Yan structure function in our "scalar photon" model, i.e., using the six-legged box diagram mentioned before and performing an expansion in the number of instantons, i.e.,

$$L = L_0 + \int d^4z \int d\rho\, d(\rho) (L_1 - L_0) + \cdots \tag{13}$$

instanton free contribution

correction due to one instanton and one anti-instanton

We find that $L \neq P$ (top) P (bottom) where P is the corresponding structure function for "deep inelastic scattering" in the same model, i.e., is given by:

$$P = P_0 + \int d^4z \int d\rho\, d(\rho) \left(P_1 - P_0\right) + \cdots \tag{14}$$

We find an additional contribution, which in general, is not expected to vanish at large Q^2. Its form and behaviour are model dependent and thus cannot be reliably calculated.

Because of this we do not claim that the inclusion of instantons definitely violates factorization. We only point out that the question of factorization is more complicated and that the existence of additional vacuum-like states in QCD can provide a source of factorization breakdown.

In conclusion: we have studied the effects of some non-perturbative phenomena (those associated with the existence of instantons) on physical processes at large momenta, i.e., where perturbative calculations are expected to be valid. We found that in a process where no large distances are involved ($\sigma(e^+e^- \rightarrow$ anything)) the non-pertubation effects we have studied vanish very rapidly, thus supporting the use of conventional perturbation theory. For the processes where some large distances are involved the instanton corrections are strickly speaking, not calculable without further assumptions. When we performed the analysis in a very simple model we found a breakdown of the factorization observed in perturbation theory. We regard this as a hint of what may happen in the true theory.

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