

RECEIVED: January 24, 2011

REVISED: February 1, 2011

ACCEPTED: February 2, 2011

PUBLISHED: February 9, 2011

Maximal electric dipole moments of nuclei with enhanced Schiff moments

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ABSTRACT: The electric dipole moments (EDMs) of heavy nuclei, such as ^{199}Hg , ^{225}Ra and ^{211}Rn , can be enhanced by the Schiff moments induced by the presence of nearby parity-doublet states. Working within the framework of the maximally CP-violating and minimally flavour-violating (MCPMFV) version of the MSSM, we discuss the maximal values that such EDMs might attain, given the existing experimental constraints on the Thallium, neutron and Mercury EDMs. The maximal EDM values of the heavy nuclei are obtained with the help of a differential-geometrical approach proposed recently that enables the maxima of new CP-violating observables to be calculated exactly in the linear approximation. In the case of ^{225}Ra , we find that its EDM may be as large as 6 to $50 \times 10^{-27} e \cdot \text{cm}$.

KEYWORDS: Supersymmetry Phenomenology

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1 Introduction

Electric dipole moments (EDMs) are among the most promising potential signatures for CP-violating physics beyond the Standard Model (SM), and one of the most promising options for extending the SM is provided by supersymmetry (SUSY) (for recent reviews, see [1, 2]). The minimal SUSY extension of the SM (the MSSM) already contains many possible CP-violating phases, even in its minimally flavour-violating (MFV) version. The maximally CP-violating MFV version of the MSSM, the MCPMFV model ([3]; for related approaches, see [4–9]), has six CP-violating phases, to which may be added the QCD vacuum phase θ_{QCD} . These phases are tightly constrained by the present experimental upper limits on the EDMs of ^{205}Tl , the neutron and ^{199}Hg . Nevertheless, one could in principle envisage (accidental) cancellations [10, 11] between the contributions to the measured EDMs of the six (seven) phases of the MCPMFV model (including θ_{QCD}), which might leave open the possibility of large net contributions to other EDMs. However, for any fixed values of the CP-conserving MCPMFV parameters, the compact ranges of the CP-violating parameters imply that the value of any other CP-violating observable, e.g., an EDM, is necessarily bounded. The question then arises whether the prospective sensitivity of any proposed experiment reaches below the maximum value attainable in any given theoretical framework, such as the MCPMFV model (with or without the possibility that $\theta_{\text{QCD}} \neq 0$). Clearly, any experiment that has insufficient sensitivity to search below the maximum value is not interesting for testing the MCPMFV model, whereas any experiment capable of reaching below the maximum value may either make a measurement or exclude part of the MCPMFV parameter space.

In a recent paper [12] we proposed a novel analytical technique, based on a differential-geometrical construction (see also [13]), for finding the maximal values of CP-violating observables subject to the existing EDM constraints, which is exact in the linear approximation. We applied this technique to find maximal values of the EDMs of the Deuteron and

muon, the CP-violating asymmetry in $b \rightarrow s\gamma$ decay, A_{CP} , and the B_s mixing phase [12]. We found that, whereas the EDM of the Deuteron in the MCPMFV model (allowing also for $\theta_{\text{QCD}} \neq 0$) might be one (two) orders of magnitude larger than the prospective experimental sensitivity, and A_{CP} might also be detectable, the EDM of the muon and the contribution to the B_s mixing phase in the MCPMFV model are likely to be too small to be observable in the near future.

In this paper we extend the applications of our analytical method [12, 13] to calculate the maximal values in the MCPMFV of the EDMs of some nuclei that are enhanced by the Schiff moment contributions associated with nearby parity-doublet states [14]. An experimental campaign is now being considered for HIE-ISOLDE that could search for EDMs of radium isotopes [15], accompanied by measurements of octupole collectivity in radium isotopes [16] that would be needed to interpret EDM measurements in terms of time-reversal violating interactions.

We find that values of the ^{225}Ra EDM that are considerably larger than $10^{-27} e \cdot \text{cm}$ (well within the estimated sensitivity of the proposed HIE-ISOLDE experiment) are possible in the MCPMFV model. How much larger depends quite sensitively on the implementation of the experimental constraint on the EDM of ^{199}Hg . As discussed below, several theoretical calculations of the EDM are available, and most of them give very similar allowed ranges for the EDM of ^{225}Ra , in the range 6 to $10 \times 10^{-27} e \cdot \text{cm}$. However, one theoretical calculation of the ^{199}Hg EDM yields much weaker constraints on the CP-violating parameters of the MCPMFV, and hence yields a much larger maximal value of the ^{225}Ra EDM, namely $50 \times 10^{-27} e \cdot \text{cm}$.

2 Schiff moments for selected nuclei

A CP-violating atomic EDM may arise from intrinsic EDMs of the constituent nucleons and atomic electrons, CP-odd electron-nucleon interactions, and the CP-odd nuclear moment known as the Schiff moment. The Schiff moments of several atoms have been calculated in the literature, and expressed in terms of sums of products of CP-even and CP-odd pion-nucleon-nucleon (πNN) couplings. We recall that the CP-even strong πNN interaction is given by [17]

$$\mathcal{L}_{\pi NN}^{\text{strong}} = g_{\pi NN} \left[\sqrt{2} (\bar{p}i\gamma_5 n \pi^+ + \bar{n}i\gamma_5 p \pi^-) + (\bar{p}i\gamma_5 p - \bar{n}i\gamma_5 n) \pi^0 \right], \quad (2.1)$$

where $g_{\pi NN} = 13.45$. On the other hand, the CP-odd (T-odd) πNN interactions are given by [17, 18]

$$\begin{aligned} \mathcal{L}_{\pi NN}^T &= \bar{g}_{\pi NN}^{(0)} \bar{N} \tau^a N \pi^a + \bar{g}_{\pi NN}^{(1)} \bar{N} N \pi^0 + \bar{g}_{\pi NN}^{(2)} (\bar{N} \tau^a N \pi^a - 3 \bar{N} \tau^3 N \pi^0) \\ &= \bar{g}_{\pi NN}^{(0)} \left[\sqrt{2} (\bar{p}n \pi^+ + \bar{n}p \pi^-) + (\bar{p}p - \bar{n}n) \pi^0 \right] \\ &\quad + \bar{g}_{\pi NN}^{(1)} (\bar{p}p + \bar{n}n) \pi^0 \\ &\quad + \bar{g}_{\pi NN}^{(2)} \left[\sqrt{2} (\bar{p}n \pi^+ + \bar{n}p \pi^-) - 2 (\bar{p}p - \bar{n}n) \pi^0 \right] \end{aligned} \quad (2.2)$$

in terms of the isoscalar $\bar{g}_{\pi NN}^{(0)}$, isovector $\bar{g}_{\pi NN}^{(1)}$, and isotensor $\bar{g}_{\pi NN}^{(2)}$ T-violating pion-nucleon couplings.

Atom	Ref.	Interaction	$-a_0$	$-a_1$	a_2	$-b$
^{199}Hg	[20]	—	0.0004	0.055	0.009	—
	[24]	SkO'	0.010	0.074	0.018	—
		(average)	0.007	0.071	0.018	—
	[19]	SLy4 (HF)	0.013	-0.006	0.022	0.003
		SIII (HF)	0.012	0.005	0.016	0.004
		SV (HF)	0.009	-0.0001	0.016	0.002
		SLy4 (HFB)	0.013	-0.006	0.024	0.007
SkM* (HFB)		0.041	-0.027	0.069	0.013	
^{225}Ra	[25]	SkO' (zero range)	-5.1	10.4	-10.1	—
	[26]	SkO'	-1.5	6.0	-4.0	—
^{211}Rn	[27]		0.019	-0.061	0.053	—
	[19]	SLy4	0.042	-0.018	0.071	0.016
		SkM*	0.042	-0.028	0.078	0.015
		SIII	0.034	-0.0004	0.064	0.015

Table 1. The coefficients $a_i, i = 0, 1, 2$ and b of the Schiff moments of ^{199}Hg , ^{225}Ra , and ^{211}Rn , expressed in units of $e \cdot \text{fm}^3$. The labels HF and HFB stand for calculations in the Hartree-Fock and Hartree-Fock-Bogoliubov approximations, respectively: see ref. [19] for details. We have changed the signs of the coefficients a_0 , a_1 , and b to follow the conventions of ref. [18].

The Schiff moment is linear in the CP-odd πNN couplings $\bar{g}_{\pi NN}^{(i)}$, and may be written as [19]

$$S = (a_0 + b) g_{\pi NN} \bar{g}_{\pi NN}^{(0)} + a_1 g_{\pi NN} \bar{g}_{\pi NN}^{(1)} + (a_2 - b) g_{\pi NN} \bar{g}_{\pi NN}^{(2)}. \quad (2.3)$$

Here the coefficients a_i specify the dependence of the Schiff moment on the CP-odd interactions and the coefficient b specifies its dependence on the nucleon dipole moments. The coefficients a_i with $i = 0, 1, 2$ and b depend on the type of atom of interest, and some theoretical estimates for ^{199}Hg , ^{225}Ra and ^{211}Rn , in units of $e \cdot \text{fm}$ are collected in table 1.

In the case of the ^{199}Hg Schiff moment, the results [20] adopted in [18, 21–23] are significantly different from the more recent calculations of [19, 24]. In particular, the coefficient a_0 may be enhanced by a factor of ~ 30 , whilst the coefficient a_1 may be reduced by a factor of ~ 10 and could even take the opposite sign. After considering the different numbers from the most recent calculations [19] in table 1, we have used the SIII (HF) calculation for our numerical illustration. As motivation, we note that the first SLy4 and the SV calculations do not yield the right quantum numbers for the ground state, that the second SLy4 calculation was found to be unable to project HFB states, and that the SkM* calculation gives a spherical minimum.

The CP-odd couplings $\bar{g}_{\pi NN}^{(i)}$ may be generated by chromoelectric dipole moments (CEDMs) of the quarks and/or dimension-six four-fermion interactions. In ref. [28], one may find full expressions for the contributions to the CP-odd isoscalar $\bar{g}_{\pi NN}^{(0)}$ and isovector

$\bar{g}_{\pi NN}^{(1)}$ couplings from the CEDMs of the light quarks:¹

$$\begin{aligned}\bar{g}_{\pi NN}^{(0)} &= (-0.5 \text{ to } 1.5) \times 10^{-12} \frac{(d_u^C + d_d^C)/g_s}{10^{-26} \text{ cm}} \frac{|\langle \bar{q}q \rangle|}{(225 \text{ MeV})^3}, \\ \bar{g}_{\pi NN}^{(1)} &= 2_{-1}^{+4} \times 10^{-12} \frac{(d_u^C - d_d^C)/g_s}{10^{-26} \text{ cm}} \frac{|\langle \bar{q}q \rangle|}{(225 \text{ MeV})^3}.\end{aligned}\tag{2.4}$$

We note that a ‘best value’ is not available for $\bar{g}_{\pi NN}^{(0)}$. For definiteness, we have taken $\bar{g}_{\pi NN}^{(0)}/\bar{g}_{\pi NN}^{(1)} = 0.2 (d_u^C + d_d^C)/(d_u^C - d_d^C)$, as follows from assuming that the couplings $\bar{g}_{\pi NN}^{(0)}$ and $\bar{g}_{\pi NN}^{(1)}$ are proportional to the matrix elements of $\bar{u}u - \bar{d}d$ and $\bar{u}u + \bar{d}d$ in the nucleon state, respectively. It is known that the matrix element with the minus (−) sign is smaller than the one with the plus (+) sign by a factor ~ 5 to 10 [29].

We have also included the contribution of the dimension-six four-fermion interactions to the isovector coupling which can be enhanced for a large value of $\tan \beta$ [30, 31]:

$$\begin{aligned}\bar{g}_{\pi NN}^{(1)} &= \frac{\langle \bar{q}q \rangle}{2f_\pi} \langle N | C_{dd} \bar{d}d + C_{sd} \bar{s}s + C_{bd} \bar{b}b | N \rangle \\ &= -8 \times 10^{-3} \text{ GeV}^3 \left[\frac{0.5 C_{dd}}{m_d} + 3.3 \kappa \frac{C_{sd}}{m_s} + (1 - 0.25 \kappa) \frac{C_{bd}}{m_b} \right],\end{aligned}\tag{2.5}$$

where the couplings appearing in (2.5) are defined via the interaction Lagrangian

$$\mathcal{L}_{4f} = \sum_{f, f'} C_{ff'} (\bar{f}f) (\bar{f}'i\gamma_5 f') ,\tag{2.6}$$

and $\kappa \equiv \langle N | m_s \bar{s}s | N \rangle / 220 \text{ MeV} \simeq 0.50 \pm 0.25$. The contribution of the four-fermion interactions to the isoscalar coupling $\bar{g}_{\pi NN}^{(0)}$ is ignored and the isotensor coupling $\bar{g}_{\pi NN}^{(2)}$, which changes the isospin by two units, is neglected in this work (as in ref. [28]), since it can be generated only at the expense of an additional $m_u - m_d$ suppression.

In the following, we concentrate on the EDMs of ¹⁹⁹Hg and ²²⁵Ra atoms, because there is already a stringent experimental upper limit on the former, and there is a proposal to measure the latter at HIE-ISOLDE [15]. Moreover, we note that the estimates of the contributions to the Schiff moment of ²¹¹Rn shown in table 1 are considerably less favourable.

The experimental constraint on the EDM of ¹⁹⁹Hg [32, 33],

$$|d_{\text{Hg}}^{\text{EXP}}| < 3.1 \times 10^{-29} e \cdot \text{cm} \text{ (95 \% C.L.)},\tag{2.7}$$

is among the most stringent constraints on CP violation imposed by atomic EDM experiments. There are several calculations of the ¹⁹⁹Hg EDM in terms of the Schiff moment in the literature, which can be cast in the general form

$$d_{\text{Hg}}[S] = 10^{-17} C_{\text{Hg}}^S e \cdot \text{cm} \times \left(\frac{S}{e \cdot \text{fm}^3} \right),\tag{2.8}$$

with the coefficient taking the values $C_{\text{Hg}}^S = -4$ [34, 35], -2.8 [36] and $+5.07$ [37]. We note that we have changed the sign of the most recent calculation based on relativistic coupled-cluster theory [37], so as to match the conventions in the previous calculations [34–36]

¹We adopt the conventions and notations of [21–23].

based on coupled perturbed Hartree-Fock calculations, and that it has a different sign. For our numerical calculations we take the estimates of the coefficient $\mathcal{C}_{\text{Hg}}^S$ that have negative signs. To be specific, we have used the following four calculations of the Mercury EDM:

- From ref. [18],

$$d_{\text{Hg}}^{\text{I}}[S] \simeq 1.8 \times 10^{-3} e \bar{g}_{\pi NN}^{(1)} / \text{GeV}, \quad (2.9)$$

- Taking $\mathcal{C}_{\text{Hg}}^S = -2.8$ [36] and the coefficients from ref. [20],

$$d_{\text{Hg}}^{\text{II}}[S] \simeq 7.6 \times 10^{-6} e \bar{g}_{\pi NN}^{(0)} / \text{GeV} + 1.0 \times 10^{-3} e \bar{g}_{\pi NN}^{(1)} / \text{GeV}, \quad (2.10)$$

- Taking $\mathcal{C}_{\text{Hg}}^S = -2.8$ [36] and the average coefficients from ref. [24],

$$d_{\text{Hg}}^{\text{III}}[S] \simeq 1.3 \times 10^{-4} e \bar{g}_{\pi NN}^{(0)} / \text{GeV} + 1.4 \times 10^{-3} e \bar{g}_{\pi NN}^{(1)} / \text{GeV}, \quad (2.11)$$

- Taking $\mathcal{C}_{\text{Hg}}^S = -2.8$ [36] the SIII(HF) coefficients from ref. [19],

$$d_{\text{Hg}}^{\text{IV}}[S] \simeq 3.1 \times 10^{-4} e \bar{g}_{\pi NN}^{(0)} / \text{GeV} + 9.5 \times 10^{-5} e \bar{g}_{\pi NN}^{(1)} / \text{GeV}. \quad (2.12)$$

Compared to the expression $d_{\text{Hg}}^{\text{I}}[S]$ used in refs. [21–23], $d_{\text{Hg}}^{\text{II}}[S]$ is slightly smaller, and has a negligible isoscalar contribution, which is actually ignored in $d_{\text{Hg}}^{\text{I}}[S]$. Whilst the isoscalar contribution is still small in $d_{\text{Hg}}^{\text{III}}[S]$, it can be larger than the isovector contribution by a factor ~ 3 in $d_{\text{Hg}}^{\text{IV}}[S]$. However, we observe that $|d_{\text{Hg}}^{\text{IV}}[S]|$ is smaller than $|d_{\text{Hg}}^{\text{I}}[S]|$ by about an order of magnitude.

On the other hand, the EDM of ^{225}Ra is related to its Schiff moment by [36]

$$d_{\text{Ra}}[S] = -8.5 \times 10^{-17} e \cdot \text{cm} \times \left(\frac{S}{e \cdot \text{fm}^3} \right). \quad (2.13)$$

Taking the coefficients given in ref. [26], see table 1, we obtain

$$d_{\text{Ra}}[S] \simeq -8.7 \times 10^{-2} e \bar{g}_{\pi NN}^{(0)} / \text{GeV} + 3.5 \times 10^{-1} e \bar{g}_{\pi NN}^{(1)} / \text{GeV}. \quad (2.14)$$

We note that the $\bar{g}_{\pi NN}^{(1)}$ contribution the EDM of ^{225}Ra is about 200 times larger than to the Mercury EDM $d_{\text{Hg}}^{\text{I}}[S]$, an enhancement due to the existence of a nearby parity-doublet states [25].

3 Differential-geometrical optimization method

We briefly review our powerful analytical approach for finding the optimal choice of CP-odd phases which maximize the size of a given CP-violating observable O , while remaining compatible with the present EDM constraints [13]. We have applied this approach previously to estimate maximal values of the Deuteron and muon EDMs, the CP asymmetry in $b \rightarrow s\gamma$, and the phase in B_s mixing [12], and it may be applied similarly to the case where the observable O is the EDM of ^{225}Ra .

We consider a theory such as the MCPMFV SUSY model with six CP-odd phases, Φ , represented by a 6D phase vector, subject to three EDM constraints denoted by $E^{a,b,c} = 0$, corresponding to the non-observation of the Thallium, neutron and Mercury EDMs. For any given value of the CP-conserving parameters in the MCPMFV model, we may expand these EDMs and the observable O in the small-phase approximation, defining the four 6D vectors $\mathbf{E}^{a,b,c} = \nabla E^{a,b,c}$ and $\mathbf{O} = \nabla O$, and we assume that the four vectors $\mathbf{E}^{a,b,c}$ and \mathbf{O} are linearly independent.

We then introduce the triple exterior product

$$A_{\alpha\beta\gamma} \equiv E_{[\alpha}^a E_{\beta}^b E_{\gamma]}^c, \quad (3.1)$$

where the Greek indices label the components of the vectors in the 6D space, i.e., $\alpha, \beta, \gamma = 1, 2, \dots, 6$. The square brackets on the r.h.s. of (3.1) indicate that the tensor $A_{\alpha\beta\gamma}$ is obtained by fully antisymmetrizing the vectors $E_{\alpha}^a, E_{\beta}^b$ and E_{γ}^c in the indices α, β, γ , i.e., $A_{\alpha\beta\gamma} = -A_{\beta\alpha\gamma} = -A_{\alpha\gamma\beta}$, etc. Borrowing a term from the calculus of differential forms, $A_{\alpha\beta\gamma}$ is a 3-form. We also introduce the 2-form

$$B_{\mu\nu} = \varepsilon_{\mu\nu\lambda\rho\sigma\tau} O_{\lambda} E_{\rho}^a E_{\sigma}^b E_{\tau}^c, \quad (3.2)$$

where summation over repeated indices is implied and $\varepsilon_{\mu\nu\lambda\rho\sigma\tau}$ is the usual Levi-Civita tensor generalized to 6D. In the language of differential forms, $B_{\mu\nu}$ is, up to an irrelevant overall factor, the Hodge-dual product between the 1-form O_{λ} , representing the CP-violating observable, and the 3-form $A_{\alpha\beta\gamma}$.

The components Φ_{α}^* of the optimal EDM-free direction maximizing O can now be obtained from the Hodge-dual product of the 3-form $A_{\beta\gamma\delta}$ and the 2-form $B_{\mu\nu}$. Explicitly,

$$\Phi_{\alpha}^* = \mathcal{N} \varepsilon_{\alpha\beta\gamma\delta\mu\nu} A_{\beta\gamma\delta} B_{\mu\nu} = \mathcal{N} \varepsilon_{\alpha\beta\gamma\delta\mu\nu} \varepsilon_{\mu\nu\lambda\rho\sigma\tau} E_{\beta}^a E_{\gamma}^b E_{\delta}^c O_{\lambda} E_{\rho}^a E_{\sigma}^b E_{\tau}^c, \quad (3.3)$$

where we have included an unknown overall normalization factor \mathcal{N} . By construction, the 6D phase vector Φ^* is orthogonal to the three vectors $\mathbf{E}^{a,b,c}$, and therefore satisfies the desired EDM constraints, $E^a = E^b = E^c = 0$, in the small-phase approximation. We observe that the magnitude $\phi^* \equiv |\Phi^*|$, and hence the overall normalization factor \mathcal{N} , can only be determined by a numerical analysis of the actual experimental limits on the three EDMs. As in the 3D example, the maximum allowed value of the CP-violating observable O is given in the small-phase approximation by

$$O = \phi^* \widehat{\Phi}_{\kappa}^* O_{\kappa} = \pm \mathcal{N} \left| \varepsilon_{\mu\nu\alpha\beta\gamma\delta} \varepsilon_{\mu\nu\lambda\rho\sigma\tau} O_{\alpha} O_{\lambda} E_{\beta}^a E_{\gamma}^b E_{\delta}^c E_{\rho}^a E_{\sigma}^b E_{\tau}^c \right|, \quad (3.4)$$

where the caret denotes the components of a unit-norm vector. As discussed in [12], quadratic and higher-order derivative terms with respect to the CP-odd phases will generically prefer a particular sign for the optimal value of O .

We can also allow for the possible presence of a non-zero strong CP phase θ_{QCD} in the theory, in which case the corresponding CP-odd phase vector Φ becomes seven-dimensional (7D) in the MCPMFV SUSY model. The generalization of the above construction of the optimal value of the observable O is discussed in section 5 of [12].

4 Results

In this section we use the above the differential-geometrical approach to analyze the maximal value of the ^{225}Ra EDM obtainable in CP-violating variants of the following representative CMSSM benchmark scenario which predicts the mass spectrum of SUSY particles in the sub-TeV region:

$$\begin{aligned}
 |M_{1,2,3}| &= 350 \text{ GeV}, \\
 M_{H_u}^2 &= M_{H_d}^2 = \widetilde{M}_Q^2 = \widetilde{M}_U^2 = \widetilde{M}_D^2 = \widetilde{M}_L^2 = \widetilde{M}_E^2 = (100 \text{ GeV})^2, \\
 |A_u| &= |A_d| = |A_e| = 100 \text{ GeV},
 \end{aligned}
 \tag{4.1}$$

at the GUT scale, introducing non-zero CP-violating phases and varying $\tan\beta$ (M_{SUSY}). We adopt the convention that $\Phi_\mu = 0^\circ$, and we vary independently the following six MCPMFV phases at the GUT scale: $\Phi_1, \Phi_2, \Phi_3, \Phi_{A_u}, \Phi_{A_d}$, and Φ_{A_e} . In the 7D case, in addition to the 6 CP phases, we consider a non-zero strong CP phase θ_{QCD} . The scenario (4.1) considered in this work is similar to one with $|M_{1,2,3}| = 250 \text{ GeV}$ that we considered previously [3, 12, 21]. The somewhat larger value $|M_{1,2,3}| = 350 \text{ GeV}$ is chosen here for consistency with the recent SUSY search results reported by the CMS Collaboration at the LHC [38]. When $\tan\beta = 10$, $\Phi_{1,2,3} = 0^\circ$, and $\Phi_{A_u, A_d, A_e} = 180^\circ$, the previous scenario with $|M_{1,2,3}| = 250 \text{ GeV}$ became the well-known SPS1a point [39–42] (also known as benchmark B). The scenario (4.1) is more similar to benchmarks C, G and I of [39–42] when $\tan\beta = 10, 20$ or 35 , respectively. Our calculations of the EDMs are based on refs. [12, 21], which include the two-loop diagrams mediated by the γ - H^\pm - W^\mp and γ - W^\pm - W^\mp couplings, and we take into account the effects of the different computations of the Schiff moment of the Mercury nucleus as explained in section 2.

4.1 The MCPMFV SUSY model with 6 CP phases

In order to analyze the scenario (4.1), we first make Taylor expansions of the following four EDMs in terms of the MCPMFV CP-violating phases:

$$d_{\text{Tl}}/d_{\text{Tl}}^{\text{EXP}}, \quad d_n/d_n^{\text{EXP}}, \quad d_{\text{Hg}}/d_{\text{Hg}}^{\text{EXP}}, \quad d_{\text{Ra}}/(10^{-27} e \cdot \text{cm}), \tag{4.2}$$

where we choose the following normalization factors:

$$d_{\text{Tl}}^{\text{EXP}} = 9 \times 10^{-25} e \cdot \text{cm}, \quad d_n^{\text{EXP}} = 3 \times 10^{-26} e \cdot \text{cm}, \quad d_{\text{Hg}}^{\text{EXP}} = 3.1 \times 10^{-29} e \cdot \text{cm}, \tag{4.3}$$

which correspond to the current experimental limits on the EDMs of Thallium [43], the neutron [44], and Mercury [32, 33]. The normalization factor for the ^{225}Ra EDM, namely $10^{-27} e \cdot \text{cm}$, is typical of the estimated experimental sensitivity.

In figure 1, we show the absolute values of the components of the three 6D MCPMFV vectors characterizing the existing EDM constraints and the 6D vector representing the d_{Ra} observable, for the scenario (4.1) varying $\tan\beta$ in a small-phase expansion around the CP-conserving point $\Phi_1 = \Phi_2 = \Phi_3 = \Phi_{A_u} = \Phi_{A_d} = \Phi_{A_e} = 0^\circ$.² The solid lines represent

²We also display (as magenta lines) the corresponding components for 7th components corresponding to the QCD phase $\bar{\theta}$, which we discuss later.

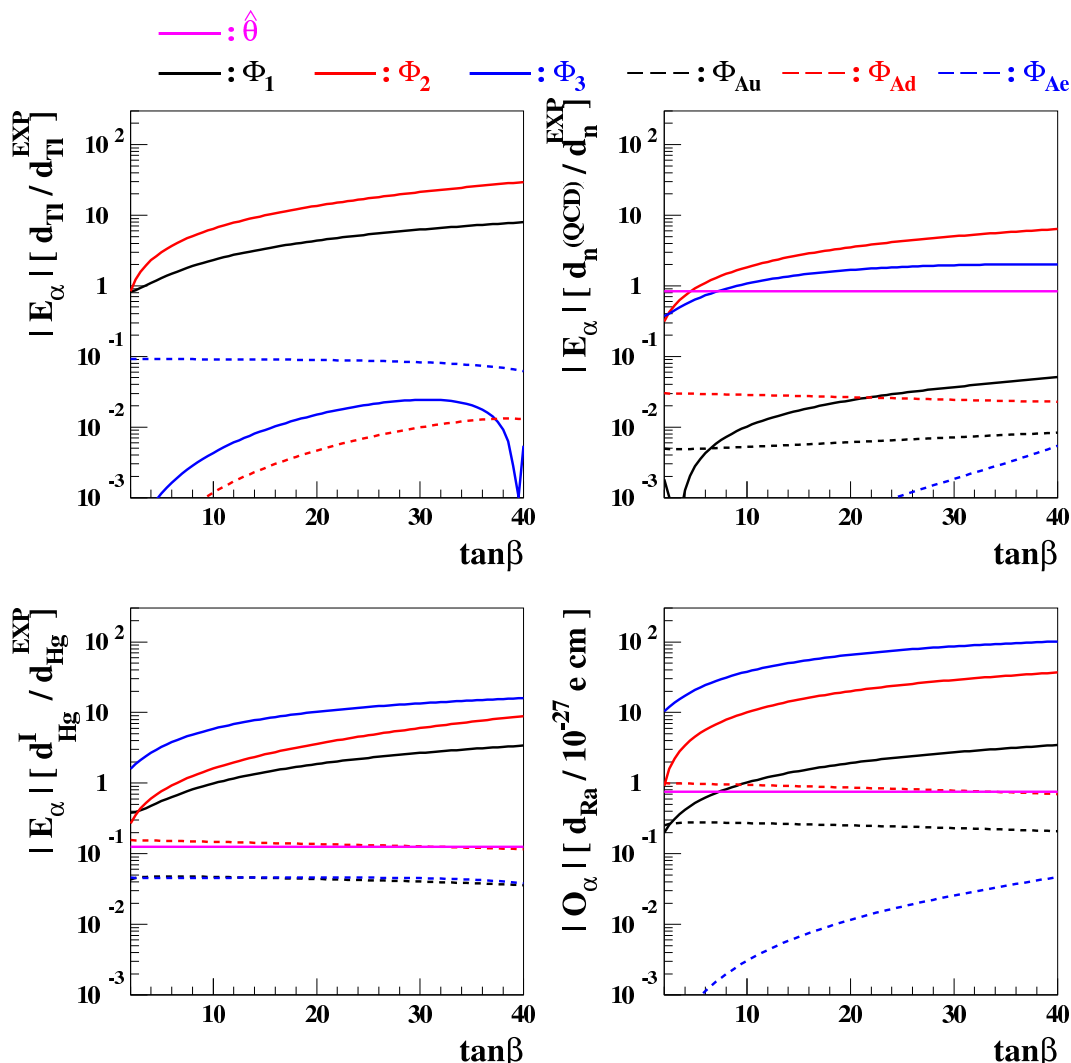


Figure 1. The absolute values of the components of the three vectors \mathbf{E}_α of MCPMFV CP-violating phases representing the present EDM constraints on d_{Tl} (upper left), d_n (upper right), and d_{Hg} (lower left), and those of the vector representing the EDM of ^{225}Ra (lower right) in small-phase expansions around the CP-conserving point, as functions of $\tan\beta$ for the scenario (4.1). We use the computation employing the QCD sum rule technique and the estimate d_{Hg}^I (2.9) for the neutron and Mercury EDMs, respectively. The CP-violating phases are measured in degrees.

the CP-violating phases of the gaugino mass parameters, and the dashed lines the trilinear A parameters, respectively. The components corresponding to the CP-violating gaugino phases dominate in all cases, increasingly as $\tan\beta$ grows, with the exception of the Φ_3 component of $\mathbf{E}^{d_{Tl}}$. For example, when $\tan\beta = 40$, $|d_{Tl}|$ and $|d_n|$ are larger than the current experimental limits by factors ~ 30 and ~ 5 , respectively, even when $\Phi_2 = 1^\circ$, whereas $|d_{Hg}^I|$ is larger than the current limit by a factor ~ 15 (~ 8) when $\Phi_3 = 1^\circ$ ($\Phi_2 = 1^\circ$).³ We see that $|d_{Ra}|$ can be as large as $\sim 100 \times 10^{-27} e\text{ cm}$ ($\sim 40 \times 10^{-27} e\text{ cm}$) if $\Phi_3 = 1^\circ$ ($\Phi_2 = 1^\circ$).

³Throughout this work, the CP-violating phases are measured in units of degrees.

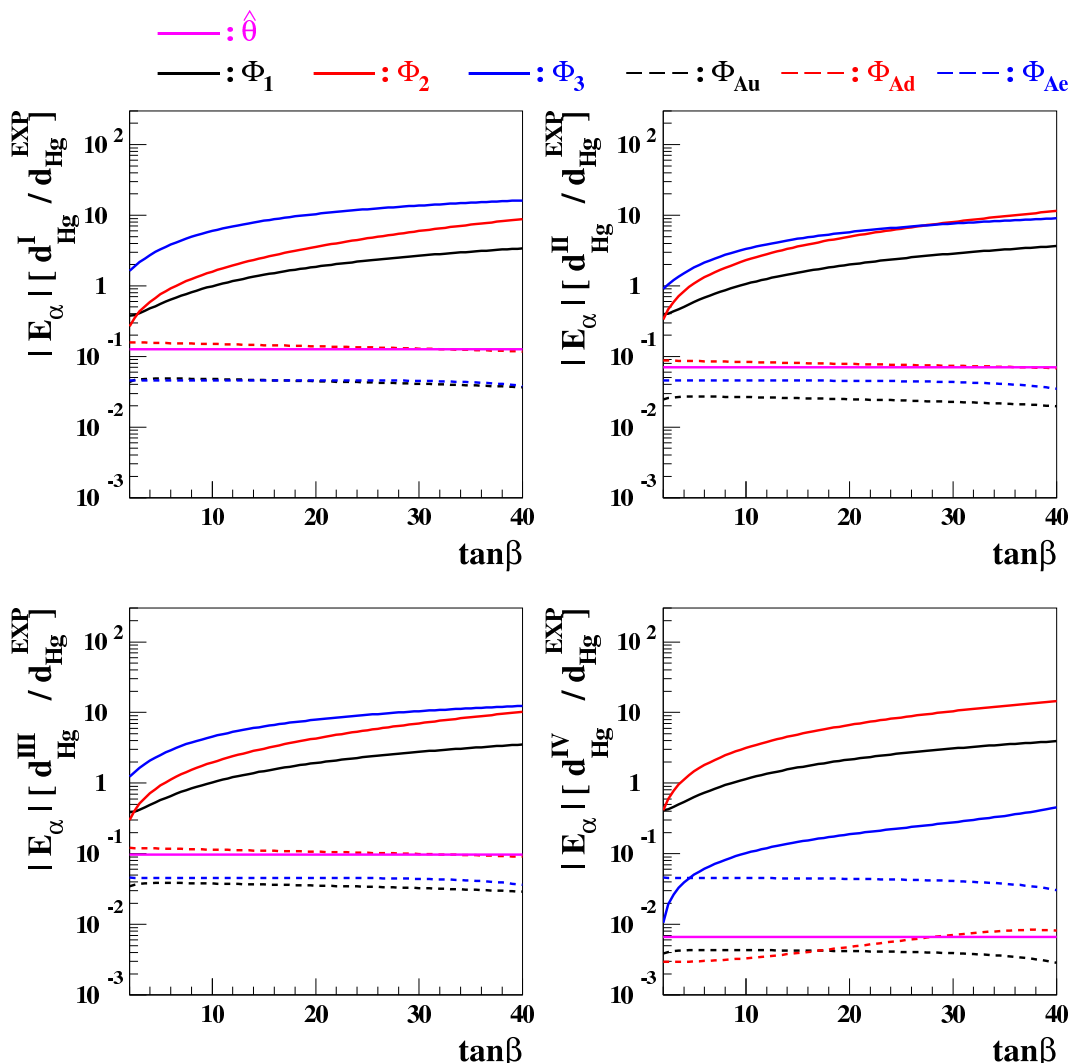


Figure 2. The absolute values of the components of the 6D vector \mathbf{E}_α of MCPMFV CP-violating phases representing the present EDM constraints on the Mercury EDM, as obtained using the four different calculations of d_{Hg} discussed in section 2: d_{Hg}^{I} (upper left), $d_{\text{Hg}}^{\text{II}}$ (upper right), $d_{\text{Hg}}^{\text{III}}$ (lower left), and $d_{\text{Hg}}^{\text{IV}}$ (lower right). The line styles are the same as in figure 1. The CP-violating phases are measured in degrees.

The relative contributions of the different CP-violating MCPMFV phases to d_{Hg} vary according to the choice of theoretical calculation, as shown in figure 2.⁴ Specifically we observe the Φ_3 contribution exhibits strong variations, and that it is much suppressed if the $|d_{\text{Hg}}^{\text{IV}}|$ calculation is used.

In figure 3 we show the cosines of the angles between the 6D MCPMFV vector representing the observable $\mathbf{O}^{d_{\text{Ra}}}$ and the EDM vectors, which are defined by

$$C_{\mathbf{O}, \mathbf{E}^a} = \frac{\mathbf{O} \cdot \mathbf{E}^a}{|\mathbf{O}| |\mathbf{E}^a|} . \quad (4.4)$$

⁴We again display (as magenta lines) the corresponding components for possible 7th components corresponding to the QCD phase $\bar{\theta}$, which we discuss later.

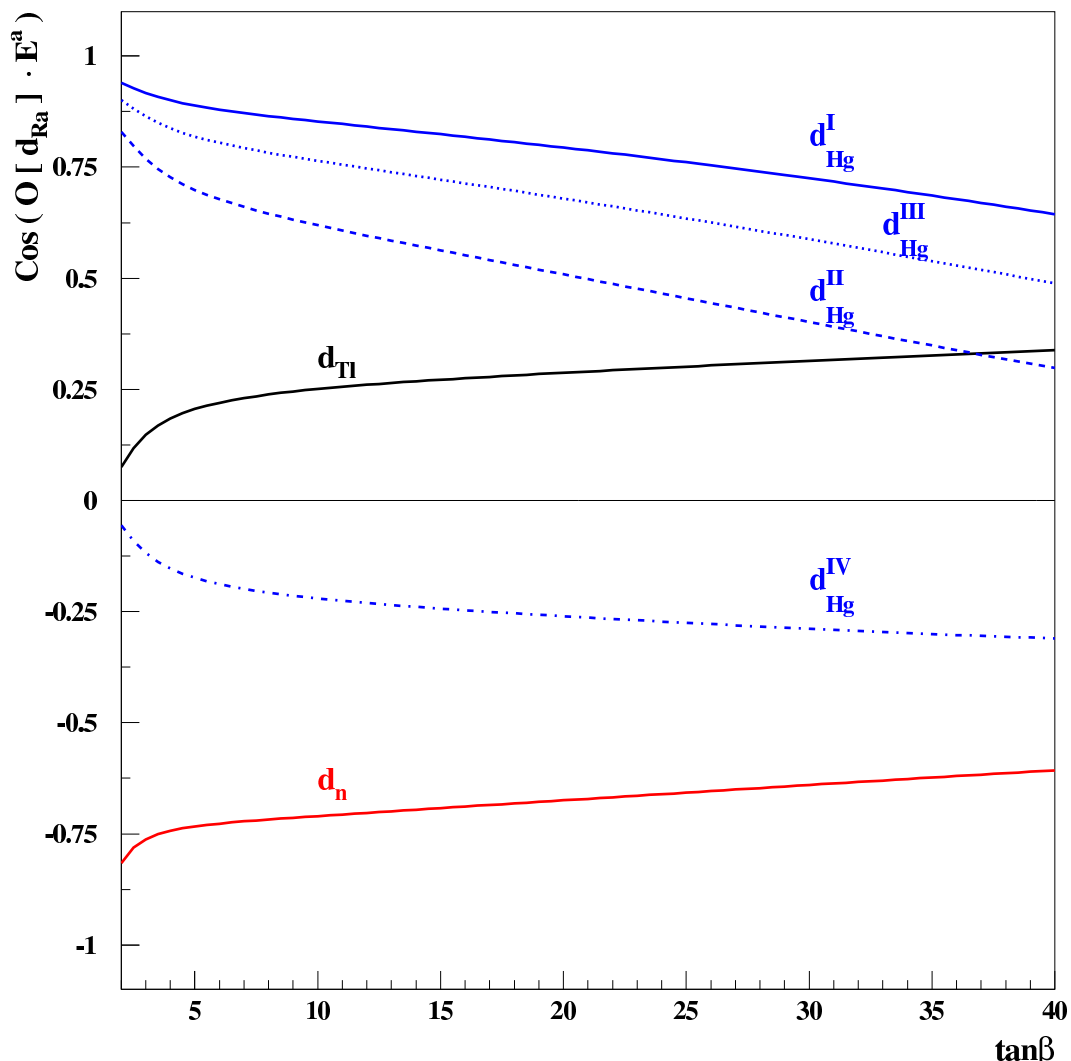


Figure 3. The cosines of the angles between the observable 6D vector $\mathbf{O}^{d_{Ra}}$ and the EDM-constraint 6D vectors $\mathbf{E}^{d_{Tl}}$ (black solid), \mathbf{E}^{d_n} (red solid), $\mathbf{E}^{d_{Hg}^I}$ (blue solid), $\mathbf{E}^{d_{Hg}^{II}}$ (blue dashed), $\mathbf{E}^{d_{Hg}^{III}}$ (blue dotted), and $\mathbf{E}^{d_{Hg}^{IV}}$ (blue dash-dotted) as functions of $\tan\beta$.

These quantities display the degrees of degeneracy, or alignment, between the observable $\mathbf{O}^{d_{Ra}}$ and the EDM vectors E^a . We see that the alignments between d_{Ra} and d_{Tl} and d_{Hg}^{IV} are quite small. On the other hand, the alignments between d_{Ra} and $d_{Hg}^{I,II,III}$ are larger, though decreasing as $\tan\beta$ grows. This can be understood by observing that the additional contributions to the Mercury EDM, beyond those from the couplings $\bar{g}_{\pi NN}^{(0),(1)}$, increase with $\tan\beta$.

Having the vectors representing the EDM constraints and the observable d_{Ra} in hand, we now combine them to construct the optimal directions in the 6D space of CP-violating MCPMFV phases, using (3.3), so as to maximize d_{Ra} in the linear approximation. For comparison, we also consider two reference directions, which have $\Phi_1 = \Phi_{A_e} = 0$ and $\Phi_2 = \Phi_3 = 0$, respectively. These two reference directions can be constructed by defining

$$\Phi_\alpha^* \equiv \mathcal{N} \varepsilon_{\alpha\beta\gamma\delta\mu\nu} E_\beta^a E_\gamma^b E_\delta^c N_\mu^{(1)} N_\nu^{(2)}, \quad (4.5)$$

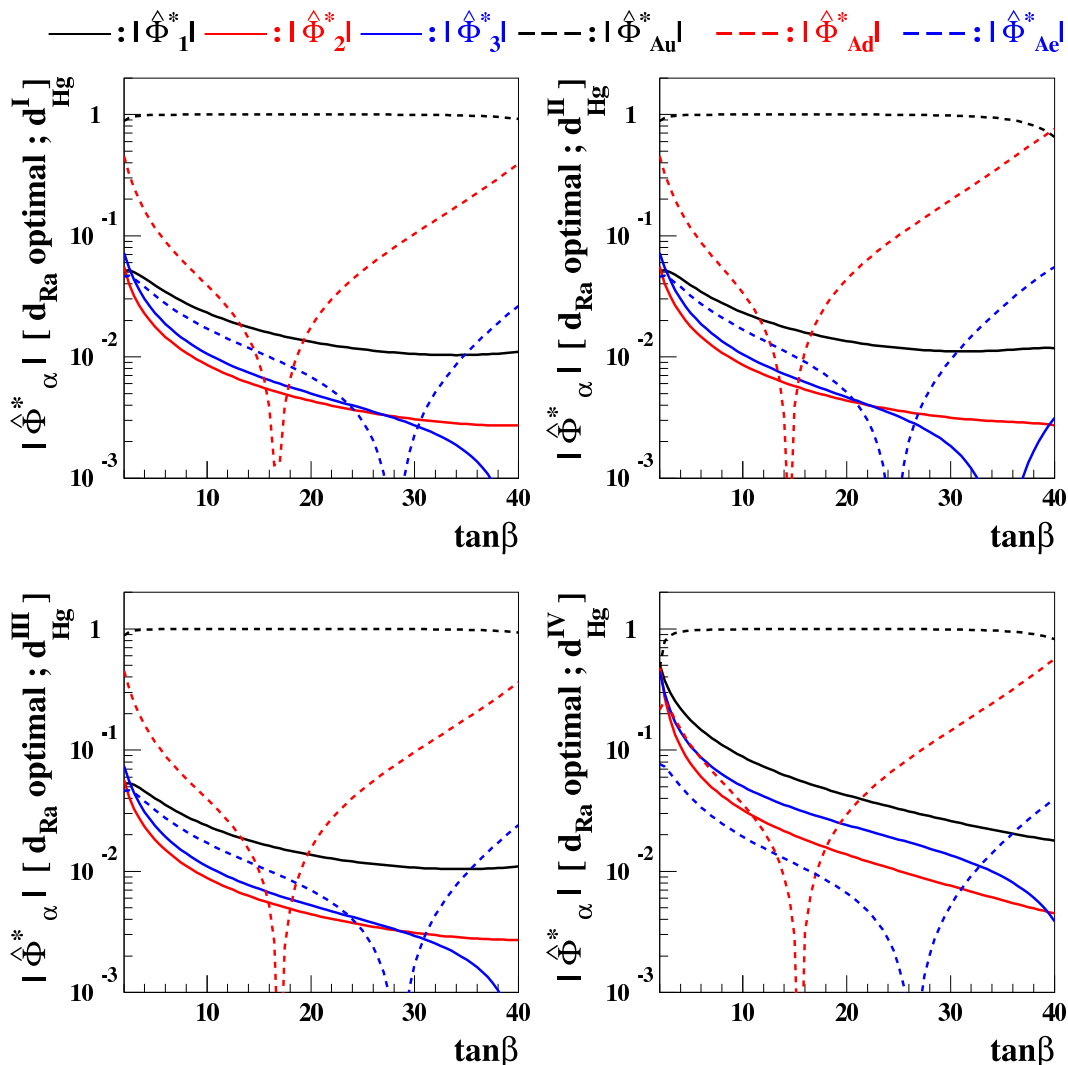


Figure 4. The absolute values of the six components of the normalized vectors in the directions where the size of d_{Ra} is optimized, as obtained using the calculations d_{Hg}^{I} (upper left), $d_{\text{Hg}}^{\text{II}}$ (upper right), $d_{\text{Hg}}^{\text{III}}$ (lower right), and $d_{\text{Hg}}^{\text{IV}}$ (lower left) for the Mercury EDM. The CP-violating phases are measured in degrees.

where, for each direction, the two null directions $N_{\mu}^{(1,2)}$ are chosen as

$$N_{\mu}^{(1)} = (1, 0, 0, 0, 0, 0), \quad N_{\mu}^{(2)} = (0, 0, 0, 0, 0, 1) \quad (4.6)$$

for the direction $\Phi_1 = \Phi_{Ae} = 0$, and

$$N_{\mu}^{(1)} = (0, 1, 0, 0, 0, 0), \quad N_{\mu}^{(2)} = (0, 0, 1, 0, 0, 0) \quad (4.7)$$

for the direction $\Phi_2 = \Phi_3 = 0$.

We display in figure 4 the absolute values of the six components of the normalized optimal vectors in the direction along which d_{Ra} is maximized, as obtained using d_{Hg}^{I} (upper left), $d_{\text{Hg}}^{\text{II}}$ (upper right), $d_{\text{Hg}}^{\text{III}}$ (lower right), and $d_{\text{Hg}}^{\text{IV}}$ (lower left) for the Mercury EDM. We first observe that the $\Phi_{1,2,3}$ components (solid lines) are relatively small, and

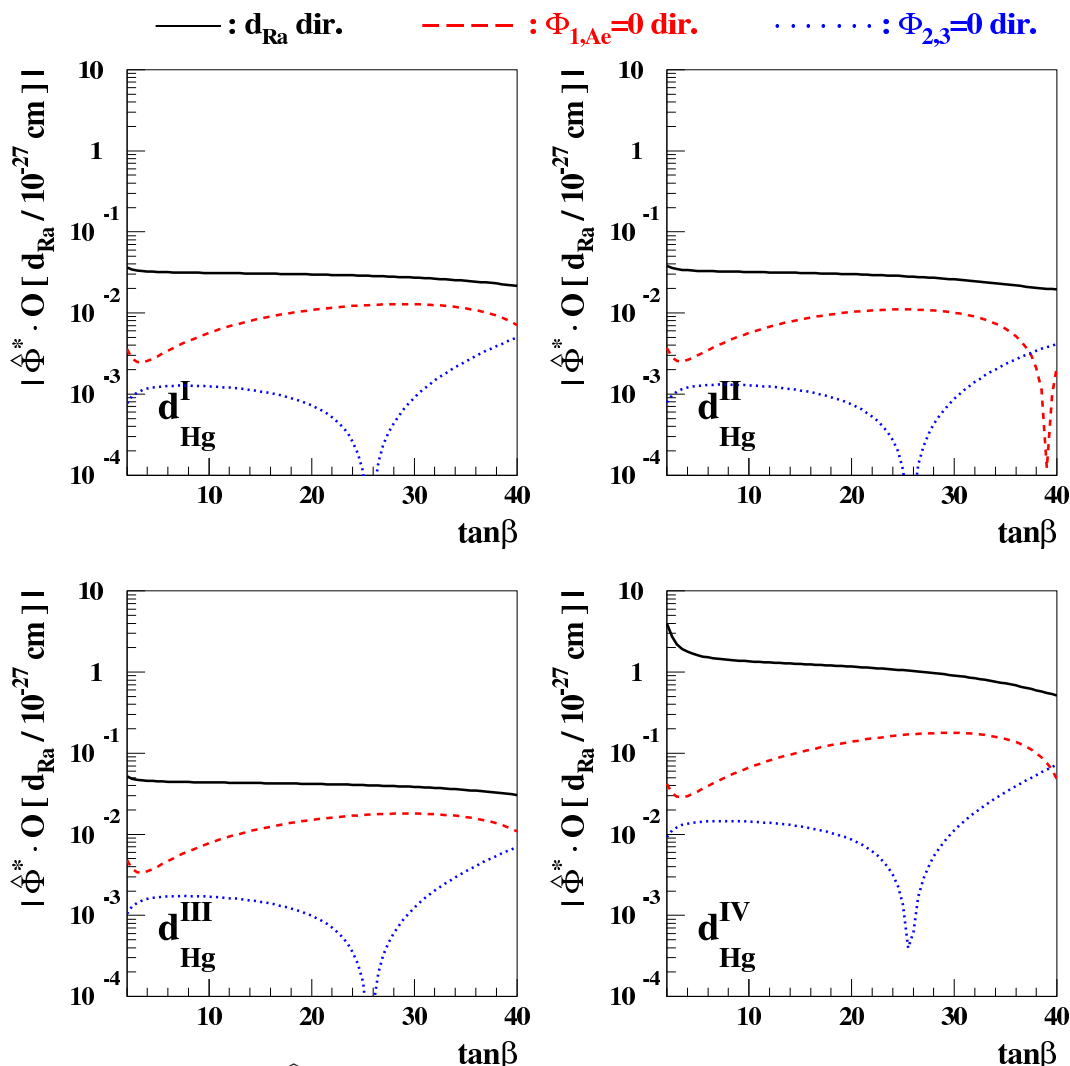


Figure 5. The products $\hat{\Phi}^* \cdot \mathbf{O}$ (black solid) for the optimal d_{Ra} direction, as obtained using the calculations d_{Hg}^I (upper left), d_{Hg}^{II} (upper right), d_{Hg}^{III} (lower right), and d_{Hg}^{IV} (lower left) for the Mercury EDM. For comparison purposes, we also show the products along the two reference directions with $\Phi_{1,Ae} = 0$ (red dashed) and $\Phi_{2,3} = 0$ (blue dotted).

decrease as $\tan\beta$ increases. Hence, all the optimal directions are mostly given by some combination of Φ_{Au} (black dashed line) and Φ_{Ad} (red dashed line) directions implying, for $\tan\beta = 40$, that $(\Phi_{Au,A_d})^{\max} \sim \phi^*$ whereas $(\Phi_{1,2,3})^{\max} \sim \phi^* \times 10^{-2}$, as will be shown in the following.

In figure 5, we consider the products $\hat{\Phi}^* \cdot \mathbf{O}$ of the 6D vectors in the normalized optimal directions for d_{Ra} , and the Radium EDM, taking account of the uncertainty of the Mercury EDM calculation. The products determine the sizes of d_{Ra} along its optimal direction through the relations given in (3.4) when $\phi^* = 1^\circ$. As shown below, ϕ^* could be as large as $\sim 100^\circ$ before the small-phase approximation breaks down and one of the three EDM constraints is violated. We observe that the direction constructed using the geometric prescription given in section 3 indeed gives the larger values of d_{Ra} than do the

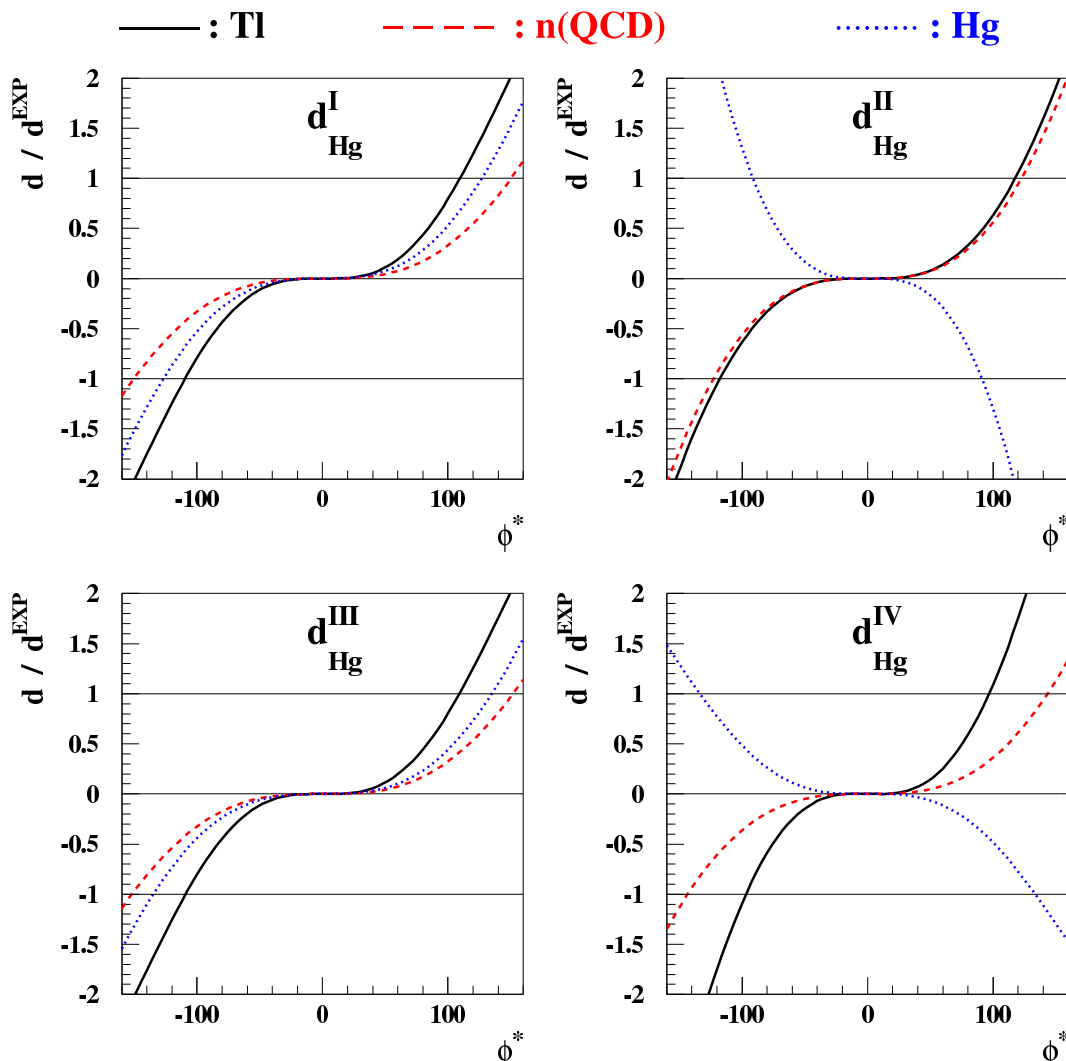


Figure 6. The values of the three EDMs along the directions optimized for d_{Ra} , as obtained using d_{Hg}^I (upper left), d_{Hg}^{II} (upper right), d_{Hg}^{III} (lower right), and d_{Hg}^{IV} (lower left) for the Mercury EDM. The ratios d_{TI}/d_{TI}^{EXP} , d_n/d_n^{EXP} , and d_{Hg}/d_{Hg}^{EXP} are denoted by the black solid, red dashed, and blue dotted lines, respectively. The scenario (4.1) is assumed, with the choice $\tan \beta = 40$.

two reference directions with $\Phi_{1,A_e} = 0$ and $\Phi_{2,3} = 0$. We note in particular that, in the case of d_{Hg}^{IV} , the Radium EDM may become about an order of magnitude larger than is possible with another choices for the Mercury EDM calculation.

Figure 6 shows the Thallium (solid), neutron (dashed) and Mercury (dotted) EDMs along the directions chosen to optimize d_{Ra} for the four choices of d_{Hg} when $\tan \beta = 40$. We see that ϕ^* could be as large as about 100° , with the most important constraints being provided by the Thallium (Mercury) EDM in the cases where the $d_{Hg}^{I,III,IV}$ (d_{Hg}^{II}) are used, respectively.

In figure 7, we show the maximal values of d_{Ra} attainable in the 6D case after imposing the three EDM constraints. We see that small-phase approximations for $d_{Hg}^{I,II,III}$ break down for $\phi^* \sim 40^\circ$, as seen by comparing with the calculations along the two reference

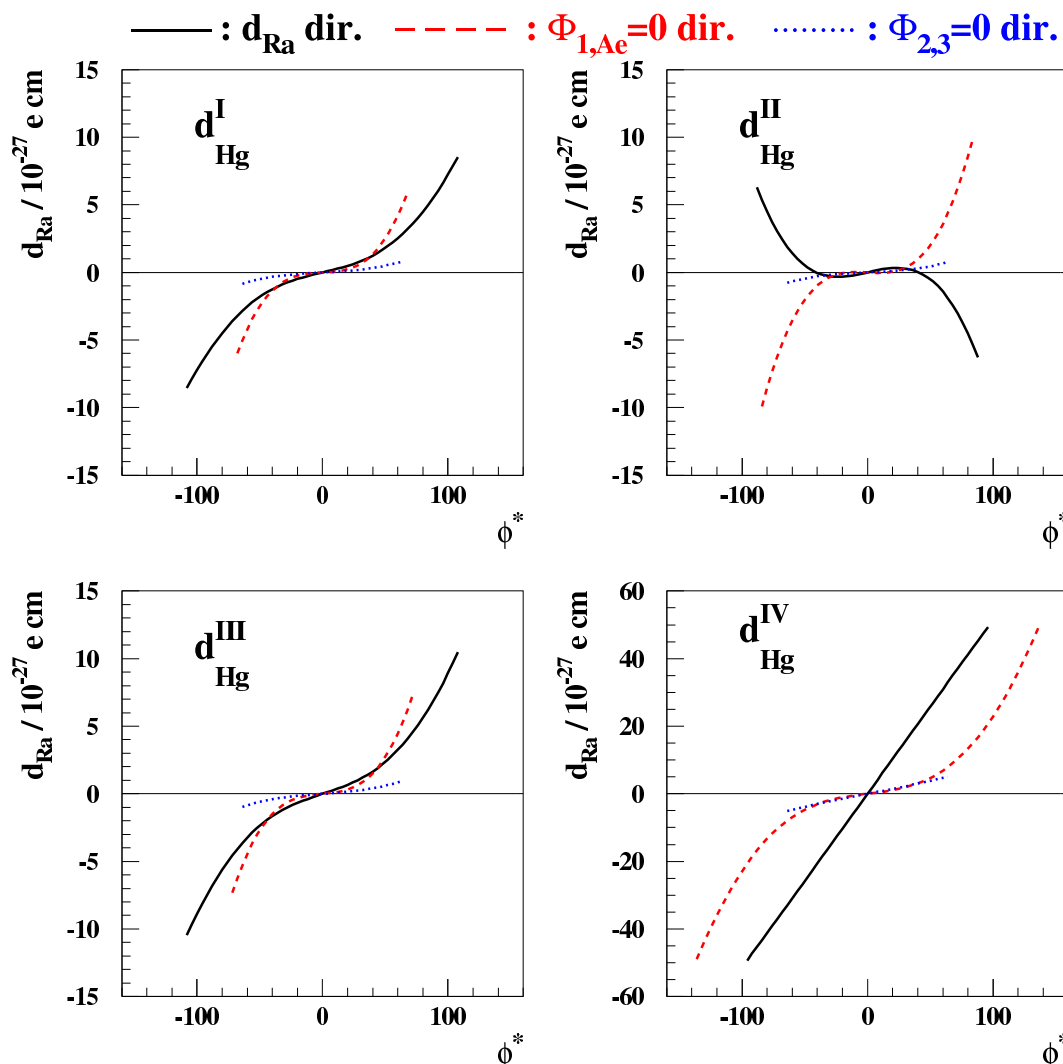


Figure 7. The EDM of ^{225}Ra in units of $10^{-27} e \text{ cm}$ for the scenario (4.1) with $\tan\beta = 40$ along the directions optimized for d_{Ra} (black solid) taking d_{Hg}^{I} (upper left), $d_{\text{Hg}}^{\text{II}}$ (upper right), $d_{\text{Hg}}^{\text{III}}$ (lower right), and $d_{\text{Hg}}^{\text{IV}}$ (lower left) for the Mercury EDM. We have imposed the EDM constraints $|d_{\text{Tl}}/d_{\text{Tl}}^{\text{EXP}}| \leq 1$, $|d_{\text{n}}/d_{\text{n}}^{\text{EXP}}| \leq 1$, and $|d_{\text{Hg}}/d_{\text{Hg}}^{\text{EXP}}| \leq 1$. For comparison, we also show d_{Ra} along two reference directions with the choices $\Phi_{1,Ae} = 0$ (red dashed) and $\Phi_{2,3} = 0$ (blue dotted).

directions, and that d_{Ra} can be as large as $\sim 6 \times 10^{-27} e \cdot \text{cm}$ or more. On the other hand, in the case of $d_{\text{Hg}}^{\text{IV}}$, d_{Ra} can be as large as $\sim 50 \times 10^{-27} e \cdot \text{cm}$.

Finally, in figure 8 we show the 6 CP-violating phases at the GUT scale (top and middle) and the 3 CP-violating phases of the third-generation A parameters at the SUSY scale (bottom). We observe that the CP phases of the gaugino mass parameters Φ_1 , Φ_2 and Φ_3 can only be as large as 2° , 0.4° and 0.4° , respectively, whereas Φ_{A_u} (Φ_{A_d}) at the GUT scale can be as large as $\sim 100^\circ$ (60°), as previously seen in figure 4. These CP-violating phases are suppressed at the SUSY scale by RG running from the GUT scale [3], but sizeable non-trivial CP-violating phases are still allowed at the SUSY scale: $|\Phi_{A_t} - 180^\circ| \sim 2^\circ$, $|\Phi_{A_b} - 180^\circ| \sim 4^\circ$, and $|\Phi_{A_\tau} - 180^\circ| \sim 30^\circ$.

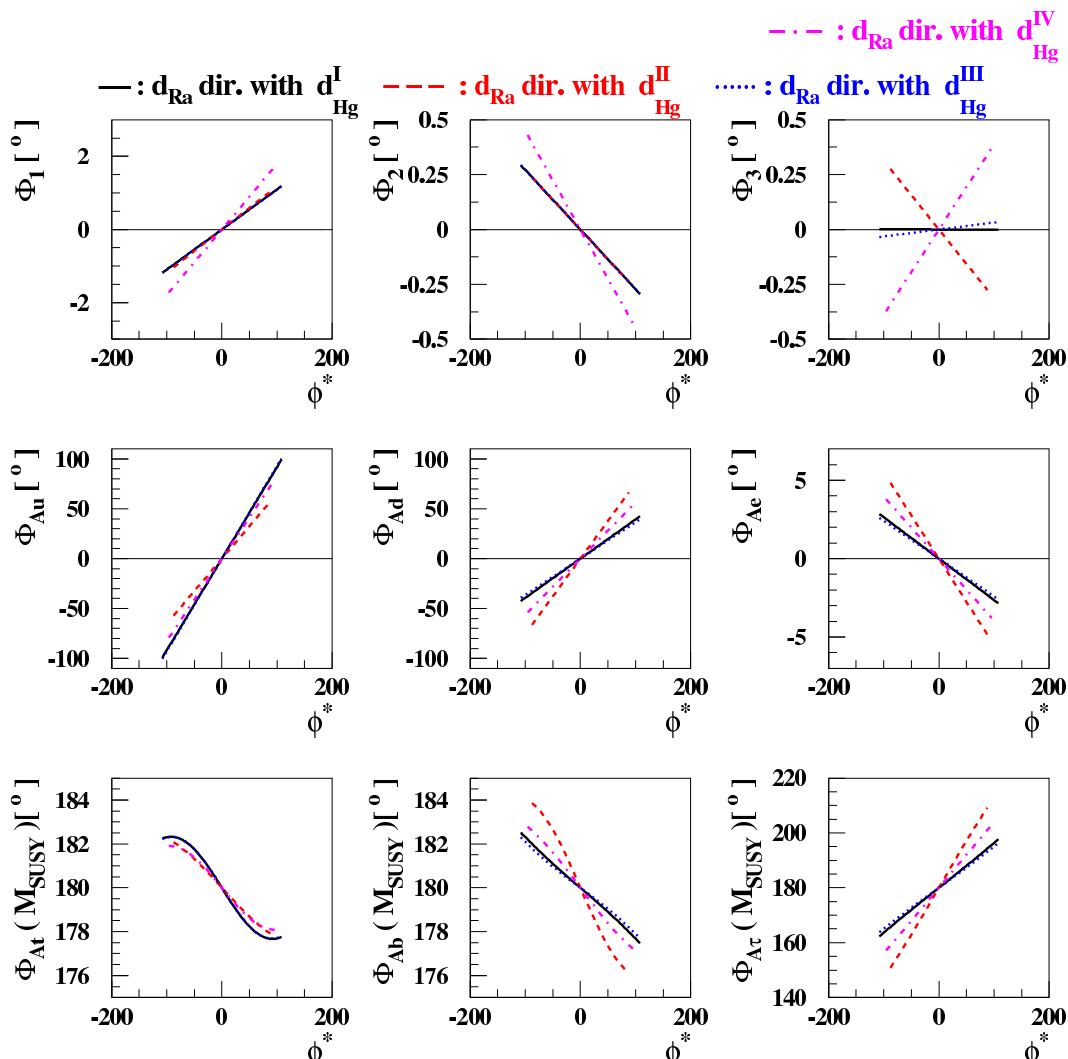


Figure 8. The 6 CP-violating phases at the GUT scale (upper and middle) and the 3 CP-violating phases of the third-generation A parameters at the SUSY scale (lower) along the directions optimized for d_{Ra} , as obtained using d_{Hg}^{I} (solid black), $d_{\text{Hg}}^{\text{II}}$ (red dashed), $d_{\text{Hg}}^{\text{III}}$ (blue dotted), and $d_{\text{Hg}}^{\text{IV}}$ (magenta dash-dotted) for the Mercury EDM. We have imposed the EDM constraints $|d_{\text{Tl}}/d_{\text{Tl}}^{\text{EXP}}| \leq 1$, $|d_{\text{n}}/d_{\text{n}}^{\text{EXP}}| \leq 1$, and $|d_{\text{Hg}}/d_{\text{Hg}}^{\text{EXP}}| \leq 1$. The scenario (4.1) is assumed, with the choice $\tan\beta = 40$.

4.2 The 7D case of non-zero θ_{QCD}

Hitherto, we have implicitly assumed that the CP-violating QCD θ -term:

$$\mathcal{L} = \frac{\alpha_s}{8\pi} \bar{\theta} G_{\mu\nu}^a \tilde{G}^{\mu\nu,a} \tag{4.8}$$

is negligible, where $\tilde{G}^{\mu\nu,a} \equiv \epsilon^{\mu\nu\rho\sigma} G_{\rho\sigma}^a / 2$ and the parameter $\bar{\theta}$ is given by the sum of the QCD θ_{QCD} and the strong chiral phase for the quark mass matrix:

$$\bar{\theta} = \theta_{\text{QCD}} + \text{Arg Det } M_q. \tag{4.9}$$

In the weak basis where $\text{Arg Det } M_q = 0$, we have $\bar{\theta} = \theta_{\text{QCD}}$.

The dimension-four operator (4.8) would in general contribute to the neutron, Mercury and Radium EDMs, e.g., through the CP-odd pion-nucleon-nucleon interactions (2.2). Explicitly, for the neutron EDM, we use the estimate [12, 18]

$$d_n(\bar{\theta}) \simeq 2.5 \times 10^{-16} \bar{\theta} e \cdot \text{cm}. \quad (4.10)$$

For the $\bar{\theta}$ -induced Mercury EDM, we neglect the contribution from the $\bar{g}_{\pi NN}^{(0)}$ coupling and use

$$d_{\text{Hg}}^{I,II,III,IV}(\bar{\theta}) \simeq (C_{\text{Hg}}^{I,II,III,IV} \times 10^{-3} \text{ GeV}^{-1}) e \bar{g}_{\pi NN}^{(1)}(\bar{\theta}), \quad (4.11)$$

where $C_{\text{Hg}}^I = 1.8$, $C_{\text{Hg}}^{II} = 1.0$, $C_{\text{Hg}}^{III} = 1.4$ and $C_{\text{Hg}}^{IV} = 9.5 \times 10^{-2}$, with [12, 45]

$$\bar{g}_{\pi NN}^{(1)}(\bar{\theta}) \simeq 1.1 \times 10^{-3} \bar{\theta}. \quad (4.12)$$

Finally, for the Radium EDM, we use

$$d_{\text{Ra}}(\bar{\theta}) \simeq (3.5 \times 10^{-1} \text{ GeV}^{-1}) e \bar{g}_{\pi NN}^{(1)}(\bar{\theta}). \quad (4.13)$$

Henceforward, we normalize $\bar{\theta}$ in units of 10^{-10} :

$$\hat{\theta} \equiv \bar{\theta} \times 10^{10}. \quad (4.14)$$

With this normalization, when $\hat{\theta} = 1$, we have $d_n(\bar{\theta}) = 2.5 \times 10^{-26} e \cdot \text{cm}$, which is very close to the current experimental bound $d_n^{\text{EXP}} = 3 \times 10^{-26} e \cdot \text{cm}$, and we find $d_{\text{Ra}} \simeq 0.8 \times 10^{-27} e \cdot \text{cm}$.

We have analyzed the possible maximal values of d_{Ra} in this 7D case including $\hat{\theta}$ following a procedure similar to that we used in the 6D case. Looking again at figure 1, we note the horizontal magenta lines representing the 7th components of the vectors representing the present EDM constraints on d_{Tl} (upper left), d_n (upper right), and d_{Hg} (lower left), and of the vector representing the EDM of ^{225}Ra (lower right). We observe that the 7th component is missing in the d_{Tl} case, because this observable has no contribution from the $\hat{\theta}$ term in our approach. On the other hand, the $\hat{\theta}$ component is close to unity for d_n (cf. the discussion at the end of the previous paragraph), $\sim 10^{-1}$ for d_{Hg}^I , and somewhat less than unity for d_{Ra} . This implies that a measurement of d_{Ra} at the level of $10^{-27} e \text{ cm}$ would already be a competitive measurement of $\hat{\theta}$, even in the absence of the other MCPMFV phases.

As seen in figure 2, we have made similar analyses using the $d_{\text{Hg}}^{II,III,IV}$ calculations, finding similar $\hat{\theta}$ components in the first two cases, but a value about an order of magnitude smaller in the d_{Hg}^{IV} case. We have also analyzed (not shown) the cosines of the angles between the observable vector $\mathbf{O}^{d_{\text{Ra}}}$ and the EDM-constraint vectors $\mathbf{E}^{d_{\text{Tl}}}$, \mathbf{E}^{d_n} and $\mathbf{E}^{d_{\text{Hg}}^{I,II,III,IV}}$ in the 7D model, as functions of $\tan \beta$. We find results that are very similar to the 6D case shown in figure 3, the most significant difference being quite small and limited to $\tan \beta < 10$ in the d_n case.

We turn now to figure 9, which is the analogue of figure 4, but including the QCD θ term (4.8). We see significant differences at intermediate and large $\tan \beta$, where we

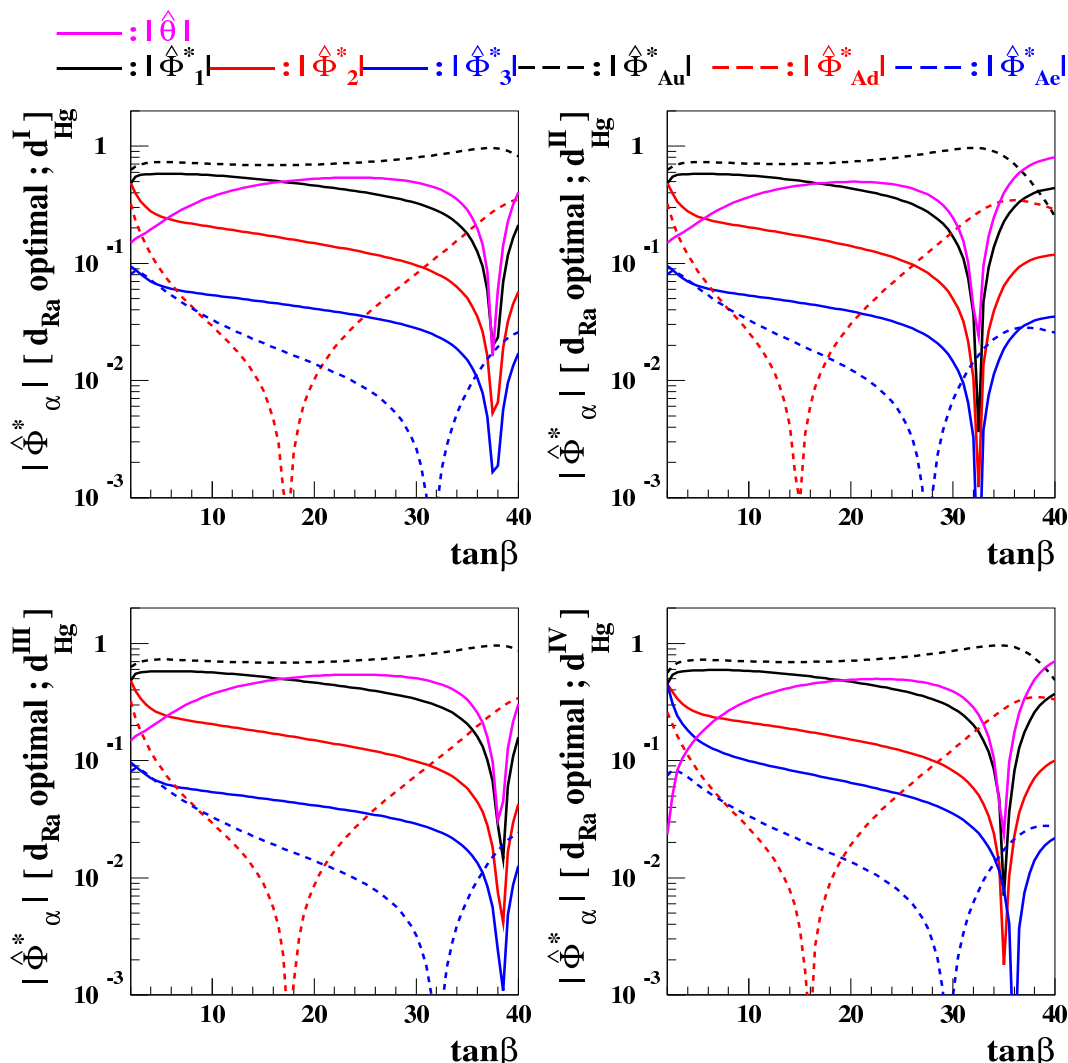


Figure 9. As in figure 4, but including the QCD θ term (4.8). The 7th components of the normalized 7D optimal vectors are denoted by the solid magenta lines.

note that the optimal vectors in all four models for d_{Hg} exhibit not only relatively large $\hat{\theta}$ components, but also larger components for the gaugino mass phases than in the 6D case. We have also analyzed (not shown) the products $\hat{\Phi}^* \cdot \mathbf{O}$ for the optimal d_{Ra} direction in the 7D space, as obtained using the calculations $d_{\text{Hg}}^{\text{I, II, III, IV}}$ for the Mercury EDM, finding results that are generally very similar to those shown in figure 5 with a little rise around $\tan \beta = 40$.

Figure 10 displays the values of the Thallium, neutron and Mercury EDMs along the directions optimized for d_{Ra} , analogously to figure 6 but this time in 7D. As before, the different panels display results obtained using d_{Hg}^{I} (upper left), $d_{\text{Hg}}^{\text{II}}$ (upper right), $d_{\text{Hg}}^{\text{III}}$ (lower right), and $d_{\text{Hg}}^{\text{IV}}$ (lower left) for the Mercury EDM, and the ratios $d_{\text{Tl}}/d_{\text{Tl}}^{\text{EXP}}$, $d_{\text{n}}/d_{\text{n}}^{\text{EXP}}$, and $d_{\text{Hg}}/d_{\text{Hg}}^{\text{EXP}}$ are denoted by the black solid, red dashed, and blue dotted lines, respectively. Also as before, the scenario (4.1) is assumed, with the choice $\tan \beta = 40$. Comparing with figure 6, we see that the magnitudes of the Thallium and Mercury EDMs increase quickly as ϕ^* deviates from 0 and it could be as large as only about 50° .

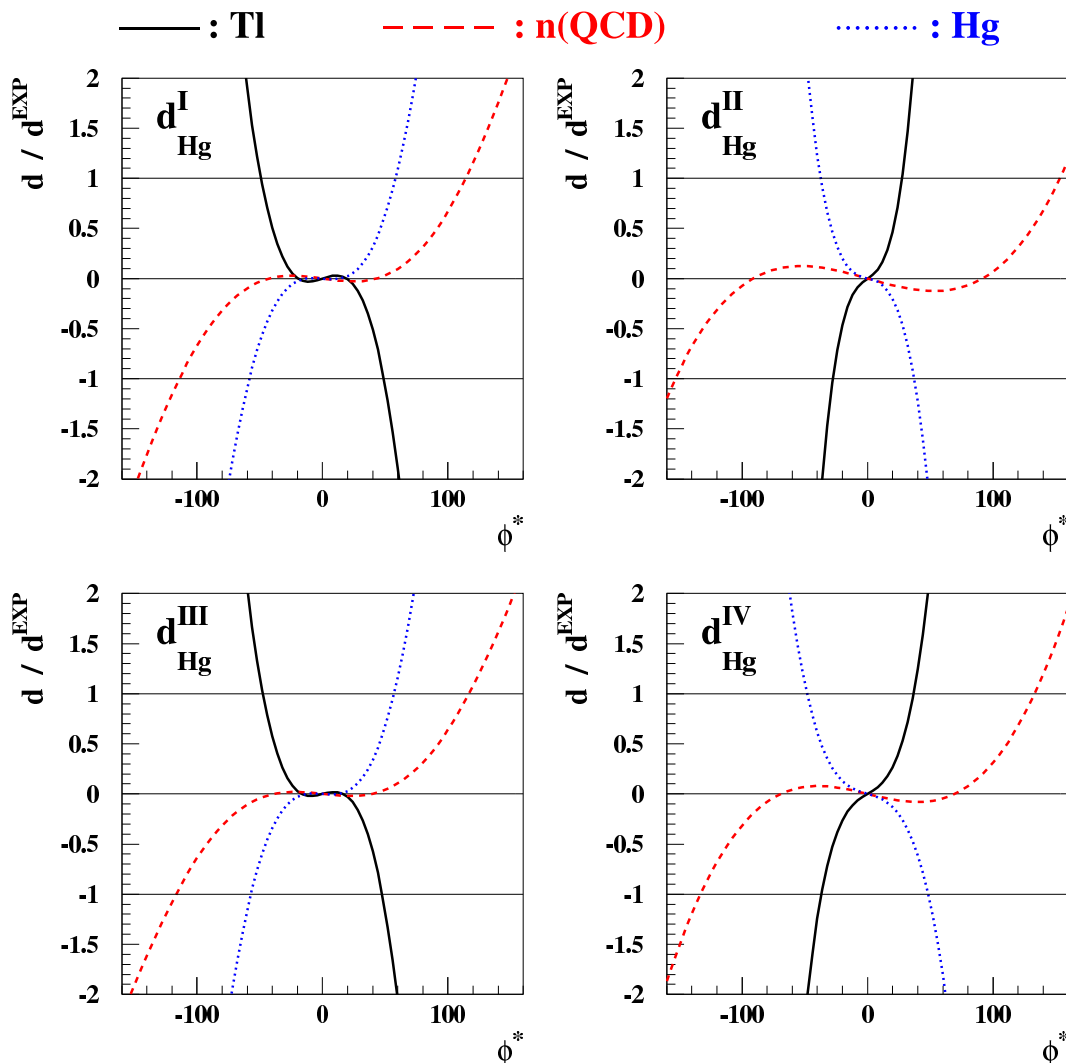


Figure 10. As in figure 6, but including the QCD θ term (4.8).

Turning to the resulting 7D estimates of d_{Ra} shown in figure 11, we see that they have somewhat smaller magnitudes than the 6D estimates shown in figure 7 in the linear approximation. However, larger values of the CP-violating gaugino phases are allowed in the 7D case than in the 6D case, as seen by comparing figures 8 and 12. Specifically, we observe that Φ_1 , Φ_2 and Φ_3 could be as large as $\sim 15^\circ$, $\sim 4^\circ$ and $\sim 1^\circ$, respectively. We also see in figure 13 that sizeable $\bar{\theta}$ could be much larger than the upper limit of $\sim 10^{-10}$ usually quoted, with values as large as $\bar{\theta} \sim 2.5 \times 10^{-9}$ becoming possible in the presence of non-zero MCPMFV phases.

5 Conclusions

In this paper we have extended our previous analyses of the MCPMFV model, with its 6 CP-violating phases to determine (in the linear approximation) the largest value of d_{Ra} that is allowed by the present constraints on the neutron, Thallium and Mercury EDMs, using

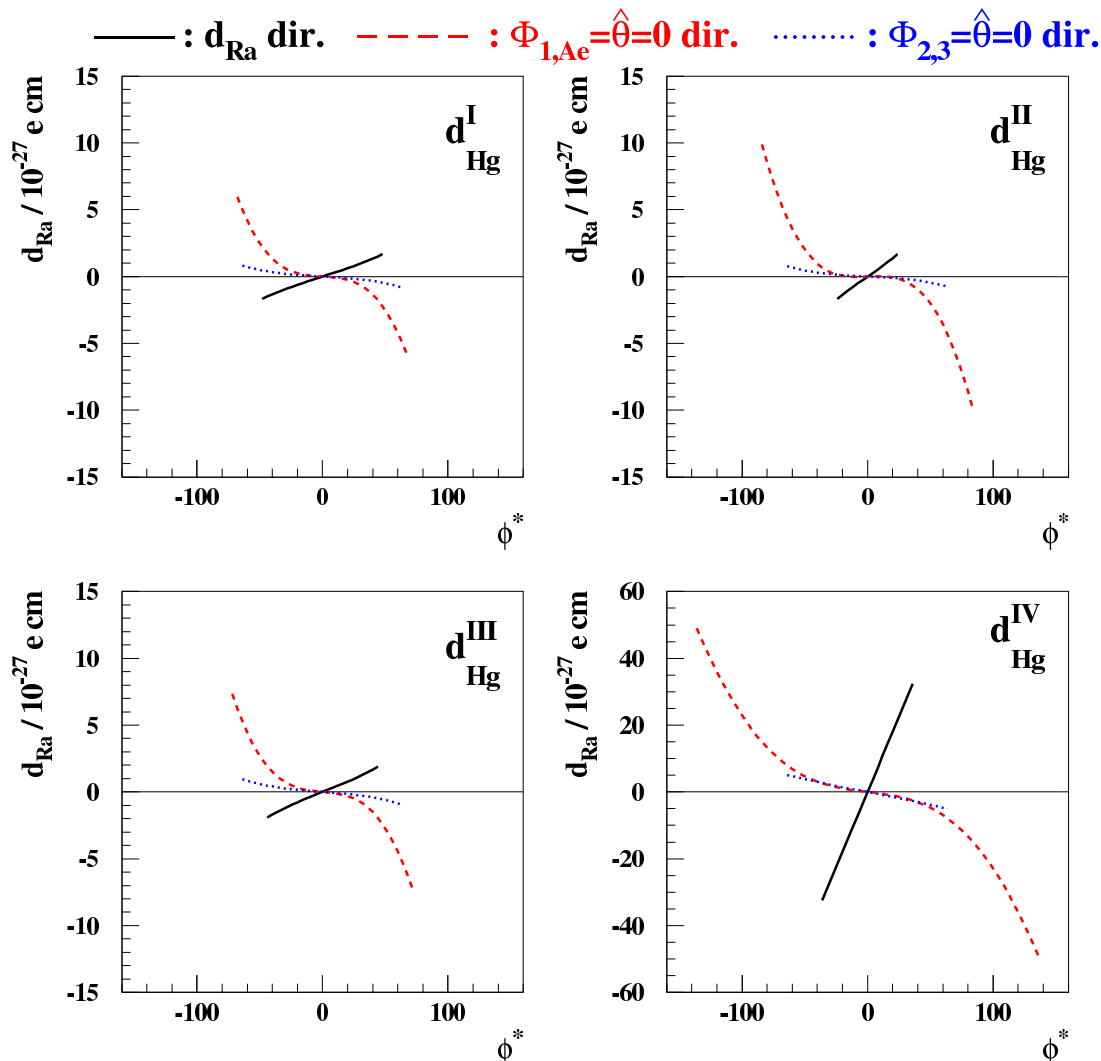


Figure 11. As in figure 7, but including the QCD θ term (4.8).

the differential-geometric approach developed in [12]. Numerically, we obtain rather similar results whether we include the CP-violating QCD vacuum phase $\bar{\theta}$ in the analysis, or not.

The results are much more sensitive to the theoretical treatment of the Mercury EDM constraint, and we compare the results obtained with four different calculations of d_{Hg} . Three of them yield quite similar results for d_{Ra} , but one calculation indicates a smaller dependence of d_{Hg} on the CP-violating phases of the MCPMFV model, and hence allows larger numerical values of these phase and, in general, larger values of d_{Ra} become possible. The maximal values we find for d_{Ra} using three of the d_{Hg} calculations are typically $\sim 6 \times 10^{-27} e \cdot \text{cm}$ or more, whereas the fourth calculation allows $d_{\text{Ra}} \sim 50 \times 10^{-27} e \cdot \text{cm}$.

For comparison, we recall that there is a proposal to measure d_{Ra} with an sensitivity approaching $\sim 10^{-27} e \cdot \text{cm}$ in one day of data-taking. This experiment would clearly have interesting potential to probe regions of the MCPMFV parameter space that have not been explored by previous EDM experiments.

This potential surely extends to many other models with several sources of CP violation, which could also be analyzed using the differential-geometric approach [12] exploited

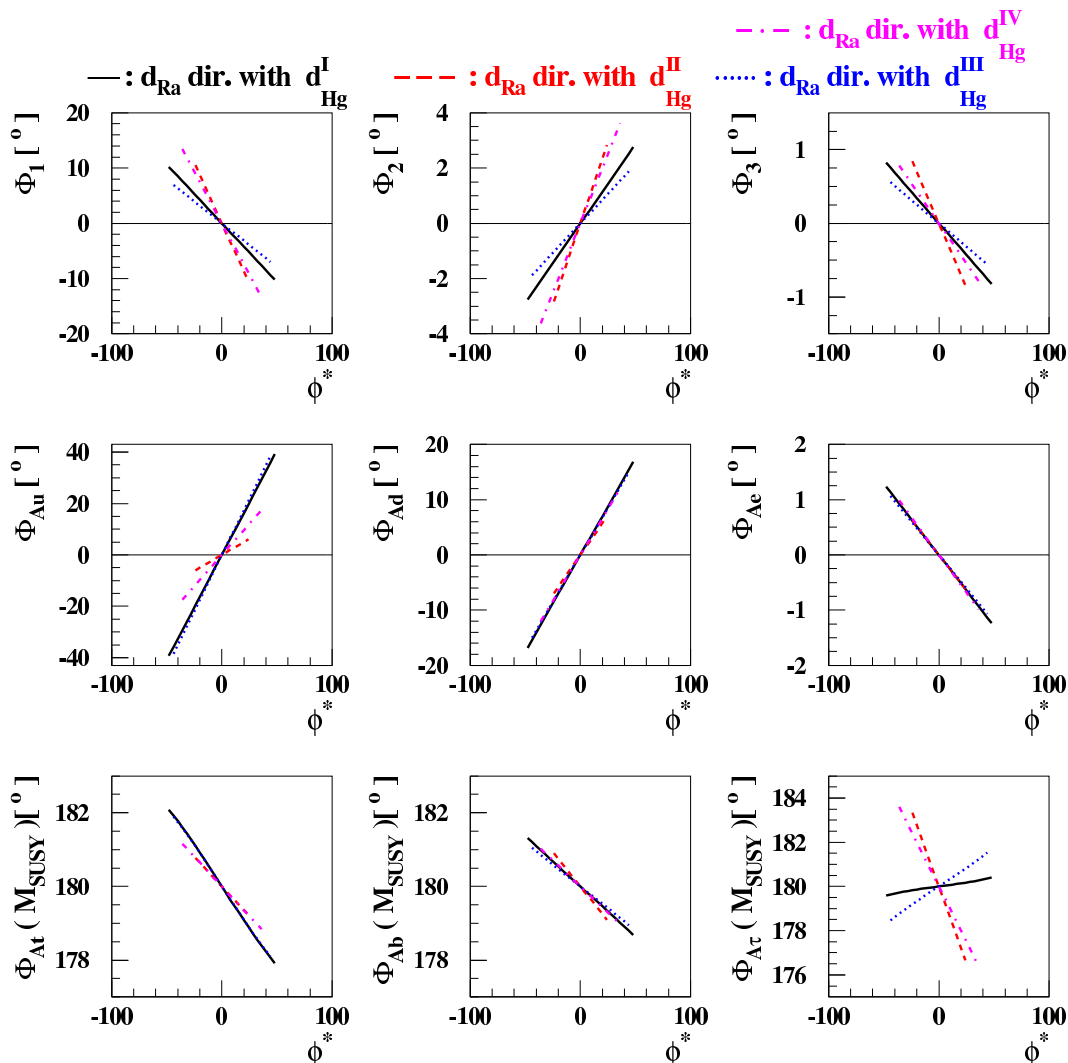


Figure 12. As in figure 8, but including the QCD θ term (4.8).

here. As long as a limited number n of EDMs have been bounded (or measured) by experiments, any model with $N > n$ CP-violating parameters will be underconstrained, and (partial) cancellations [10, 11] are possible that would allow large values for other CP-violating observables. The MCPMFV model is one such example, in which $N = 6$ (or 7 if $\bar{\theta}$ is included) and $n = 3$, so far. As we have shown in this paper using this differential-geometric approach, constraining or measuring d_{Ra} at the level of $\sim 10^{-27} e \cdot \text{cm}$ or better would be a valuable addition to the existing arsenal of experimental probes of CP violation.

However, it would still not complete the set of constraints needed for a model with $N > 4$, such as the MCPMFV. For this reason, other measurements, e.g., of the CP-violating asymmetry in $b \rightarrow s\gamma$ decay. We have discussed elsewhere the maximal value that this observable might take in the MCPMFV, and it would be an interesting and complementary measurement to that of d_{Ra} .

We conclude by drawing reader's attention to potential caveats in the physics of CP violation. The baryon asymmetry of the Universe is one of the strongest pieces of evidence

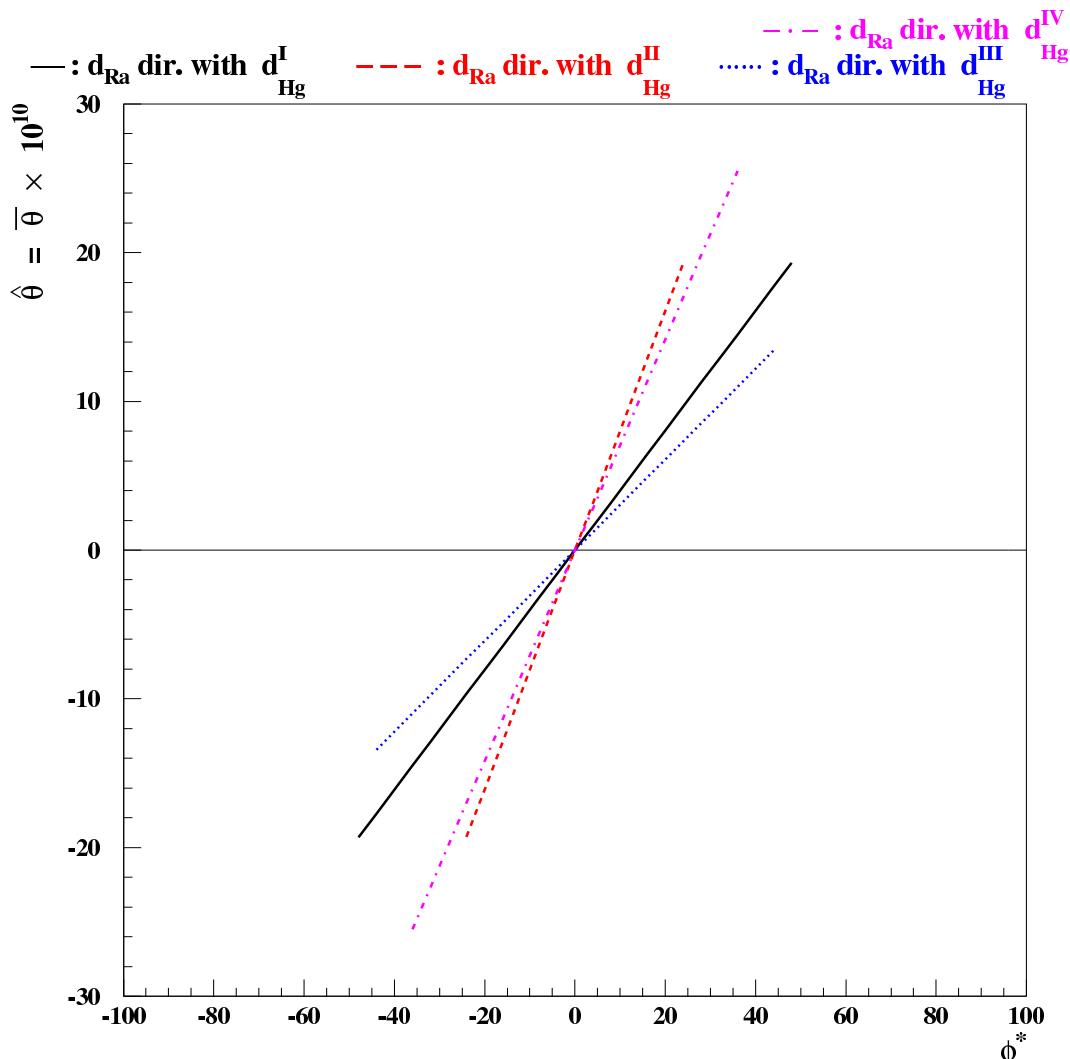


Figure 13. The values of $\hat{\theta} \equiv \bar{\theta} \times 10^{10}$ along the directions that maximizer d_{Ra} in the linear approximation, as found by using the calculations d_{Hg}^I (solid black), d_{Hg}^{II} (red dashed), d_{Hg}^{III} (blue dotted), and d_{Hg}^{IV} (magenta dash-dotted) for the Mercury EDM. We have imposed the EDM constraints $|d_{Tl}/d_{Tl}^{EXP}| \leq 1$, $|d_n/d_n^{EXP}| \leq 1$, and $|d_{Hg}/d_{Hg}^{EXP}| \leq 1$. The scenario (4.1) is assumed, fixing $\tan\beta = 40$.

for physics beyond the Standard Model, as it cannot be generated successfully within the standard Kobayashi-Maskawa model of CP violation. There must be new sources of CP violation beyond the Kobayashi-Maskawa phase, and it behooves experiments to chase down all those within reach. Some CP-violating phases may manifest themselves at the TeV scale and be accessible to contemporary collider experiments, e.g. at the LHC. However, baryogenesis could equally well be achieved via CP-violating phases appearing at higher energy scales, and EDMs have the potential to probe beyond the TeV scale, in particular because the Standard Model Kobayashi-Maskawa predictions for EDMs are quite small. As our analysis exemplifies, new EDM observables probe complementary region of parameter space including the strong CP phase θ_{QCD} . Therefore, constraining or even measuring d_{Ra} is an interesting experimental objective.

Acknowledgments

We thank Jon Engel, Maxim Pospelov and Adam Ritz for useful comments and suggestions.

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