JOINT INSTITUTE FOR NUCLEAR RESEARCH, DUBNA

Report P9 - 4728



# ON COHERENT ENERGY LOSSES OF A RELATIVISTIC BUNCH IN LINEAR ACCELERATOR SYSTEMS

A.B. Kuznetsov and S.B. Rubin

Dubna, 1969

Translated by B. Schnizer
(Original: Russian)

(CERN Trans. 70-11)

Geneva
November 1970

Abstract - The effect of the coherent radiation on the acceleration process of highly charged bunches is considered. The energy dependence of the energy losses of a relativistic charge passing various periodic structures is analysed.

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At present the question of energy losses of highly charged relativistic bunches passing through the various structures presents some interest. The investigation of the dependence of energy losses on the relativistic factor $\gamma = (1 - v_0^2/c^2)^{-\frac{1}{2}}$ while the bunch passes through a linear accelerator having in general periodic structure, appears particularly important.

Already a long time ago[1] it has been explained that in such cases the energy losses have a resonant character. But up to now even in the most recent works one has not tried to explain with sufficient completeness the asymptotic $\gamma$-dependence of the losses[2-5]. The only exception, as far as we know, is the case of the excitation of a "comb" of half planes for which an exact solution has been obtained by Bolotovskii and Voskresenskii and for which in certain approximations a dependence of the losses on $1/\gamma$ has been found. It is not clear whether this law is universal.

The difficulty in solving problems of this kind rests in the necessity to account for all frequencies for which radiation is excited. It has been possible to find an approach for the three periodic systems considered below (one among them is the "comb" of Bolotovskii and Voskresenskii which is given for comparison), offering the possibility to estimate the asymptotic behaviour of the losses for large $\gamma$.

The structure of the systems is presented in Fig. 1. Cases of point charges (cases I,II) and of a rod of charge (case III) are treated. The velocity of these sources is $v_0$ = const. The functional relation between the values of the current density in the m-th (m arbitrary) and the zero-th element of the structure follows from the assumptions (on the symmetry) of the problems as:

$$j_z^m\left(\xi + mD, t + \frac{Dm}{v_0}\right) = j_z^0(\xi, t), \quad 0 \le \xi \le d \qquad \text{in case I}$$

$$j_r^m(r, t + \frac{Dm}{v_0}) = j_r^0(r, t); \quad j_y^m(y, t + \frac{Dm}{v_0}) = j_y^0(y, t) \quad \text{in cases II, III}$$

The Fourier-components of the Hertz vectors, created by these currents of the secondary fields, may be presented in the form:

$$\Pi^I_{z\omega} = -\frac{\pi a e^{-i\omega t}}{D\omega} \int_0^d j^0_{z\omega}(\xi)d\xi \sum_{s=-\infty}^{+\infty} e^{i(z-\xi)w_s} \begin{cases} J_0(\chi_s r)H_0^{(1)}(\chi_s a), \; r < a; \\ J_0(\chi_s a)H_0^{(1)}(\chi_s r), \; r > a; \end{cases} \tag{1.1}$$

$$\Pi^{II}_{r\omega} = -\frac{\pi e^{-i\omega t}}{D\omega} \int_a^\infty j^0_{r\omega}(r')r'dr' \sum_{s=-\infty}^{+\infty} e^{izw_s} \begin{cases} J_1(\chi_s r)H_1^{(1)}(\chi_s r'), \; r<r'; \\ J_1(\chi_s r')H_1^{(1)}(\chi_s r), \; r>r'. \end{cases} \tag{1.2}$$

$$\Pi^{III}_{y\omega} = -\frac{e^{-i\omega t}}{D\omega} \int_0^\infty j^0_{y\omega}(y')dy' \sum_{s=-\infty}^{+\infty} e^{izw_s} \frac{e^{i\chi_s|y-y'|}}{\chi_s} \tag{1.3}$$

where

$$w_s = \frac{\omega}{v_0} - \frac{2\pi s}{D}, \quad \chi_s = \sqrt{k^2 - w_s^2}, \quad k = |\frac{\omega}{c}|.$$

The work done in the period on the source by the $\omega$-component of the secondary field may be found from the formula:

$$W_\omega = -q \int_0^{D/v_0} \mathrm{Re}\{E_{z\omega}|_{z=v_0 t}\} v_0 dt. \tag{2}$$

In the evaluation of the corresponding expression it turns out that all terms with $s \neq 0$, i.e. all waves with phase velocities different from $v_0$, become zero and that only the surface wave propagation with the phase velocity of the source gives the final result. The expressions for $W_\omega$ will assume for the three cases the form:

$$W^{I}_{\omega} = -\frac{2qa}{c}\,\frac{k}{\gamma^2\beta^2}\,K_0\left(\frac{ka}{\gamma\beta}\right)\mathrm{Jm}\left\{\int_0^d j^0_{z\omega}(\zeta)\,e^{-i\zeta\frac{\omega}{v_0}}\,d\zeta\right\}, \tag{3.1}$$

$$W^{II}_{\omega} = \frac{2qk}{c\gamma\beta^2}\,\mathrm{Re}\left\{\int_a^\infty j^0_{r\omega}(r)\,K_1\left(\frac{kr}{\gamma\beta}\right)r\,dr\right\}, \tag{3.2}$$

$$W^{III}_{\omega} = \frac{\kappa}{c\beta}\,\mathrm{Re}\left\{\int_0^\infty j^0_{y\omega}(y)\,e^{-\frac{k}{\gamma\beta}(y+a)}\,dy\right\}. \tag{3.3}$$

Formulae (3) are fundamental. Estimates for $W_\omega$ will be obtained from the corresponding estimates for the current amplitudes. The latter may be found, as usual, from integral equations obtained from the boundary conditions on the screens. For brevity, problem II is presented below, for the others only the results are quoted.

The sum in Eq. (1.2) at z = 0 may be represented by an integral with contour $\mathcal{L}$, shown in Fig. 2 *).

$$M = \sum_{s=-\infty}^{+\infty}\begin{cases} J_1(\chi_s r)\,H^{(1)}_1(\chi_s r') \\ J_1(\chi_s r')\,H^{(1)}_1(\chi_s r)\end{cases} = \frac{D}{4\pi i}\int_{\mathcal{L}}\begin{cases} J_1\left(r\sqrt{k^2-\left(\frac{\omega}{v_0}-\nu\right)^2}\right)H^{(1)}_1\left(r'\sqrt{k^2-\left(\frac{\omega}{v_0}-\nu\right)^2}\right)\mathrm{ctg}\left(\frac{\nu D}{2}\right)d\nu, \\ J_1\left(r'\sqrt{k^2-\left(\frac{\omega}{v_0}-\nu\right)^2}\right)H^{(1)}_1\left(r\sqrt{k^2-\left(\frac{\omega}{v_0}-\nu\right)^2}\right).\end{cases} \tag{4}$$

Deforming the contour $\mathcal{L}$ into the contour $(\mathcal{L}^{I} + \mathcal{L}^{II})$ (assuming for the root the positive sign on the right of the cuts) and further putting $\nu = k(1 + \beta + i\beta v)/\beta$ on $\mathcal{L}^{I}$ and $\nu = k(1 - \beta - i\beta v)/\beta$ on $\mathcal{L}^{II}$, we get:

$$M = \frac{-kD}{2\pi}\int_0^\infty J_1\left(kr\sqrt{v^2-2iv}\right)J_1\left(kr'\sqrt{v^2-2iv}\right)\left[\mathrm{ctg}\frac{kD}{2\beta}(1+\beta+i\beta v)+\mathrm{ctg}\frac{kD}{2\beta}(1-\beta-i\beta v)\right]dv. \tag{5}$$

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*) As usual, we assume that k has a small imaginary part.

It is easy to see that the functions under the integral in (5) tend exponentially to zero for $v \to \infty$. After inserting (5) into (1.2) the result is further transformed by partial integration with respect to $r'$ taking into account the boundary condition for the current:

$$j^{0}_{r\omega}(r)\Big|_{r=a} = 0. \tag{6}$$

The integral equation for $j^{0}_{r\omega}$ is written down as:

$$-E^{0}_{r\omega} = \frac{k}{2c}\int_{a}^{\infty}\frac{d(r'j^{0}_{\omega})}{dr'}\,dr'\cdot\int_{0}^{\infty}G(v)\,\frac{1-v^{2}+2iv}{\sqrt{v^{2}-2iv}}\,J_{1}(kr\sqrt{v^{2}-2iv})J_{0}(kr'\sqrt{v^{2}-2iv})\,dv \tag{7}$$

where $E^{0}$ is the external field*), and where

$$G(v) = \operatorname{ctg}\frac{kD}{2\beta}(1+\beta+iv\beta) + \operatorname{ctg}\frac{kD}{2\beta}(1-\beta-i\beta v).$$

It is convenient to differentiate (7) again with respect to r and to substitute: $v = \gamma^{2}x^{2}/\sqrt{1+\gamma^{2}x^{2}}$. In this form we apply to the double integral the method of stationary phase[6] taking into account that the domain where $r \approx r'$ gives the main contribution. (One may check this using the asymptotic expansions for the Bessel functions and taking into account the presence of the large parameter $\gamma$.) Restricting ourselves to the main terms we obtain as a result:

$$-\frac{dE^{0}_{r\omega}}{dr} = \frac{2(1+i)}{kcD}\,\frac{\gamma^{2}}{r}\cdot\frac{d}{dr}(r\,j^{0}_{r\omega}). \tag{8}$$

From this, together with (6) we find the estimate for the current. Finally, after inserting this into (3.2) and integrating with respect to frequency, the estimate for the total energy lost by the charge in one period of the

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*) The Fourier components into which the Hertz vector of the external field is decomposed are introduced in Fig. 1. ($\kappa$ charge per unit length of the wire.)

structure is obtained:

$$W^{II} = \frac{5}{24\pi} \frac{q^2 D}{a^2} . \tag{9}$$

An analogous procedure performed for the "comb" gives:

$$W^{III} = \frac{\kappa^2 D}{16 a} . \tag{10}$$

Finally, for case I we give only a crude upper estimate which may be obtained with the help of (3.1) in the following way: Let us assume that $j^0_{z\omega}$ is simply proportional to the amplitude of the "incoming" field $E^0_{z\omega}$ which would enter into the corresponding integral equation. By this we take into account the effect of the periodicity only partially, inasmuch as formula (3.1) is exact and has been derived with due consideration of the periodicity of the structure, while in the expression for $j^0_z$ we neglect the screening of the charge by the neighbouring elements, i.e. to a certain extent we take the most unfavourable case. As a result, the following expression for the total loss is obtained:

$$W^{I} \approx A \frac{q^2 d}{a^2 \gamma}, \tag{11}$$

where A is a numerical coefficient which remains unknown in the given approximation.

As a conclusion, we may note that the obtained estimates show that in the high-relativistic limit the losses tend at least to a constant limit with indefinite increase of $\gamma$. The deviation of the result for the system of half planes from that by Bolotoskii and Voskresenskii who state the formula (Ref. 3):

$$W = \frac{2\kappa^2 D}{a} \frac{\beta}{\gamma} \tag{12}$$

for the loss, may be explained by the fact that in the asymptotic formula (12) only the frequency domain ka $\geq \gamma$ has been taken into account, while in formula (10) ka $\gtrsim$ 1. It is difficult to compare the numerical results given in Ref. 3 calculated from the exact formula, with (10), since corresponding points on the calculated curve lie too near to zero.

Finally, we give the formula for the losses in case II, when the system is excited by an infinitely thin charged ring of radius b [with (a - b)/b << 1]:

$$W^{II} \approx 10^{-2} \frac{q^2 \Pi}{a(a-b)} . \tag{13}$$

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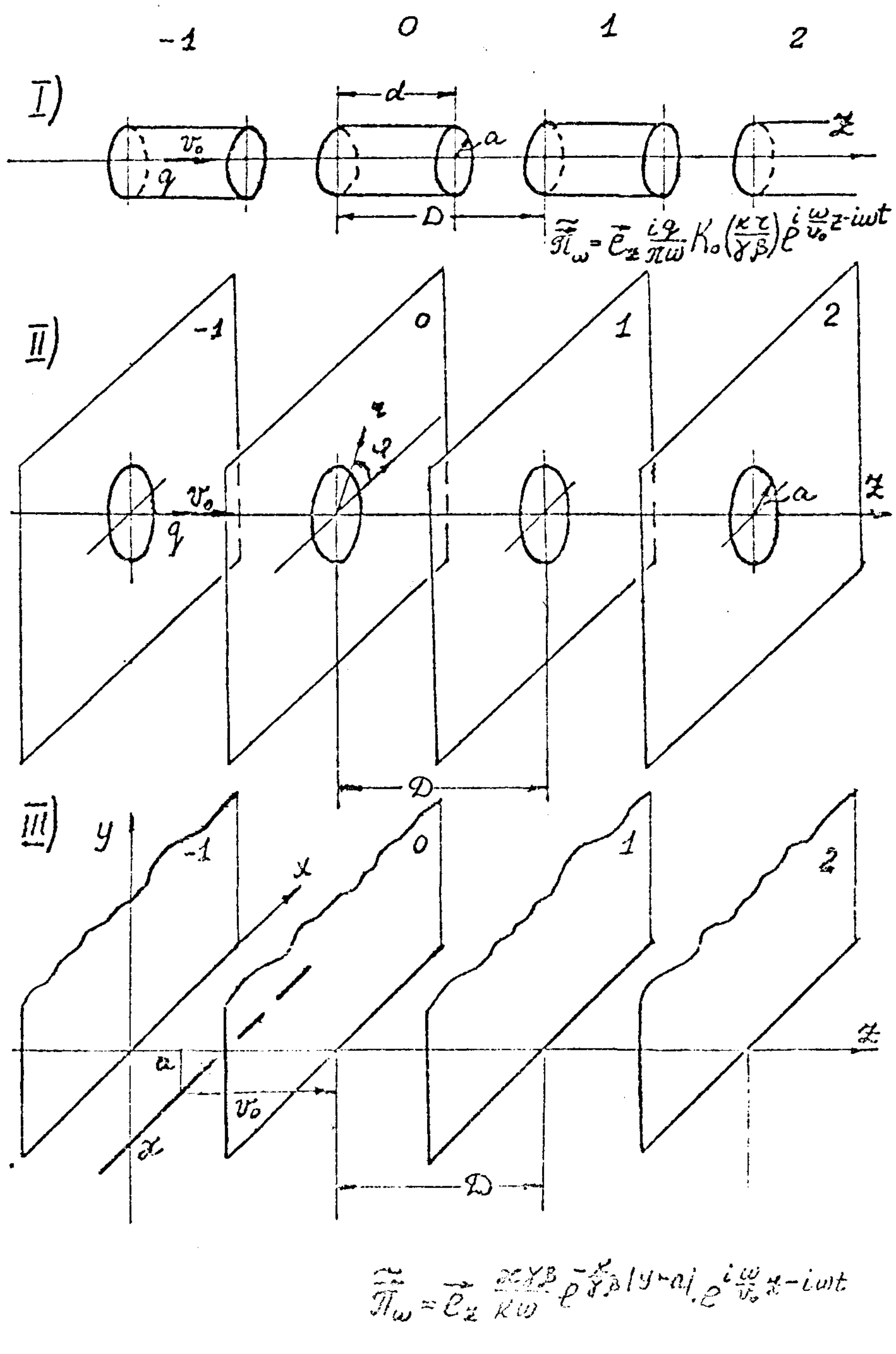
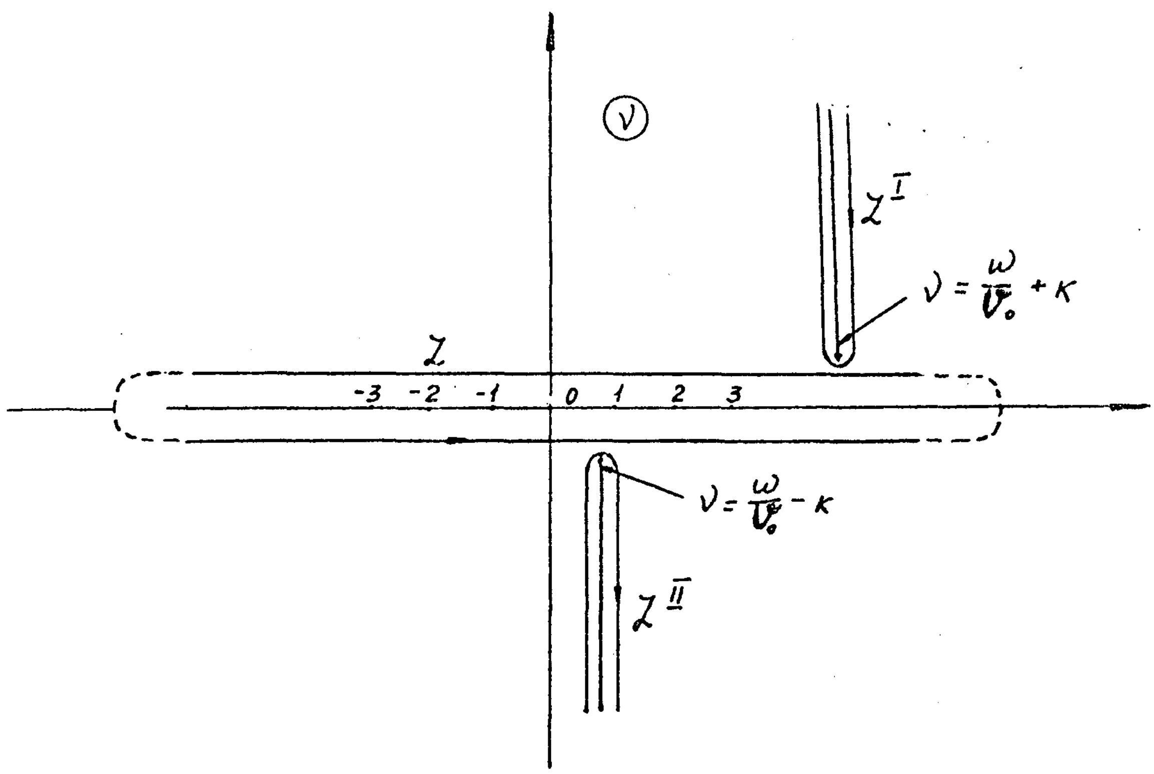

-1
0
1
2
I)
d
$v_0$
q
a
z
D
$\tilde{\vec{\Pi}}_\omega = \vec{e}_z \frac{iq}{\pi\omega} K_0\left(\frac{\kappa r}{\gamma\beta}\right) e^{i\frac{\omega}{v_0}z - i\omega t}$
II)
r
φ
a
z
D
III)
y
x
z
a
$v_0$
D

Fig. 1



ν
$\mathcal{L}^{I}$
$\nu = \frac{\omega}{v_0} + k$
$\mathcal{L}$
-3 -2 -1 0 1 2 3
$\nu = \frac{\omega}{v_0} - k$
$\mathcal{L}^{II}$


Fig. 2