Nucleon statistics in holographic QCD: Aharonov-Bohm effect in a matrix model

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We show that the Aharonov-Bohm effect in the nuclear matrix model [K. Hashimoto, N. Iizuka, and P. Yi, J. High Energy Phys. 10 (2010), 3.] derives the statistical nature of nucleons in holographic QCD. For N_c = odd (even), the nucleon is shown to be a fermion (boson).

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The statistics of baryons depends on the number of colors in QCD; in particular for large N_c QCD, as the baryons are bound states of N_c quarks, they are fermions for odd N_c, while bosons for even N_c. The purpose of this paper is to show this statistics in holographic QCD, by using the nuclear matrix model [1].

The nuclear matrix model [1] derived in holographic QCD offers a simple effective description of multibaryon systems, where we can compute baryon spectra, short-distance two-body nuclear forces, and even three-body nuclear forces [2]. This model is a $U(k)$ matrix model which describes generic k-body interaction of nucleons. The rank of gauge group $U(k)$ is not at all related to the number of colors N_c, but just the number of nucleons k. The action is quite simple,

$$S=\frac{\lambda N_c M_{\rm KK}}{54\pi}\int dt\,{\rm tr}_k\Big[(D_0X^M)^2-\frac{2}{3}M_{\rm KK}^2(X^4)^2 + D_0\bar{w}_i^{\dot\alpha}D_0w_{\dot\alpha i}-\frac{1}{6}M_{\rm KK}^2\bar{w}_i^{\dot\alpha}w_{\dot\alpha i}+\frac{3^6\pi^2}{4\lambda^2M_{\rm KK}^4}(\vec{D})^2 + \vec{D}\cdot\vec{\tau}^{\dot\alpha}{}_{\dot\beta}\bar{X}^{\dot\beta\alpha}X_{\alpha\dot\alpha}+\vec{D}\cdot\vec{\tau}^{\dot\alpha}{}_{\dot\beta}\bar{w}_i^{\dot\beta}w_{\dot\alpha i}\Big]+N_c\int dt\,{\rm tr}_kA_0.$$

The field content is summarized in Table I. The dynamical fields are only X^M and $w_{\dot\alpha i}$, while A_0 and D_s are auxiliary fields. In writing these fields, the indices for the gauge group $U(k)$ are implicit. The symmetry of this matrix quantum mechanics is $U(k)_{\rm local}\times SU(N_f)\times SO(3)$, where the last factor $SO(3)$ is the spatial rotation, which, together with a holographic dimension, forms a broken $SO(4)\simeq SU(2)\times SU(2)$ shown in the table. The breaking is due to the mass terms for X^4 and $w_{\dot\alpha i}$. In the action, the trace is over these $U(k)$ indices, and the definition of the covariant derivatives is $D_0X^M\equiv\partial_0X^M-i[A_0,X^M]$, $D_0w\equiv\partial_0w-iwA_0$, and $D_0\bar{w}\equiv\partial_0\bar{w}+iA_0\bar{w}$. The spinor indices of X are defined as $X_{\alpha\dot\alpha}\equiv X^M(\sigma_M)_{\alpha\dot\alpha}$ and $\bar{X}^{\dot\alpha\alpha}\equiv X^M(\bar\sigma_M)^{\dot\alpha\alpha}$ where $\sigma_M=(i\vec\tau,1)$ and $\bar\sigma_M=(-i\vec\tau,1)$, with Pauli matrices τ. The model has a unique scale $M_{\rm KK}$, and $\lambda=N_cg_{\rm QCD}^2$ is the 'tHooft coupling constant of QCD, with the number of colors N_c. For the derivation of this matrix model via holography, and also for the computation of the baryon spectrum ($k=1$), two-body nuclear forces ($k=2$), and three-body forces ($k=3$) at short distances, see [1,2].

Let us turn to our question. Since this nuclear matrix model has only bosonic variables, it is natural to ask how the fermionic nature of baryons comes out from the matrix model. In chiral soliton models, this question was answered from the properties of Wess-Zumino term [3].

To identify the statistics (fermionic/bosonic) of nucleons in the nuclear matrix model, we consider a 2π rotation in the target space of the matrix model. The target space index is carried by X^M and $w_{\dot\alpha i}$. The effect on X^M is trivial, since X decouples from the system in the matrix model for a single baryon ($k=1$) once the ADHM (Atiyah-Drinfeld-Hitchin-Manin) constraint is solved. However, since we have a nontrivial gauge field A_0, there is a nontrivial effect on the $w_{\dot\alpha i}$ sector. In fact, this gauge field A_0 turns out to be responsible for the statistics of the baryons, as we will see.

A long time ago, it was shown by Witten [3] that the Wess-Zumino term in a pion effective Lagrangian is essential for showing the nucleon statistics, in the picture of solitonic nucleon of the system. Now, in holographic QCD, this Wess-Zumino term is known to be from the 4-form Ramond-Ramond flux in the gravity background in the D4–D8 model of holographic QCD. In the nuclear matrix model [1], the Ramond-Ramond flux generates a Chern-Simons term in 1 dimension, which is just a term consisting of a single gauge field A_0. The $w_{\dot\alpha i}$ field is charged under the gauge symmetry, so it is natural to expect that the gauge dynamics in this 1 dimension with the Chern-Simons term gives the nucleon statistics.

In the nuclear matrix model, the terms including the fundamental field $w_{\dot\alpha i}$, except for the ADHM potential terms and the mass term, are

TABLE I. Fields in the nuclear matrix model.

field	index	$U(k)$	$SU(N_f)$	$SU(2)\times SU(2)$
$X^M(t)$	$M=1,2,3,4$	adj.	**1**	**(2, 2)**
$w_{\dot\alpha i}(t)$	$\dot\alpha=1,2;\ i=1,\ldots,N_f$	**k**	$\mathbf{N_f}$	**(1, 2)**
$A_0(t)$		adj.	**1**	**(1, 1)**
$D_s(t)$	$s=1,2,3$	adj.	**1**	**(1, 3)**

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$$S = \frac{\lambda N_c M_{\rm KK}}{54\pi} \int dt D_0 \bar{w}_i^{\dot{\alpha}} D_0 w_{\dot{\alpha}i} + N_c \int dt A_0. \quad (1)$$

Note that A_0 is a gauge field for $U(k)$ gauge symmetry of the matrix model, so, for $k = 1$ (single baryon), A_0 does not carry any non-Abelian index.

Let us make a spatial rotation, for example, along the x^3 axis, by an angle 2π. We look at how a wave function of a baryon transforms under this rotation, and if it acquires a phase $n\pi$ with an odd integer n, i.e. it changes a sign, then the state is determined to be a fermion.

Since $w_{\dot{\alpha}i}$ carries a spinor index of the target space, it is obvious that the spatial rotation acts for the case of the rotation around the x^3 axis with an angle θ, as

$$w_{\dot{\alpha}i} \rightarrow U_{\dot{\alpha}}^{\dot{\beta}} w_{\dot{\beta}i}, \qquad U = \exp\left[i\frac{\theta}{2}\tau^3\right]. \quad (2)$$

Here τ^3 is the third component of Pauli matrices. Our spatial rotation by 2π means that the angle θ moves in the period $0 \leq \theta \leq 2\pi$.

As shown in [1], the vacuum of the matrix model for $k = 1$ is quite simple,

$$w = \begin{pmatrix} \rho_0 & 0 \\ 0 & \rho_0 \end{pmatrix}. \quad (3)$$

After minimizing the Hamiltonian, we obtain a certain nonzero value for this ρ_0. So the spatial rotation (2) corresponds to a certain path in the target space of $w_{\dot{\alpha}i}$. In the following, we would like to compute an Aharonov-Bohm phase with this path. For that, it is inconvenient that two nonzero entries in (3) move simultaneously. So, we combine a gauge transformation $\exp[-i\theta/2]$ together with the spatial rotation (2), so that we find a path

$$w_{\dot{\alpha}i} \rightarrow U_{\dot{\alpha}}^{\dot{\beta}} w_{\dot{\beta}i}, \qquad U = \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\theta} \end{pmatrix}. \quad (4)$$

With this, we find that only the lower-right corner of $w_{\dot{\alpha}i}$ in (3) rotates. Indeed, the same change of the parameterization of the path was used in [3] for the soliton in the pion effective field theory.

We are interested in a phase change of a baryon wave function. The argument of the wave function is the moduli of this matrix model, and it is a part of $w_{\dot{\alpha}i}$ configuration space. If we think of the path of $w_{\dot{\alpha}i}$ defined by (4), then the phase of the wave function of our concern is in fact an Aharonov-Bohm (AB) phase, for the path (4), as if we regard the lower-right entry of the matrix field $w_{\dot{\alpha}i}$ as a position of a charged particle.

Let us write down the Lagrangian for this charged particle, to compute the AB phase. Writing the lower-right component of $w_{\dot{\alpha}i}$ as $w_{\dot{\alpha}=2,i=2} = u + iv$ where u and v are real, then the relevant part of the matrix model is

$$S = \frac{\lambda N_c M_{\rm KK}}{54\pi} \int dt |\partial_t(u + iv) - iA_0(u + iv)|^2. \quad (5)$$

It was shown in [1] that solving the equation of motion for A_0 gives $A_0 = -27\pi/2\lambda M_{\rm KK}\rho_0^2$, which is a real constant. Then the action (5) can be rewritten with conjugate momenta in real coordinates as

$$S = \frac{1}{2M} \int dt [(P_u + A_0 vM)^2 + (P_v - A_0 uM)^2]. \quad (6)$$

Here we have defined the "mass" M of the hypothetical particle moving in the u-v space as $M = \lambda N_c M_{\rm KK}/27\pi$. The expression shows that the particle is in a minimally-coupled gauge potential in the u–v space, defined by

$$\tilde{A}_u \equiv -A_0 M v = \frac{N_c}{2\rho_0^2} v, \qquad \tilde{A}_v \equiv A_0 M u = -\frac{N_c}{2\rho_0^2} u. \quad (7)$$

The magnetic flux made by this gauge potential is constant.

The path of this hypothetical charged particle is given by (4), which is

$$u + iv = \rho_0 e^{-i\theta} \qquad (0 \leq \theta \leq 2\pi) \quad (8)$$

so the circle encloses the area $\pi\rho_0^2$, in a counterclockwise way. The AB phase Φ is given by an integration of the gauge potential (7) along this path,

$$\Phi = -\rho_0 \oint \tilde{A}_\theta d\theta = N_c \pi. \quad (9)$$

In the last equality, we have used (7) in a polar coordinate, $\tilde{A}_\theta = -N_c/2\rho_0$. The negative sign is from the orientation of the path.

This AB phase means that, when N_c is odd, the spatial rotation by the angle 2π results in a sign (-1) multiplied to the baryon wave function. Therefore, when N_c is odd (even), the baryon is a fermion (boson).

It is intriguing that a simple mechanism, the AB phase, is encoded in the nuclear matrix model naturally to ensure the baryon statistics in holographic QCD.

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