Relativistic Chiral Mean Field Model for Finite Nuclei

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Abstract
Pion is important in Nuclear Physics. Yukawa introduced pion as a mediator of nucleon-nucleon interaction in 1934 [1]. However, Nuclear Physics started by shell model with strong spin-orbit interaction in 1949 by Meyer and Jensen. The shell model is a phenomenological model and it has to be explained from more basic dynamics by explicitly using pion. The pion had not played the central role in nuclear physics until recent years. In this paper, we would like to discuss the recent development on the role of pion in nuclei.

1 Introduction

Ikeda proposed in 2000 that the pion should be playing the dominant role in nuclear physics. He then suggested the introduction of the pion explicitly in the study of nuclear structure, whose strength should be the strongest in ^{4}He the amount of which is indicated by the d-wave probability. This statement triggered me to think deeply and interpreted his statement as pion condensation in finite nuclei. This was difficult for me to accept at the first instance, since pion condensation was considered to happen only at high density as more than the twice of the saturation density in nuclear matter ($\rho_c \geq 2\rho_0$) [2]. However, a nucleus has a surface which may allow pion mean field to be developed at the nuclear surface in order to make the source term of the pion mean field have the spatial derivative on the spin-isospin density finite. This was the starting point of our study of nuclei with the pionic degree of freedom to be considered explicitly. Hence, we started to study a possibility of surface pion condensation in finite nuclei.

2 Surface pion condensation

We study the pionic effect in the relativistic mean field (RMF) model with the pion term, which interacts with the nucleon by the pseudovector coupling [3],

$$\mathcal{L} = \bar{\psi}[i\gamma^\mu\partial_\mu - M - g_\pi\gamma_5\gamma^\mu\tau^a\partial_\mu\pi^a - g_\sigma\sigma - g_\omega\gamma^\mu\omega_\mu - g_\rho\gamma_\mu\tau^a\rho^{a\mu}]\psi + \mathcal{L}_{meson} \,. \tag{1}$$

Here, we write other mesons as σ, ω and ρ mesons with proper couplings to the nucleon. The meson Lagrangian $\mathcal{L}_{meson}$ contains the meson masses and kinetic terms in the standard form [4]. The mean field approximation provides the following Dirac equation for the nucleon,

$$[i\gamma^\mu\partial_\mu - M - g_\sigma\sigma - g_\omega\omega\gamma^0 - g_\rho\rho\gamma^0\tau^0 - g_\pi\vec{\nabla}\pi\gamma_5\vec{\gamma}\tau_0]\psi = 0 \,. \tag{2}$$

The equation of motion for the pion mean field is written as

$$(\vec{\nabla}^2 - m_\pi^2)\pi = -g_\pi\vec{\nabla}\langle\bar{\psi}\gamma_5\vec{\gamma}\tau_0\psi\rangle \,. \tag{3}$$

Here, the bracket $\langle\cdots\rangle$ above denotes the ground state expectation value. The pion term in the Dirac equation has the negative parity because $\vec{\nabla}\cdot\gamma_5\vec{\gamma} \sim \vec{\nabla}\cdot\vec{\sigma}$. Hence, when the wave function for the nucleon has a definite parity, the pion term becomes zero. This is the reason that the pion does not enter in the RMF model. We have to mix the parity for the single particle wave function,

$$\psi_{jm} = \alpha_{jm}\psi_{jm}^+ + \beta_{jm}\psi_{jm}^- \,. \tag{4}$$

Here the superscript $\pm$ denotes the parity of single particle wave function with total angular momentum jm, which is conserved. Hence, we call this ansatz as spherical ansatz. We should further note that the derivative $\vec{\nabla}$ operates on the pion mean field. This fact indicates that the source term should vary spatially. As for the pion mean field, the spin-isospin density has to be non-zero. At the same time, in order for pion mean field to be finite, it is necessary to make the spatial derivative of the spin-isospin density to be finite. These facts make us try to calculate the surface pion condensation in finite nuclei.

We have calculated the nuclear ground states in a wide mass range using the spherical ansatz in the RMF model. The coupling constant of the pion with the nucleon g_π is taken from the Bonn potential [5]. Other parameters are taken from the TM1 parameter set [4]. We have found that the surface pion condensation takes place [3]. We show in Fig.1 the pionic energy per nucleon as functions of mass number. The pionic energy increases with mass as $A^{2/3}$. At the same time we have found that the amounts of the pion contribution are much larger for the jj-closed shell nuclei as compared with the LS-closed shell nuclei.

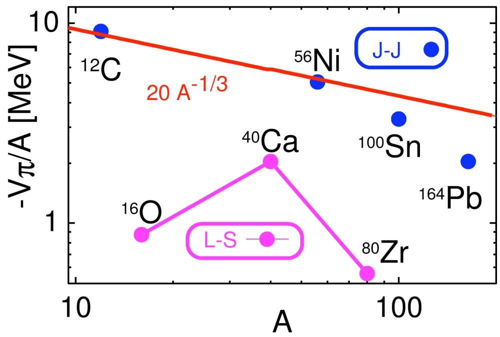

Fig. 1: The pionic energy per nucleon as functions of the mass number A. The pionic energy increases with mass as $A^{2/3}$ (surface property). This means that the pionic energy per nucleon decreases with $A^{-1/3}$. The pionic energy contributions for the jj-closed shell nuclei are much larger than those for the LS-closed shell nuclei.

Around the same time ($\sim$ 2000), there appeared several interesting findings. Pieper *et al.* [6] showed that their variational method can reproduce nuclear ground states with a few excited states up to the mass number $A \leq 10$ by using the bare nucleon-nucleon interaction together with a properly chosen three-body interaction. A surprising finding was that the matrix elements of the pion exchange interaction for those states were about 80% of the entire two body matrix element, $\langle V_\pi \rangle \sim 0.8 \langle V_{NN} \rangle$. A new high resolution experimental method was developed at RCNP with the use of (^{3}He, t) reactions. With this method of the order of $\sim$ 35 keV energy resolution, Fujita *et al.* [7] showed that the Gamow-Teller (GT) strengths were highly fragmented. Considering a simple operator structure $\vec{\sigma}\vec{\tau}$, these experimental data with high resolution indicate that the ground state configurations are much more complicated than the shell model predictions.

3 The pion and the chiral symmetry

The important fact about the pion is its relation with the chiral symmetry, which was recognized by Nambu in 1961 [8]. The chiral symmetry is the key symmetry to connect the hadronic world with the QCD physics. Particularly important is that the nucleon acquires mass dynamically due to the spontaneous breaking of chiral symmetry. The pion is the Nambu particle of the chiral symmetry breaking and therefore it oughts to play the most important role in hadron physics. The NJL model has a similarity to BCS theory, where the pair condensate $\langle \psi\psi \rangle$ is the pairing gap and the excitation mode is the plasmon. In the NJL model, the density condensate $\langle \bar{\psi}\psi \rangle$ is related with the quark (hadron) mass and the excitation mode is the pion.

It is therefore a good starting point to take a Lagrangian with the chiral symmetry for the construction of nuclear ground state. We take the Gell-Mann-Levy linear sigma model Lagrangian [9].

$$\begin{aligned}\mathcal{L}_{\sigma\omega} &= \bar{\psi}(i\gamma_\mu\partial^\mu - g_\sigma(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi}) - g_\omega\gamma_\mu\omega^\mu)\psi \\ &+\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi} - \frac{\mu^2}{2}(\sigma^2+\vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2+\vec{\pi}^2)^2 \\ &-\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}\widetilde{g_\omega}^2(\sigma^2+\vec{\pi}^2)\omega_\mu\omega^\mu + \varepsilon\sigma.\end{aligned} \tag{5}$$

The pion field appears symmetrically with the σ filed to make the Lagrangian of Gell-Mann and Levy chiral symmetric. In order to provide a small mass to the pion after the chiral symmetry breaking, there appears an explicit chiral symmetry breaking term $\varepsilon\sigma$. We introduce the ω meson terms to introduce a proper amount of repulsion, which probably reflects from the nucleon-nucleon hard core. The symmetry breaking of the chiral symmetry provides experimental masses of the nucleon and ω meson.

The pseudo-scalar pion-nucleon coupling in the linear σ model leads to unrealistically large attractive contribution through the strong coupling between positive and negative-energy states, and we have to treat seriously the effect of the negative energy states. We thus employ the non-linear realization of the chiral Lagrangian which is obtained by the Weinberg transformation of the linear σ model [10]. We take the lowest order term in the pion field and the Lagrangian density in the non-linear representation is written as [11, 12],

$$\mathcal{L} = \mathcal{L}_{\sigma,\omega} + \mathcal{L}_\pi, \tag{6}$$

where

$$\begin{aligned}\mathcal{L}_{\sigma,\omega} &= \bar{\psi}(i\gamma_\mu\partial^\mu - M - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu)\psi \\ &+ \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}{m_\sigma}^2\sigma^2 - \lambda f_\pi\sigma^3 - \frac{\lambda}{4}\sigma^4 \\ &- \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}{m_\omega}^2\omega_\mu\omega^\mu \\ &+ \widetilde{g_\omega}^2 f_\pi\sigma\omega_\mu\omega^\mu + \frac{1}{2}\widetilde{g_\omega}^2\sigma^2\omega_\mu\omega^\mu\end{aligned} \tag{7}$$

and

$$\mathcal{L}_\pi = -\frac{g_A}{2f_\pi}\bar{\psi}\gamma_5\gamma_\mu\partial^\mu\pi^a\tau^a\psi + \frac{1}{2}\partial_\mu\pi^a\partial^\mu\pi^a - \frac{1}{2}{m_\pi}^2\pi^{a2}. \tag{8}$$

We have worked out the above chiral sigma model Lagrangian in the RMF model using the spherical ansatz as performed for the ordinary Lagrangian [12]. We have obtained the similar conclusion as made with the ordinary Lagrangian. The surface pion condensation takes place and the pion contribution is much larger for the jj-closed shell nuclei than the LS-closed shell nuclei. We also point out that the spatial distributions of particle wave functions become very compact to include the high momentum components owing to the tensor character of pion exchange interaction [12, 13].

4 Relativistic chiral mean field model

We have noticed that the pion contribution does not increase with the mass number A but rather increases with the nuclear surface due to the surface condensation physics. On the other hand, the chiral expansion model of Kaiser *et al.* [14] demonstrated the dominant contribution of 2 particle-2 hole (2p-2h) excitation diagrams of the pion exchange interaction. Hence, we ought to improve the spherical ansatz approximation. Very important physics of the pion exchange between two nucleons is that the spins of the two nucleons have to flip and therefore the two nucleons cannot stay under the Fermi surface. It is therefore necessary to include explicitly 2p-2h configurations in the process of performing the energy variation of nuclear ground state. In the non-relativistic framework, we have developed the tensor optimized shell model (TOSM) to treat the strong tensor interaction coming from the pion exchange interaction [15]. We note here that the RMF model description of the pion needs the parity projection to the positive parity state and the projected wave function can be related with the 2p-2h states with the particle pairs to be in $J^\pi = 0^-$ [18].

We solve the equations of motion for the meson and nucleon fields by introducing the following wave function in the relativistic chiral mean field model [18],

$$\Psi = \Psi_N \Psi_M \ . \tag{9}$$

The meson wave function Ψ_M is assumed to be the coherent states for the sigma and omega mesons. On the other hand, the nuclear wave function is written as

$$\Psi_N = C_0|RMF\rangle + \sum_i C_i|2p-2h\rangle_i \ . \tag{10}$$

The RMF equation provides the basic wave functions for the σ and ω mesons and the nucleon. The 2p-2h wave functions provide large contributions to the ground state energy and therefore the self-consistent calculations of the wave functions are essential for the good ground state properties. For the calculation of pion matrix elements we use the formulation of particle-hole excitation used extensively for the discussion of the effect of pionic excitation in nuclei [2]. We have to consider in addition the short range correlation, for which we use the unitary correlation operator method (UCOM) developed by Feldmeier and his collaborators [17].

We have solved the RCMF model equations for ^{4}He [18], ^{12}C and ^{16}O [19]. We have found large energy contributions from the 2p-2h configurations as expected. We should note for this that we have to take into account 2p-2h configurations up to very high pionic spin as $J^\pi \sim q_\pi^u R$ with R being the nuclear radius and q_π^u the upper pionic momentum carried by single particle nucleon states. This pionic spin amounts to $J^\pi \sim 10$ for ^{12}C. We found that the contribution of the pion exchange interaction is on the order of 80% for ^{4}He, that agrees with the variational calculation [6].

The comparison of ^{12}C and ^{16}O is very interesting, since the former is a jj-closed shell nucleus and the latter is a LS-closed shell nucleus. We show one interesting result on the pion contribution to these two nuclei in Fig.2. The pionic energies are shown as functions of the pionic quantum number J^π for ^{12}C and ^{16}O. We see that the pion contributions saturate around $J^\pi \sim 10$. The pionic contribution per nucleon is larger for ^{12}C than ^{16}O as seen in the left hand figure. In the right hand side, we show each contribution of the pionic quantum number to the pionic energy. We observe an interesting feature where more energy gain is seen for smaller pionic quantum number $J^\pi \leq 3$ for ^{12}C than ^{16}O and a similar gain is obtained for larger pionic quantum numbers. This is very interesting, because the pion makes additional contribution to the jj-closed shell nuclei over the LS-closed shell nuclei. Hence, we should make a statement that the pionic contribution has a strong shell effect. We have calculated the change of the effective nucleon mass and also the change of the chiral condensate as a function of the radial coordinate [18, 19].

We can make a clear discussion on the recovery of the chiral condensate in nuclear matter with the RCMF model. We have applied the RCMF model to nuclear matter [20]. The calculated results are in general very close to those of Kaiser et al. [14]. We show in particular the results on the chiral condensate in nuclear matter in Fig.3. In the left hand side, the full calculated result on the chiral condensate is shown as a function of the nuclear density, which is compared with the model independent result. They are very similar. However, its breakdown is very interesting as shown in the right hand side. The σ mean field result takes care about a half of the chiral condensate and the pionic cloud contribution provides about another half at small densities. On the other hand, as the density increases, the 2p-2h contribution increases with the density. As the net result, the total quark condensate behaves similarly as the model independent value. We note that the σ mean field contribution provides the reduction of the hadron masses and it is important to measure the mass change of hadrons in nuclei.

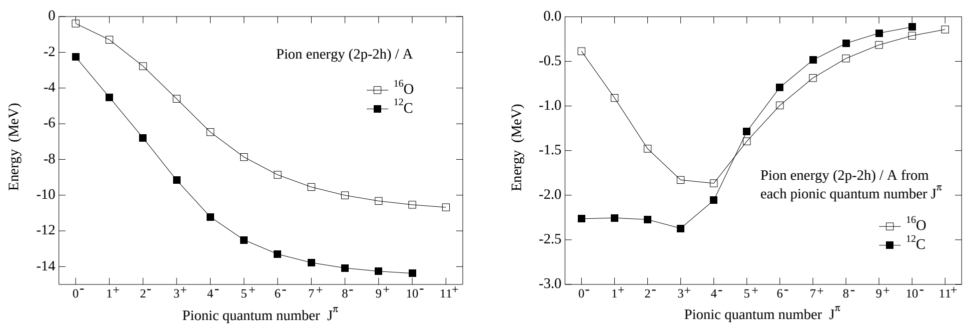

Fig. 2: The pionic energies per nucleon are shown as functions of the pionic quantum number J^π for ^{12}C and ^{16}O. The cut-off momentum Λ is taken as 1000 MeV. The other free parameters, σ meson mass and ω-nucleon coupling constant, are adjusted to reproduce the binding energy and r.m.s. matter radius.

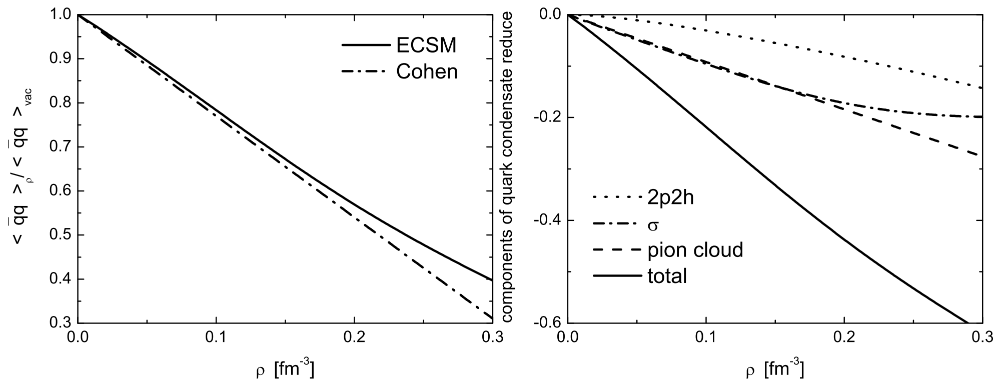

Fig. 3: In the left panel, we show the chiral condensate in nuclear matter calculated with the RCMF model including the pion cloud contribution $\Sigma_N^{(\pi)}$=20 MeV [21]. For comparison, the model independent result is shown by dot-dashed curve with $\Sigma_N = 50$ MeV. In the right panel, we show each component of the chiral condensate, where the full value is denoted by solid curve. The σ mean field contribution is shown by dash-dotted curve, the 2p-2h contribution by dotted curve. The pion cloud contribution is shown by dashed curve.

5 Deeply bound pionic atom

We have an interesting experimental data on the recovery of the chiral condensate in the systematic analysis of deeply bound pionic atoms. As the Nambu particle, the pion property is directly related with the value of the chiral condensate in nuclear matter. Toki and Yamazaki [22] proposed the existence of deeply bound pionic atoms in heavy nuclei in 1988. A systematic experimental study was performed to measure the deeply bound pionic atom states on the Pb and Sn isotopes [23].

The analysis of the experimental data on the deeply bound pionic atom states extracted the change of the isovector s-wave parameter as [23]

$$\frac{b_1}{b_1(\rho)} = 1 - 0.37\frac{\rho}{\rho_0} \; . \tag{11}$$

The Weinberg-Umezawa relation is written as $b_1 \sim 1/f_\pi^2$. On top of this, we have the GOR relation $f_\pi^2 m_\pi^2 = -2m_q\langle\bar{\psi}\psi\rangle$. The pion mass m_π is known to stay unchanged by the circumstance as the temperature and density owing to its Nambu particle character. Therefore the experimental finding states that the chiral condensate changes with the nuclear density as

$$\frac{\langle\bar{\psi}\psi\rangle_\rho}{\langle\bar{\psi}\psi\rangle} = 1 - 0.37\frac{\rho}{\rho_0} \; . \tag{12}$$

This relation is very close to the model independent relation of the chiral condensate.

$$\frac{\langle\bar{\psi}\psi\rangle_\rho}{\langle\bar{\psi}\psi\rangle} = 1 - \frac{\Sigma_N \rho_0}{f_\pi^2 m_\pi^2}\frac{\rho}{\rho_0} = 1 - 0.39\frac{\rho}{\rho_0}\ . \tag{13}$$

For getting the last number, we have used $\Sigma_N = 50MeV$. The extracted tendency of the experimental finding (12) on the recovery of the chiral condensate is quite close to the model independent relation. This may merely tell that there are non-interacting nucleons in the nucleus. However, we have to keep in mind that the experimental data is extremely precious. The nuclear matter effect on the chiral condensate has very deep physics as the pionic cloud effect, the change of the hadron masses and the 2p-2h pionic contributions on the nuclear ground state. It is now very important to extract informations on the change of the hadron masses and the 2p-2h contributions by performing further experiments.

6 Conclusion

We have studied the role of pion in nuclear structure. We have started from the relativistic mean field model with inclusion of the pion terms in the RMF Lagrangian. We have shown that the pion mean field becomes finite and the pionic effect behaves as nuclear surface. We have then introduced the chiral symmetry in the relativistic Lagrangian for nuclear many body system. To be explicit, we take the Gell-Mann-Levy linear sigma model Lagrangian and construct finite nuclei.

We have developed extended relativistic chiral mean field model, where the pion exchange interaction is treated in terms of the 2 particle-2 hole configurations in the basis of the RMF ground state. All the wave functions including those of the 2p-2h states are variationally determined by the condition of the energy minimization of the ground state. The energy gain due to the pion contribution is enormous, the amount of which agrees with the findings of the few-body calculations and chiral model calculations of nuclear matter.

We have calculated finite magic number nuclei as ^{4}He, ^{12}C and ^{16}O. The energy gain due to the pion is very large. It is about 80% for A=4 and decreases with the mass number. Very important to point out is that the energy gain due to the pion for ^{12}C is larger than that for ^{16}O. This is the Pauli blocking effect of the tensor excitations of nucleons. The extra energy gain is expected in other jj-closed shell nuclei, since the similar Pauli blocking effect happens in heavier nuclear systems. We should mention also that the particle wave functions become very compact in order to make the tensor contribution of the pion exchange interaction most effective. Hence, we may obtain the magic number effect in the whole periodic regions due to the pionic effect.

We have discussed also the change of the chiral condensate in nuclear matter as a function of the matter density. The chiral condensate decreases with the density due to the change of the hadron masses and the pionic cloud effect in addition to the 2p-2h contributions. The net result becomes similar to the model independent model value. This change of the chiral condensate seems to have been observed in the deeply bound pionic atom experiments.

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